MEscope Application Note 42

Multi-Input Multi-Output (MIMO) FRFs

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** option. Without this option, you can still carry out the steps in this App Note using the **AppNote42** project file. These steps might also require **a more recent release date** of MEscope.

APP NOTE 42 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote42.zip**

This Project contains **numbered Hotkeys & Scripts** for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click on a* Hotkey** to open its Script window

WHAT IS A MULTI-INPUT MULTI-OUTPUT (MIMO) MODEL?

Calculation of Transfer Functions, Outputs, and Inputs are all based upon use of a Multi-Input Multi-Output (**MIMO**) model of the dynamics of a structure.

A **MIMO** model is a frequency domain model where **Digital Fourier Transforms (DFTs)** of **multiple Inputs** are multiplied by elements of a **Transfer Function matrix** to yield **DFTs** of **multiple Outputs**.

The **MIMO** model is expressed with the equation:

$$\{X(f)\} = [H(f)]\{F(f)\}$$

{F(f)} ➔ **Input** DFTs (**m**-vector)

[H(f)] ➔ **Transfer Function** matrix (**n** by **m**)

{X(f)} ➔ **Output** DFTs (**n**-vector)

m ➔ number of **Inputs**

n ➔ number of **Outputs**

f ➔ frequency variable (radians per second)

In the Transfer Function matrix, Rows correspond to Outputs, and Columns correspond to Inputs.

Each Input & Output corresponds to a measurement at a **DOF** (point & direction).

Each Transfer Function is a Cross-channel function between two **DOFs**, an Input DOF and Output DOF.

TRANSFER FUNCTION

A **Transfer Function** is defined as the **DFT** of an **Output** divided by the **DFT** of an **Input**.

FREQUENCY RESPONSE FUNCTION (FRF)

An **FRF** is defined as the **DFT** of a **response Output** (displacement, velocity, acceleration) divided by the **DFT** of the **excitation force Input** that caused the response.

TRANSMISSIBILITY

A **Transmissibility** is defined as the **DFT** of an **Output** divided by the **DFT** of an **Input** with the ***same units***.

An **FRF** and **Transmissibility** are **special cases of a Transfer Function**.

MIMO CALCULATIONS

Any one of the components of the **MIMO** model, (**Inputs**, **Outputs**, **Transfer Functions**) **can be calculated from the other two components** using commands in the MEscope **Transform** menu.

A **MIMO** model is also used to calculate and display a **sinusoidal ODS** due to **multiple sinusoidal excitation forces**.

Inputs & **Outputs** can be represented by either **TWFs** or **DFTs**.

Transfer Functions can be either **experimentally derived** or **synthesized from modal parameters.**

FREQUENCY RESPONSE FUNCTION (FRF)

Each **FRF** defines the **dynamic properties** between an **excitation force Input DOF** and a **response Output DOF**.

FRF → DFT of a response / DFT of its excitation force

The excitation **force is typically measured with a load cell**. A **response is typically measured with an acceleration, velocity, or displacement sensor**.

EXPERIMENTAL MODAL ANALYSIS (EMA)

In an experimental modal analysis (**EMA**), **FRF** measurements **are usually made under controlled conditions** where the test article is artificially excited **using a broadband excitation signal**.

In an **impact** test, the impactor applies a **broadband** impulsive force to the test article to excite many modes simultaneously.

In a **shaker** test, one or more shakers are attached to the test article and driven with un-correlated **broadband** signals to excite many modes simultaneously.

MEASURING ROW & COLUMNS OF THE FRF MATRIX

In an **EMA**, **at least one row or column of the FRF matrix** must be calculated from the acquired data.

COLUMNS OF THE FRF MATRIX

If a **single excitation force** is applied at the **same fixed location** and responses are measured at multiple points & directions, this corresponds to measuring elements from a **single column of the FRF matrix.**

If **multiple excitation forces** are applied **at multiple fixed locations** and responses are measured at multiple points & directions, this corresponds to measuring elements from **multiple columns of the FRF matrix.**

ROWS OF THE FRF MATRIX

If a **single response sensor** is attached at **a single fixed location** and an excitation force is applied at multiple points & directions, this corresponds to measuring elements from **a single row of the FRF matrix**.

If **multiple response sensors** are attached at **a multiple fixed locations** and an excitation force is applied at multiple points & directions, this corresponds to measuring elements from **multiple rows of the FRF matrix**.

MULTI-REFERENCE MODAL TEST

A **multiple reference modal test** is done using either **multiple (fixed) exciters** with sensors to measure the forces, or **multiple (fixed) response sensors**.

MULTI-SHAKER MODAL TEST

In a multiple shaker modal test, **two or more (fixed) shakers** are used to **simultaneously excite** the structure.

Large structures with **non-linear dynamic behavior** are typically tested using multiple shakers, driven by **pure or burst random** or **burst chirp** (fast swept sine) excitation signals.

Multiple shakers must be driven with **uncorrelated broadband signals**.

The calculated **FRFs** are elements of **two or more columns of the FRF matrix** in a **MIMO** model of the structure

Multiple Coherences are calculated and indicate **how all shakers contribute to each response**

Partial Coherences are calculated and indicate **how each shaker contributes to each response**

Random excitation together with **spectrum averaging** is used to **"average out"** the **non-linear dynamic behavior** of the structure from Auto & Cross spectra and hence from the **FRFs.**

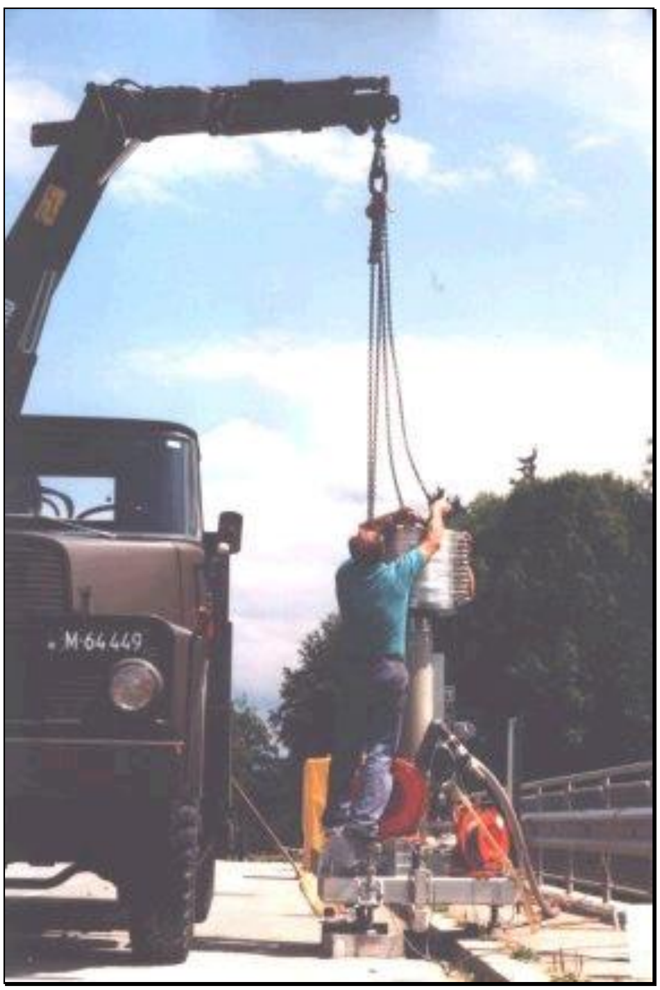
MULTIPLE MEASUREMENT SETS ACQUIRED FROM THE Z-24 BRIDGE

The data used in this App Note was acquired in **multiple Measurement Sets** from the bridge shown in the figure below. The bridge was excited using two hydraulic shakers with random forcing-functions applied to the shakers.

Because of acquisition hardware limitations, **nine Measurement Sets** of force & response data were acquired.

Each Measurement Set contains force & response data that was **simultaneously acquired** while the bridge was excited.

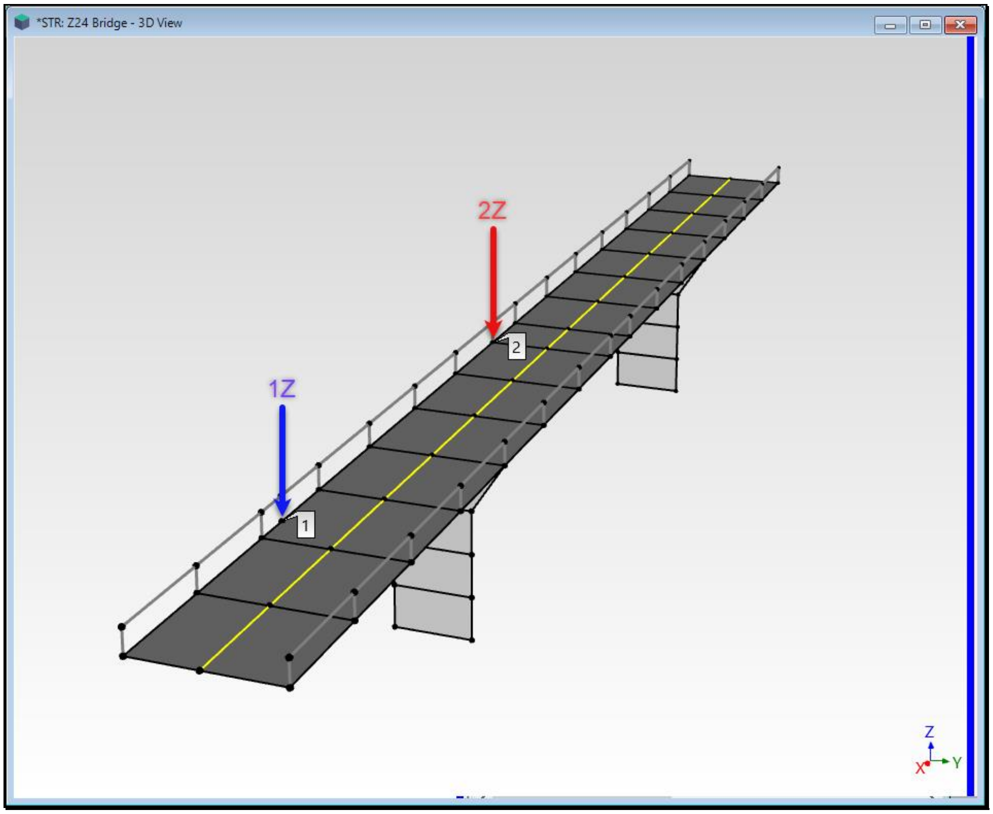
Z24 Bridge Viewed from the Bern-to-Zurich Highway A1



Installing one of the two shakers used to Excite the Bridge.

Installing the Accelerometers for one Measurement Set.

Close-up of a Seismic Accelerometer.



3D Model Showing Fixed Shaker Locations.

The two shakers had different force capacities. The **larger shaker** excited the bridge at **DOF 1Z** while the **smaller shaker** excited it at **DOF 2Z** as shown above. The shakers **operated simultaneously** and remained fixed throughout the test.

The data will show that the **larger shaker had a much stronger influence** on the response of the bridge than the smaller shaker.

NINE MEASUREMENT SETS

The data was collected in **9 Measurement Sets**. Bridge response motions were measured using seismic accelerometers at **75 different DOFs**. The accelerometers were moved to **different unique DOFs** before each Measurement Set of data was acquired.

Each Measurement Set contained the same acceleration responses at **DOFs (1Z, -2Y, 2Z)**.

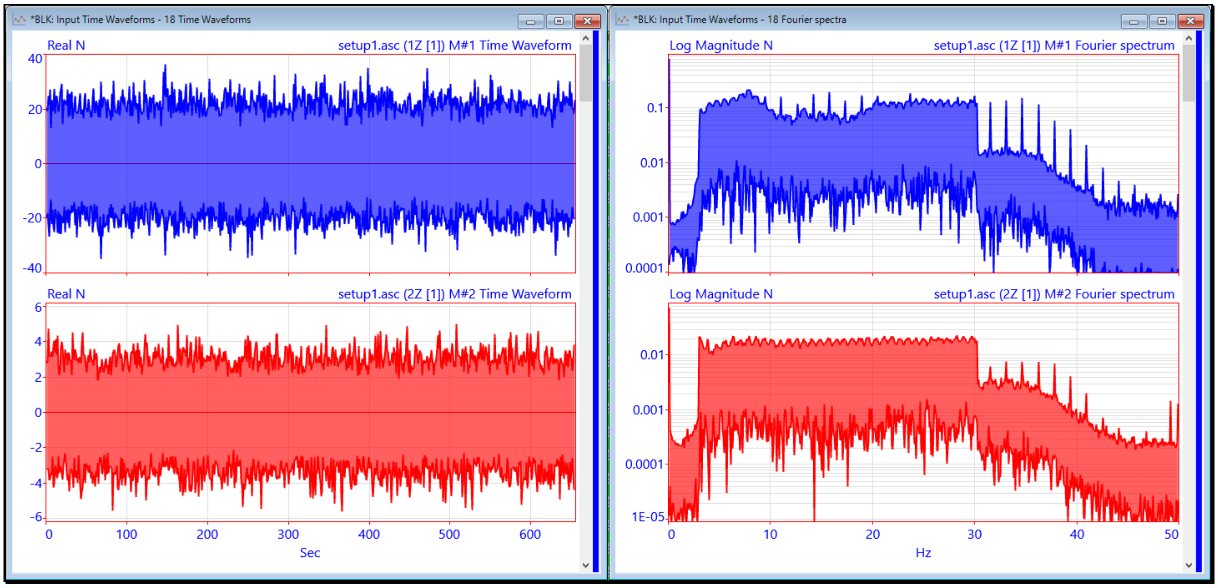
Forces were simultaneously acquired at **DOFs 1Z** & **2Z** along with accelerations at **72 *unique* DOFs** in **9** Measurement Sets.

Forces were simultaneously acquired at **DOFs 1Z** & **2Z** together with accelerations at **DOFs** (1Z, -2Y, 2Z) in all Measurement Sets.

The forces & response accelerations at the same **DOFs** (**1Z, -2Y, 2Z**) in each Measurement Set will be used to examine the consistency among the 9 Measurement Sets and also the structural reciprocity.

SHAKER FORCE SIGNALS

The shakers were driven with **computer-generated broadband random noise** spanning a **3 Hz** to **30 Hz** frequency range. The shaker **TWFs** and their **DFTs** for **Measurement Set [1]** are shown in the figure below.

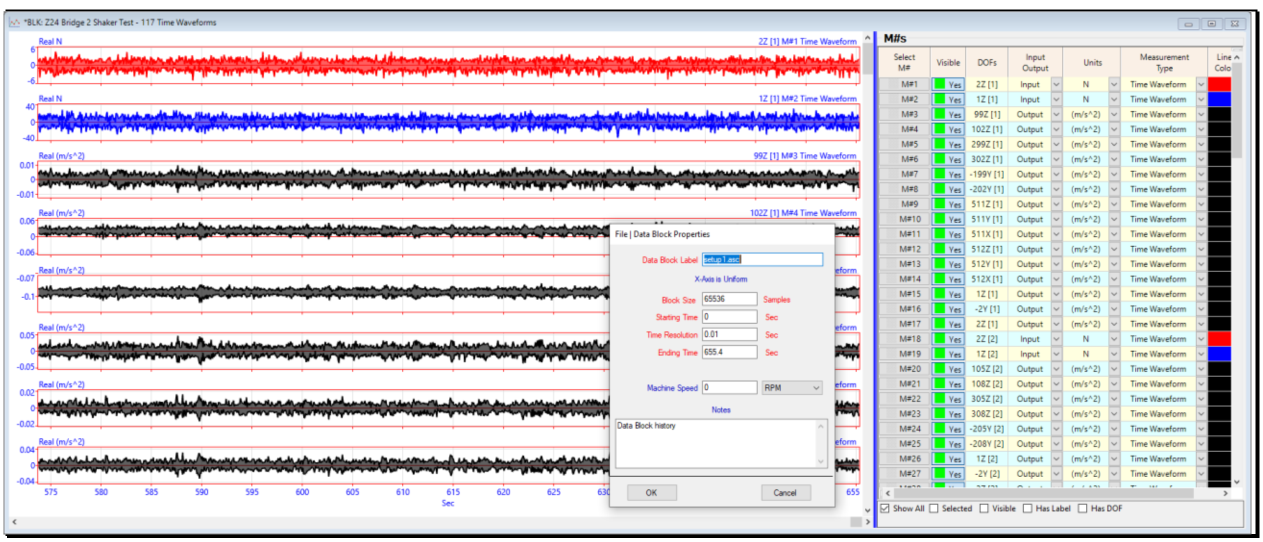


Shaker Force Signals Applied to ***DOFs*** *1Z & 2Z During Acquisition of Measurement Set [1].*

STEP 1 - ACQUIRED TWFS

- ***Press*** **Hotkey 1 Acquired TWFs**

The Data Block **BLK: Z24 Bridge 2 Shaker Test** contains all the **TWFs** acquired in **9 Measurement Sets** during the bridge test. Each **TWF** contains **65536 samples acquired over 10.92 minutes.**



Data Block **BLK: Z24 Bridge 2 Shaker Test** *with 117 Acquired* ***TWFs****.*

Each force **TWF** is labeled as an **Input** in the **Input Output** column of the **M#** properties s spreadsheet.

Each accelerometer **TWF** is labeled as an **Input** in the **Input Output** column of the **M#** properties s spreadsheet.

Each Measurement Set contains a different number of **M#s**. The table below lists the number of **TWF M#s** acquired in each Measurement Set.

Measurement Set	Force DOFs 1Z, 2Z	Common Response DOFs 1X, -2Y, 2Z	Unique Roving DOFs	TOTAL M#s
1	**2**	**3**	**12**	**17**
2	**2**	**3**	**6**	**11**
3	**2**	**3**	**9**	**14**
4	**2**	**3**	**6**	**11**
5	**2**	**3**	**6**	**11**
6	**2**	**3**	**12**	**17**
7	**2**	**3**	**3**	**8**
8	**2**	**3**	**12**	**17**
9	**2**	**3**	**6**	**11**
TOTALS	**18**	**27**	**72**	**117**

STEP 2 - CALCULATE FRFS & COHERENCE

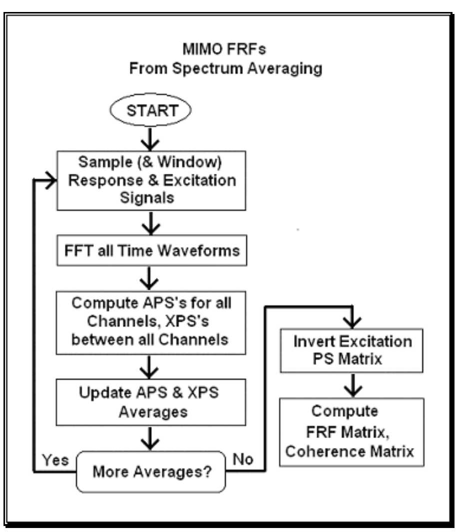
- ***Press* Hotkey 2 Calculate FRFs & Coherences**

When **Hotkey 1 is *pressed***, **FRFs & Coherences** are calculated from the force & response **TWFs** in **BLK: Z24 Bridge 2 Shaker Test**,

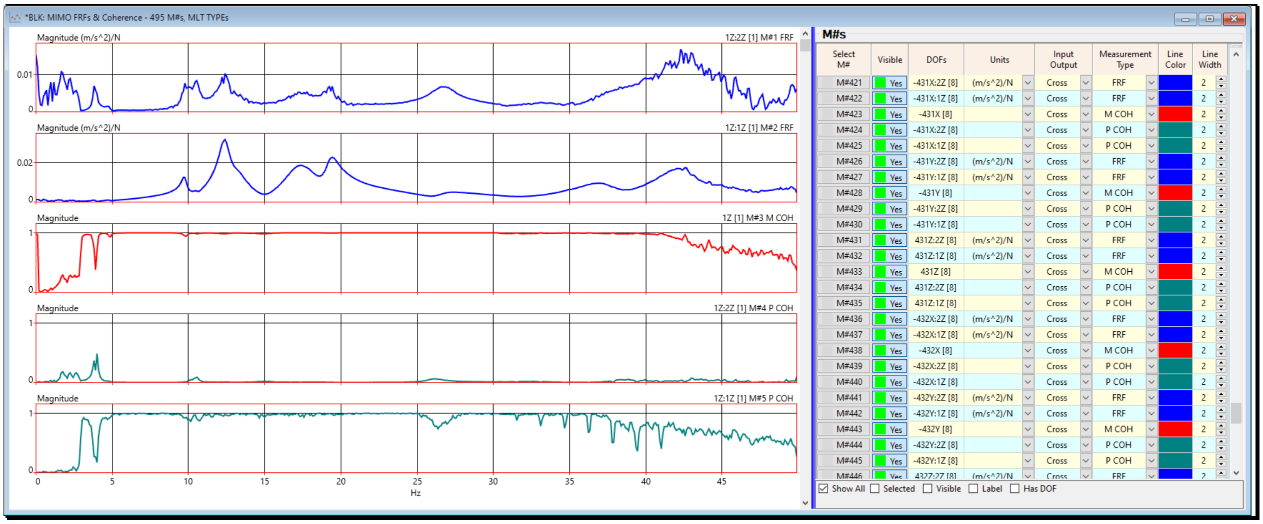
When **FRFs** are calculated from **TWFs**, estimates of the **Auto spectra** of the excitation forces and the **Cross spectrum** between each excitation force and each response are calculated.

Time domain windowing is applied to reduce leakage, spectrum averaging is used to reduce extraneous noise, and overlap processing is used during the calculation of the **Auto & Cross** spectrum estimates, *(*For more details, ***See* Spectrum Averaging** in the **Signal Processing** chapter of the Operating Manual.)

Multiple reference **FRFs** together with **Multiple & Partial Coherences** are calculated from each Measurement Set of **TWFs**. The **FRFs** are calculated using the **MIMO** processing depicted in the diagram below.



Block Diagram for Multi-Reference FRF Calculation.



*FRFs & Coherences Calculated from 9 Measurement Sets of Acquired **TWFs**.*

BLK: MIMO FRFs & Coherence window contains **495 M#s** consisting of the following,

- **198 FRFs**
  - **144 FRFs** with **unique DOFs** (**72 unique DOFs** paired with **2 force DOFs**)
  - **54 FRFs with the same DOFs** (**6 response DOFs** (**1X, -2Y, 2Z**) paired with **2 force DOFs** for each Measurement Set)
- **99 Multiple Coherences** (**one for each unique response DOF** plus one for **DOFs** (**1X, -2Y, 2Z**) from each Measurement Set
- **198 Partial Coherences** (**72 unique DOFs** paired with **2 force DOFs**)

The log magnitudes of FRF 1Z:2Z & FRF 1Z:1Z are displayed *in the upper part* of BLK: MIMO FRFs & Coherence.

The **red Multiple Coherence** is displayed *in the middle* and indicates that the force signals acquired at 1Z & 2Z caused all of the response at DOF 1Z.

The **two green Partial Coherences** of DOF 1Z are displayed *in the lower part* of BLK: MIMO FRFs & Coherence.

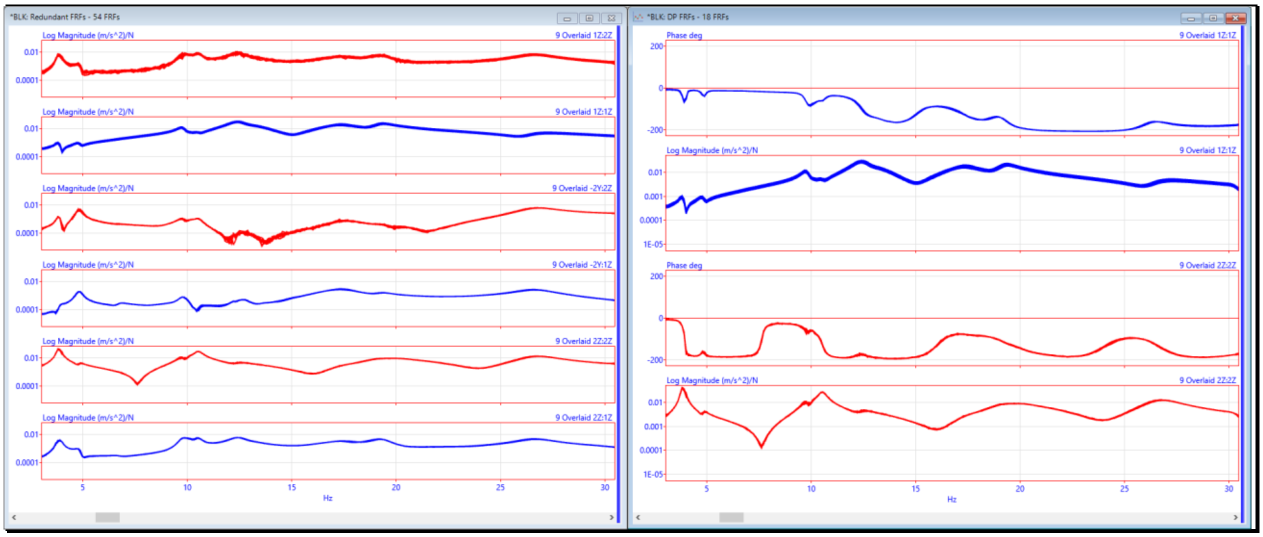
Partial Coherence 1Z:2Z is *"very low"* and Partial Coherence 1Z:1Z is *"nearly 1"* over the frequency range (3 to 30 Hz), meaning that the excitation from the shaker at DOF 1Z caused most of the response at DOF 1Z.

STEP 3 - REDUNDANT FRFS

- ***Press* Hotkey 3 Redundant & Driving Point FRFs**

When **Hotkey 3 is *pressed***, log magnitudes of the 6 groups of **FRFs with redundant DOFs** are overlaid ***on the left*** and the two **Driving Point FRFs** (**1Z:1Z & 2Z:2Z**) from all 9 Measurement Sets are overlaid ***on the right***.

The overlaid **FRF** log magnitudes ***on the right*** verify that the data acquisition in all 9 Measurement Sets was very consistent. The overlaid **FRFs** also show that the **FRFs** with **1Z** as a reference are **more consistent than** the **FRFs** with **2Z** as a reference.



Redundant FRFs Overlaid on the left and Driving Point FRFs *Overlaid on the right (3 to 30 Hz)*

The nine **2Z:2Z FRFs** (from the **smaller shaker**) have ***nearly identical log magnitudes & phases***. The differences between the log magnitudes of the nine **1Z:1Z FRFs** (from the **larger shaker**) are basically ***small gain changes*** while their ***phases are nearly identical***.

The overlaid driving point **FRFs** verify that the structure was undergoing ***linear stationary vibration*** while data was being acquired for all 9 Measurement Sets.

STEP 4 - STRUCTURAL RECIPROCITY

- ***Press* Hotkey 4 Reciprocity of FRFs**

Structural reciprocity **is usually assumed** when a modal test is performed on a structure.

Structural reciprocity means that if an **FRF** is calculated from a **response at DOF A** due to **excitation at DOF B**, it is the same as the **FRF** calculated from a **response at DOF B** due to **excitation at DOF A**.

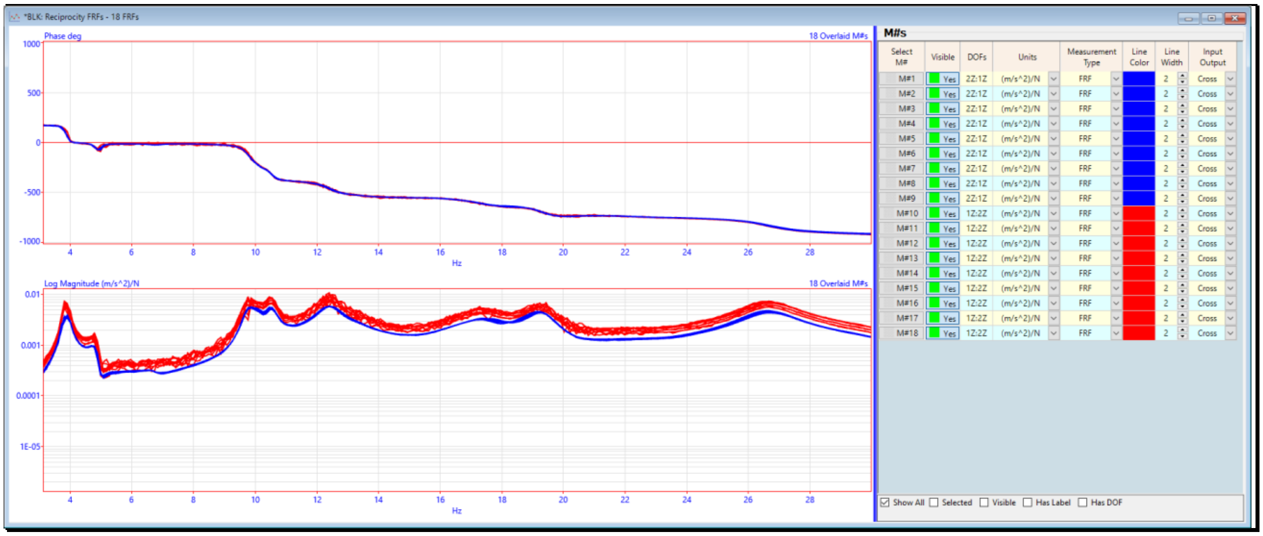
When **Hotkey 4 is *pressed***, reciprocity is checked by overlaying the nine **FRFs** with **DOFs 2Z:1Z** with the nine **FRFs** with **DOFs 1Z:2Z**. Pairs of these two **FRFs** were calculated from all 9 Measurement Sets on **TWF** data.

Structural reciprocity is a **basic property of a linear dynamic system**.

Structural reciprocity is **usually assumed during a modal test**.

When structural reciprocity **is valid**, mode shapes can be extracted **from any row or column of FRFs** in the **FRF** matrix of the **MIMO** model.

If structural reciprocity **is not valid**, ***the entire FRF matrix must be measured*** to capture all the dynamic properties of a structure. ***This is impractical from a measurement standpoint***.



Nine **2Z:1Z FRFs** *and Nine* **1Z:2Z FRFs** *Overlaid.*

The **18 overlaid FRFs** above confirm **structural reciprocity** between the two shaker locations.

The 18 unwrapped phases are the same.

The 18 overlaid log magnitudes are essentially the same. **FRFs** from the large shaker (at **1Z**) are more consistent than those **FRFs** from the small shaker at **2Z**.

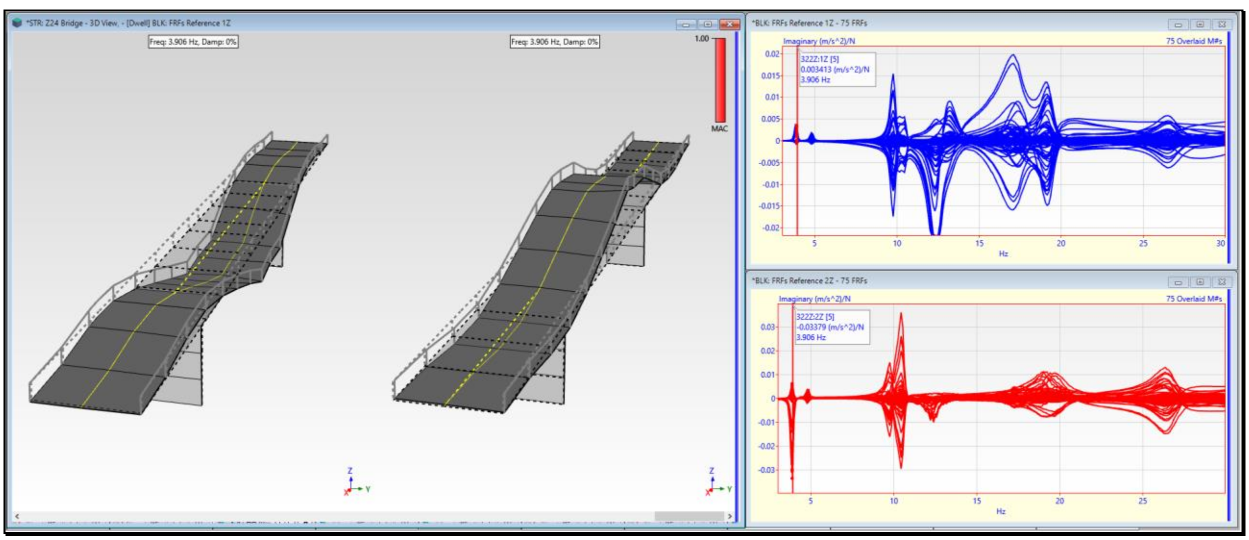
The overlaid **FRF** plots confirm that the bridge was behaving dynamically **in a linear symmetric manner** during data acquisition.

STEP 5 - ANIMATING ODS'S FROM THE MULTI-REFERENCE FRFs

- ***Press* Hotkey 5 - ODS from Reference DOF 1Z vs. 2Z**

A set of **multi-reference FRFs** is ideal for verifying that **each resonance peak corresponds to a single mode of vibration**.

When **Hotkey 5 is *pressed***, the **FRFs** for each reference **DOF** (**1Z & 2Z**) are copied into separate Data Blocks and the ODS from each reference is displayed in side-by-side animation.



ODS from Reference 1Z Versus 2Z.

- ***Drag*** the **Line** cursor to one of the peaks in either Data Block to display its ODS

The **pair of ODS's** (one from each Data Block) that have **Maximum MAC** with each other are displayed side-by-side in animation.

The **ODS at 3.9 Hz** in both Data Blocks has a **MAC** → **1.0**, indicating that a **3.9 Hz first bending mode shape is *dominating the* ODS** at that frequency.

STEP 6 - REVIEW STEPS

To review all the steps of this App Note,

- ***Press* Hotkey 6 Review Steps**