# **VIBRANT** MEscope Application Note 41

# **Extracting OMA Mode Shapes from Random Responses**

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Ad**vanced Signal Processing & VES-4600 Advanced Modal Analysis options. Without these options, you can still carry out the steps in this App Note using the **AppNote41** project file. These steps might also *require a more recent release date* of MEscope.

## **APP NOTE 41 PROJECT FILE**

• To retrieve the Project for this App Note, click here to download AppNote41.zip

This Project contains *numbered Hotkeys & Scripts* for carrying out the steps of this App Note.

• Hold down the Ctrl key and click on a Hotkey to display its Script window

# **RANDOM RESPONSES**

In this App Note, an output-only **OMA** test of the Jim Beam structure shown below is simulated using two shakers and random excitation signals. A *"round trip"* is performed starting with an **EMA** modal model of the Jim Beam. The modal model is used to synthesize **FRFs** which will then be used to calculate the acceleration responses due to two random forces applied at corners of the top plate, as shown below.

Then, **Cross spectra** are calculated from the *random acceleration responses*, and they are *curve fit* to extract **OMA** mode shapes. Finally, the **OMA** mode shapes are compared with the original **EMA** mode shapes to demonstrate that the same modes can be extracted from *output-only random response* data.

A Data Block of TWFs of the two random forces has already been created and saved in this Project. Each random force TWF contains **50,000 samples** of time domain data. These forces (**Inputs**) are applied at **DOFs 5Z & 15Z** as shown below. Random responses to these forces will be calculated using a **dynamic MIMO model** of the Jim Beam which is synthesized using its **EMA** mode shapes.



# **HOTKEY 1 - BROAD-BAND EXCITATION**

## • Press Hotkey 1

A Data Block of TWFs of the two random forces has already been created and saved in this Project. The **Input** forces are applied at **DOFs 5Z & 15Z** as shown on the left below. Each random force TWF contains **50,000 samples** of time domain data. Random responses calculated from these two force Input will also have **50,000 samples**.

This is enough data to perform spectrum averaging using 25 spectrum averages with a spectrum Block Size of 1000 samples, with no overlap grossing when the Cross spectra are calculated.

The DFTs of the random forces are displayed below on the right. Notice that the random signals are defined with a band-width 2000 Hz.

The "flat spectrum" and the band-width of the DFTs of the two random forces guarantees that all modes in the frequency range (0 to 2000 Hz) will be excited.

- In a real-world output-only data acquisition environment, where the excitation forces cannot be measured, their band-width is unknown
- The **DFTs** of the excitation forces must be assumed to be *"relatively flat"* in order to excite and extract OMA modes from output-only data



To calculate responses (**Outputs**) at each of the 33 points on the Jim Beam caused by the random excitation forces (**Inputs**), the **EMA** mode shapes are used to synthesize **FRFs** between the **2 Inputs** at **DOFs 5Z & 15Z** and **99 Outputs** at **DOFs 1X, 1Y, 1Z to 33X, 33Y, 33Z**.

# **HOTKEY 2 - CONVERTING UMM TO RESIDUE MODE SHAPES**

#### • Press Hotkey 2

To be used for **FRF** synthesis, the **EMA** mode shapes must be converted into **Residue** mode shapes that have Roving **DOFs 1X, 1Y, 1Z to 33X, 33Y, 33Z** and Reference **DOFs 5Z & 15Z**. The **Residue** mode shapes are then used to synthesize **192 FRFs** between all **99** Roving **DOFs** and the two Reference **DOFs 5Z & 15Z**.

When **Hotkey 2** is *pressed*, two Shape Tables are displayed as shown below. The original **EMA** mode shapes are displayed on the left and the **Residue** mode shapes are displayed on the right.

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M#	#2	17	in/lbf-sec	~ U	IMM Mode Shape	~	4.297	282.2	0.1119	87.07	0.6103	132.1	1.396	244.9		M#2	1Y:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	119	25.88	3.602	358.9	72.75	230.3	122.2	339.1
M	13	1Z	in/lbf-sec	~ U	IMM Mode Shape	~	5.849	282.5	2.329	286.1	0.3584	211.1	0.4562	234.5		M#3	1Z:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	162	26.24	74.99	197.9	42.72	309.2	39.93	328.7
Ma	14	2X	in/lbf-sec	~ U	MM Mode Shape	~	4.05	94.67	6.557	97.14	0.7996	93.2	0.8644	280		M#4	2X:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	112.2	198.4	211.1	8.945	95.31	191.3	75.66	14.22
M	15	27	in/lbf-sec	~ U	IMM Mode Shape	~	1.633	276.4	0.1762	154.9	0.4096	241.6	0.8581	249.4		M#5	2¥:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	45.24	20.15	5.674	66.76	48.83	339.8	75.1	343.7
Ma	P6	2Z	in/lbf-sec	~ U	MM Mode Shape	~	8.465	281.6	6.2	288.7	5.017	276.8	6.834	97.57		M#6	2Z:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	234.5	25.32	199.6	200.5	598	14.98	598.2	191.8
Ma	\$7	3X	in/lbf-sec	~ U	IMM Mode Shape	~	3.437	92.99	5.427	92.63	0.4649	84.24	0.7117	95.21		M#7	3X:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	95.2	196.7	174.7	4.442	55.41	182.4	62.3	189.4
Ma	#8 I	3Y	in/lbf-sec	~ U	IMM Mode Shape	~	1.903	94.86	0.1714	220.7	1.751	273.8	0.1798	100.2		M#8	3Y:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	52.7	198.6	5.519	132.5	208.8	11.92	15.74	194.4
Ma	¢9	3Z	in/lbf-sec	~ U	IMM Mode Shape	~	8.448	279.4	3.984	283.2	10.92	276.8	10.51	93.5		M#9	3Z:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	234	23.15	128.3	195	1302	14.91	919.9	187.7
M#	10	4X	in/lbf-sec	~ U	IMM Mode Shape	~	3.987	97.35	5.261	92.56	0.2583	96.33	1.681	89.93	100	M#10	4X:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	110.4	201.1	169.4	4.37	30.79	194.5	147.1	184.2
M#	11 4	4Y	in/lbf-sec	~ U	IMM Mode Shape	~	6.08	95.5	0.4867	163.6	2.944	276.1	0.1873	1.758		M#11	4Y:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	168.4	199.2	15.67	75.44	350.9	14.28	16.39	95.99
Me	12	4Z	in/lbf-sec	~ U	IMM Mode Shape	~	8.571	274.4	2.607	97.95	16.28	276.1	2.153	87.3	199	M#12	4Z:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	237.4	18.07	83.94	9.756	1940	14.27	188.4	181.5
M#	13	5X	in/lbf-sec	~ U	MM Mode Shape	~	3.908	97.92	5.152	94	0.0594	352.5	1.812	96.16	100	M#13	5X:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	108.2	201.6	165.9	5.812	7.081	90.61	158.6	190.4
Ma	14	5Y	in/lbf-sec	~ U	IMM Mode Shape	~	9.437	96.12	0.4224	135.3	3.667	278.3	0.476	272.1		M#14	5Y:5Z	g/lbf-sec	∼ R	esidue Mode Shape	~	261.4	199.8	13.6	47.12	437.1	16.42	41.66	6.325
M#	15	5Z	in/lbf-sec	~ U	IMM Mode Shape	~	9.466	281.6	8.079	88.48	19.34	276.4	10.71	271.4		M#15	5Z:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	262.2	25.28	260.1	0.2918	2305	14.59	937.4	5.609
M#	16 0	6X	in/lbf-sec	~ U	IMM Mode Shape	~	0.1042	116.5	5.152	91.16	0.01873	227	2.01	95.03		M#16	6X:5Z	g/lbf-sec	~ R	esidue Mode Shape	~	2.885	220.2	165.9	2.973	2.232	325.1	175.9	189.3
Ma	17 (	6V	in/lhf-sec	v U	MM Mode Shape	~	9.639	91.97	1.149	331.8	3.614	289.1	1.119	96.08		M#17	6Y:57	n/lhf-sec	V R	esidue Mode Shane	~	267	195.7	37	243.6	430.8	27.21	97.93	190.3 *
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#### EMA MODE SHAPES

- Each mode shape **M**# only has a **Roving DOF**
- Each mode shape **M**# has units of **in/(lbf-sec**). (*displacement / force-sec*)
- Each mode shape M# has Measurement Type → UMM Mode Shape. (Unit Modal Mass scaling preserves the dynamic properties of the structure)

## **RESIDUE MODE SHAPES**

- Each mode shape **M**# has both a **Roving & Reference DOF**
- Each mode shape **M**# has units of **g**/(**lbf-sec**). (*acceleration / force-sec*)
- Each mode shape **M**# has Measurement Type → **Residue Mode Shape**
- There are *twice as many* **Residue** mode shape **M#s** as **UMM** mode shape **M#s** because there are *two* **reference DOFs**

The random responses to the two random force Inputs applied at DOFs 5Z & 15Z are calculated using the **Transform** | **Outputs** command in MEscope. This calculation is depicted in the diagram below.



MIMO Calculations Block Diagram

# **HOTKEY 3 - CALCULATING RANDOM RESPONSES**

#### • Press Hotkey 3

When **Hotkey 3** is *pressed*, the **Modal Model** of **Residue** mode shapes is used to *synthesize the necessary* **FRFs** between the two random forces (**Inputs**) at **DOF 5Z & 15Z** and the responses (**Outputs**) at **99 DOFs** of the Jim Beam. The random force **Inputs** are displayed *on the left*, and the random response **Outputs** are displayed *on the right*, shown below.

• Place the mouse pointer in the graphics area of either Data Block, and *spin the mouse wheel* to Zoom in on the random data



Random Forces on the Left and Responses on the Right.

# **HOTKEY 4 - CALCULATING CROSS SPECTRA**

#### • Press Hotkey 4

When **Hotkey 4** is *pressed*, Cross Spectra are calculated from the random responses. The 99 random responses are displayed on the left, and the 99 Cross Spectra with reference response **15Z** are displayed *on the right*, as shown below.



Acceleration Random Responses on the Left and Cross Spectra on the Right.

#### **CHOOSING A REFERENCE DOF**

Notice that all **99 M#s** in the **BLK: Random Responses** are designated as **Outputs**. To calculate Cross Spectra for curve fitting, *at least one of the responses* must be a *reference response*.

Any one of the 99 responses can be chosen as a reference response (*designated as an* Input *or* Both in MEscope). The response M# at DOF 15Z was used as *both an Output & Input* by changing its Input Output property to Both.

An *active response* **DOF**, (where all resonances of interest are excited), should be chosen as the *reference response* **DOF**.

#### HANNING WINDOW

Because the response **TWFs** *are continuous random signals*, (also called *pure random* signals), to *reduce leakage* in their **DFTs**, the responses must have a **Hanning window** applied to them before applying the FFT.

If a Hanning window is applied to *pure random* **TWFs** *leakage surrounding each resonance peak* in their **DFTs** and hence their Cross Spectra *will be reduced*.

#### SPECTRUM BLOCK SIZE

The **FFT** calculates a **DFT** (**Digital Fourier Spectrum**) with a spectrum Block Size *equal to one-half* of its **TWF** Block Size before the FFT is applied to it.

When Hotkey 4 is *pressed*, Cross Spectra are calculated from the random response TWFs using by averaging 25 DFTs of 1000 samples each together.

- The Block Size of the random response TWFs is 50,000 samples
- For spectrum averaging, there is enough data in each random response **TWF** to calculate **25 DFTs** with 1000 samples in each **DFT**

These calculations *simulate the same process* that would be used in a spectrum analyzer to *post-process random output-only data* acquired from a machine or structure.

#### **HOTKEY 5 - CURVE FITTING THE CROSS SPECTRA**

#### • Press Hotkey 5

When **Hotkey 5** is *pressed*, the Cross Spectra are curve fit using a **Stability diagram** to extract **OMA** mode shapes, as shown below.



Stability Diagram Showing 10 Stable Pole Groups.

#### **DECONVOLUTION WINDOW**

All the curve fitting methods in MEscope are **FRF-based curve fitting** methods which utilize an **FRF** *analytical model* to curve fit experimental data. However, **Cross Spectra**, **Auto Spectra**, and **ODS-FRFs** do not have *the complex waveform* of an **FRF**.

A **DeConvolution** window *must be applied* to **Cross Spectra**, **Auto Spectra**, and **ODS-FRFs** to extract modal parameters from them using **FRF-based curve fitting** methods.

When Hotkey 5 is pressed, a DeConvolution window is applied to the Cross spectra prior to curve fitting them.

• Scroll though the Cross spectra on the *upper-left* to observe the red Fit Function overlaid on the *log magnitude* of each DeConvolution windowed Cross Spectrum

#### **STABLE POLE GROUPS**

The **AF Polynomial** method was used to curve fit the windowed Cross Spectra. The **Stability Diagram** is displayed in the *lower-left corner* of the curve fitting window. *Stable pole groups* are displayed on top of the **Mode Indicator** in the **Stability Diagram**.

All the pole estimates in a stable pole group are indicated with the same color

Stable pole group colors *alternate between two colors*. Those two colors are the *second & third colors* on the **Contour Colors** tab in the **File | Data Block Options** box

#### HOTKEY 6 - COMPARING OMA WITH EMA MODE SHAPES

• Press Hotkey 6

When Hotkey 5 is *pressed*, a side-by-side animated display will begin with an OMA mode shape *on the left* and an original EMA mode shapes *on the right*.

The *Modal Assurance Criterion* (MAC) is used to numerically match mode shape pairs. Each pair of mode shapes with Maximum MAC is displayed together.

- MAC has values between 0 &1
- If MAC is greater than  $0.9 \rightarrow$  two shapes are *strongly co-linear*, or correlated.
- If MAC  $\rightarrow$  is less than 0.9  $\rightarrow$  two shapes are *weakly co-linear*, or uncorrelated.

The **MAC** for each mode shape pair is also displayed in the *upper-right corner* of the display. This numerical comparison of mode shape pairs confirms that curve fitting a set of *output-only* **Cross spectra** can recover the mode shapes used to model the input-output dynamics of the Jim Beam.



**OMA** Mode Shape on the Left and **EMA** Mode Shape with Maximum MAC on the Right.

# **ROUND TRIP**`

This completes the *round-trip simulation* of the excitation of the Jim Beam using two random forces applied to corners of its upper plate. Ten **EMA** mode shapes were used to synthesis an FRF-based dynamic model of the Jim Beam. Random response **TWFs** (**Outputs**) caused by the random force **TWFs** (**Inputs**) were calculated using **Transform** | **Outputs**, one of the **MIMO** (**Multi-Input Multi-Output**) commands in MEscope.

These calculated random response **TWFs** are the same as any random response **TWFs** that would be acquired from a real-world test article using a multi-channel data acquisition system.

Then, the random response **TWFs** were used to calculate Cross Spectra between all the responses and a reference response at **DOF 15Z** on the upper plate. Finally, the Cross Spectra were curve fit to extract OMA modes of the Jim Beam. These **OMA** modal parameters *closely matched* the original **EMA** modal parameters of Jim Beam.

## **HOTKEY 7 - REVIEW STEPS**

To review all the steps of this App Note,

• Press Hotkey