MEscope Application Note 39

FRF-Based Curve Fitting

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-4000 Modal Analysis** option. Without this option, you can still carry out the steps in this App Note using the **AppNote39** project file. These steps might also require a ***more recent release date*** of MEscope.

APP NOTE 39 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote39.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down*** **the Ctrl key** and ***click on a*** **Hotkey** to open its Script window

ODS ANALYSIS VERSUS MODAL ANALYSIS

Modal analysis is used to characterize ***resonant vibration*** in machinery & structures. If a noise or vibration problem is due to the ***excitation of a structural resonance***, then the structure either must be isolated from the excitation source, or physically modified to reduce its level of vibration.

ODS Analysis shows how a machine or structure is vibrating, and where ***excessive vibration levels*** occur for various Points & directions.

Modal Analysis indicates whether the excessive vibration is due to ***excitation of a structural resonance***.

ODS's & mode shapes *can be compared* in MEscope to determine ***how a resonance participates in*** (or contributes to) the ***overall vibration level*** in a machine or structure.

WHAT IS FRF-BASED CURVE FITTING?

FRF-based curve fitting is a process of matching a ***parametric model*** of an **FRF** (in a ***least squared error*** sense) to experimental **FRF** data.

The ***unknown parameters*** of the parametric model are **modal frequency, damping & residues** (mode shape components).

The outcome of curve fitting is a set of modal parameters (**frequency, damping & mode shape**) for each mode that is identified in the experimental data.

After curve fitting is completed, mode shapes are stored into a Shape Table from which they can be displayed in animation on a 3D model of the test article.

CURVE FITTING STEPS

1. ***Determine*** the **Number of Modes** in a frequency band of measurement data
2. ***Estimate*** modal **Frequency & Damping** for all modes in the frequency band
3. ***Estimate*** modal **Residues** (mode shape components) for ***each*** mode and ***each*** measurement

CURVE FITTING GUIDELINES

1. Overlay the Measurements

A resonance peak should appear at the same frequency in every **FRF**, except where **node points** (***zero valued residues***) of the mode shape occur.

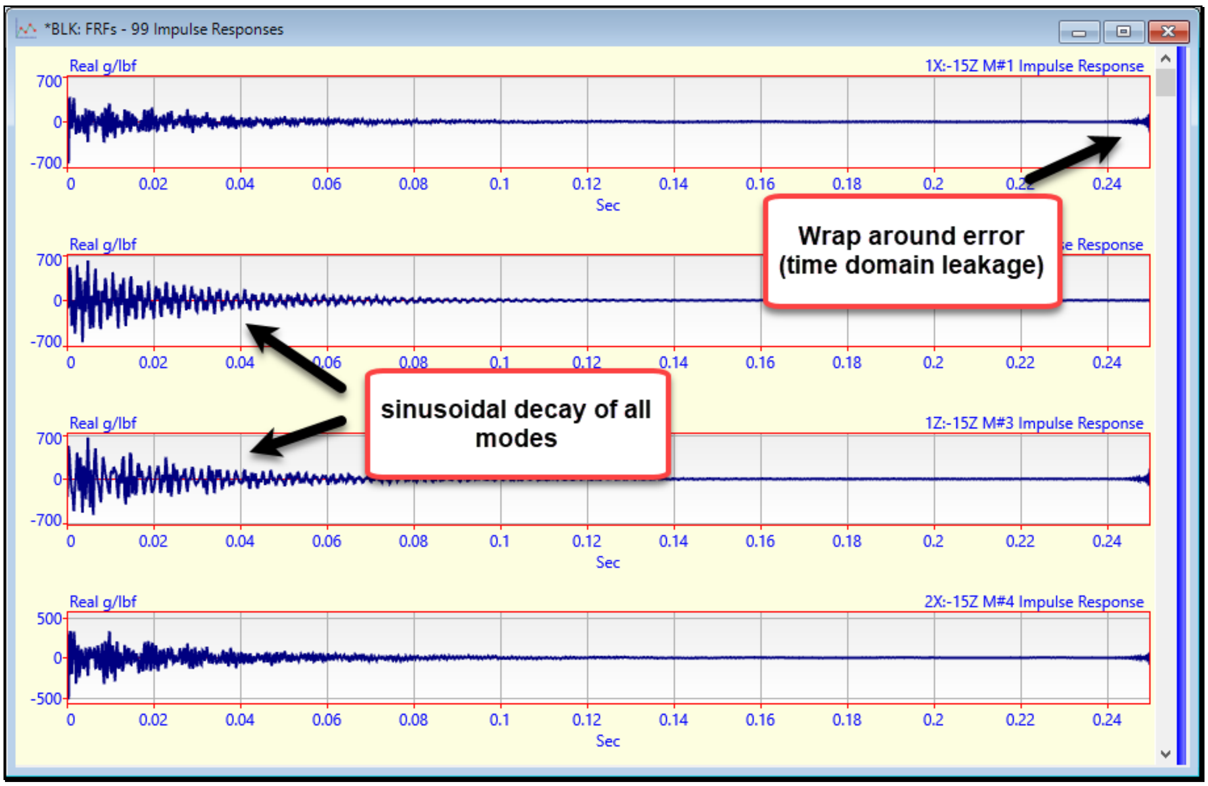
- Execute **Format | Overlay** to overlay the **FRFs** and look for ***resonance peaks*** at the ***same frequency*** in all **FRFs**

2. Inspect the Impulse Response Functions (IRFs)

- Execute **Transform | Inverse FFT** to transform ***each*** **FRF** into its corresponding IRF

All the **IRFs** should exhibit a ***damped sinusoidal decay to nearly zero*** at the end of each **M#**, as shown below.

- **Wrap around error** (shown below) is not harmful to frequency domain curve fitting



*Impulse Response Functions (**IRFs**).*

3. Use the Mode Indicator

- ***Press*** the **Count Peaks** button on the **Mode Indicator** tab to count the number of resonance peaks (modes) in a cursor band.

The number of Peaks counted is displayed in the **Peaks** box and in the **Modes** box on the **Frequency Damping** tab.

4. Use the Band cursor

Curve fit ***only those portions*** of data that contain ***valid resonance peaks.***

If the **Band** cursor is displayed, ***only*** data in the cursor band is used for curve fitting

If no **Band** or **Peak** cursor is displayed, ***all*** the data in each **M#** is used for curve fitting

5. Verify Fundamental Mode Shapes

The low frequency modes in most structures have ***simple bending*** and ***simple torsional*** (***twisting***) mode shapes.

- Estimate modal parameters for a few of the ***lower frequency*** (***fundamental***) modes
- Save the results into a Shape Table and display the mode shapes in animation to verify their validity

Points that animate ***substantially different*** from neighboring Points on the structure model are indications of ***poor* measurements**, ***poor* curve fits**, or **both**.

6. Compare Results from Different Curve Fitting Methods

Curve fit the **FRFs** using ***more than one curve fitting method*** and compare mode shapes from different methods.

- Execute **Display | MAC** (**Modal Assurance Criterion**) to numerically compare mode shapes between different curve fitting methods
- Execute **Animate | Animate a Pair | Animate Shapes** to display shapes side-by-side from two Shape Tables

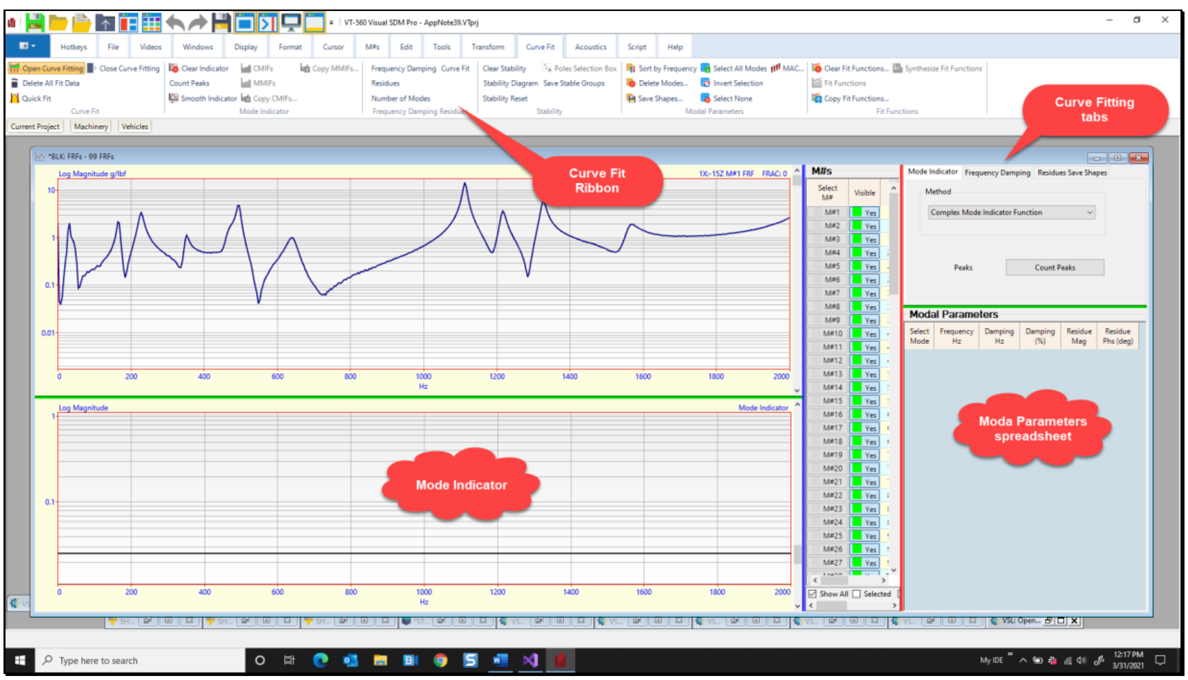
STEP 1 - CURVE FITTING IN A DATA BLOCK

- ***Press* Hotkey 1 Open Curve Fitting**

FRF-based curve fitting can only be done in a Data Block ***containing frequency domain data***.

When curve fitting is initiated, the following changes take place in a Data Block window.

- **M#s** are displayed in the ***upper-left corner*** of the Data Block window
- A **Mode Indicator** is displayed in the ***lower-left corner*** of the Data Block window
- The **Curve Fitting Tabs** are displayed on the ***upper-right corner*** of the Data Block window
- The **Modal Parameters** spreadsheet, for viewing and editing the modal parameter estimates, is displayed on the ***lower right corner*** of the Data Block window
- The **Curve Fitting Toolbar,** (containing frequently used commands), or the **Curve Fitting Ribbon** is displayed in the Toolbar area above the Data Block window



Data Block Window Open for Curve Fitting.

CURVE FITTING SPLITTER BARS

During curve fitting, two additional ***splitter bars*** are displayed in the Data Block window.

A ***vertical red splitter bar*** separates the Curve Fitting tabs from the **M#s** Display & Mode Indicator graphs

A ***horizontal blue splitter bar*** separates the **M#s** and Mode Indicator graphs

- ***Drag*** the ***red splitter bar horizontally*** or the ***horizontal blue splitter bar vertically*** to re-size the four curve fitting areas of the Data Block

DELETING ALL CURVE FITTING DATA

All curve fitting data is retained in a Data Block file when it is saved as part of a Project.

Before starting a new curve fitting session, all previous curve fitting data should be deleted from the Data Block.

- Execute **Curve Fit | Delete All Fit Data**
- ***Click*** on **Yes** in the dialog box that opens

STEP 2 - COUNTING RESONANCE PEAKS

- ***Press*** **Hotkey 2 Count Peaks**

The first step of modal parameter estimation is to determine how many modes are represented by resonance peaks in a frequency band of the **FRFs**.

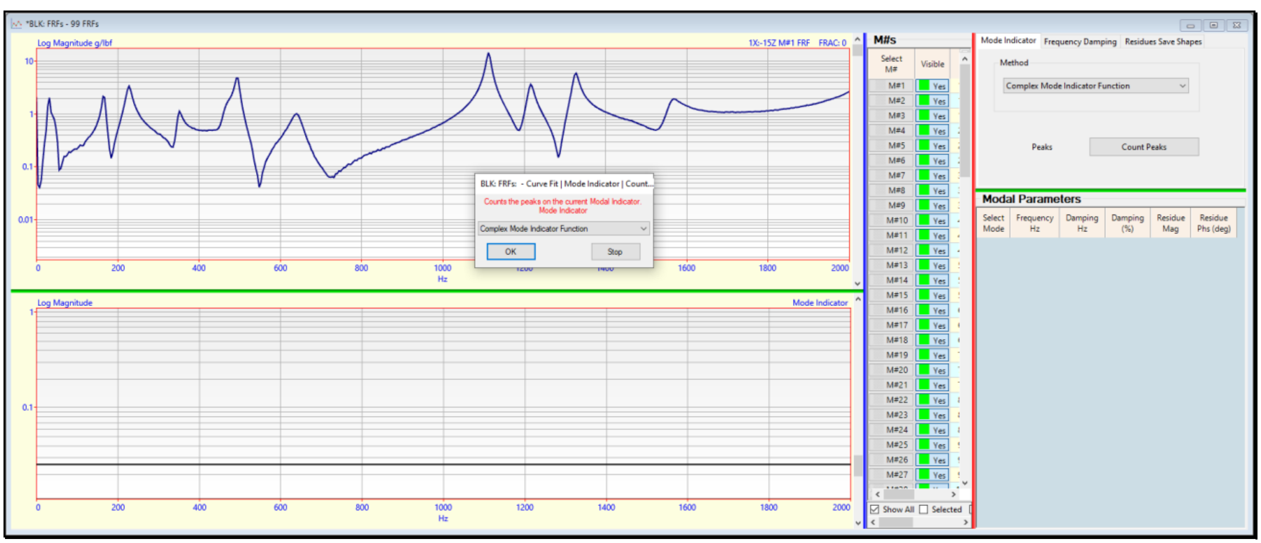
Each ***peak*** in an **FRF** is evidence of ***at least one mode***.

There are two ways to determine the number of modes represented by ***peaks*** in the **FRFs**.

1. ***Press*** the **Count Peaks** button on the **Mode Indicator** tab of the Curve Fit panel
2. **Overlay** all **M#**s, visually inspect them for resonance peaks, ***un-check*** the **Count Peaks** box and ***enter*** the number of peaks into the **Modes** box on the **Frequency Damping** tab

The **Count Peaks** button on the **Mode Indicator** tab is used to calculate a Mode Indicator function using ***all*** (or ***selected***) **M#s** and count the resonance peaks ***above*** the **Noise Threshold** line on the Mode Indicator.

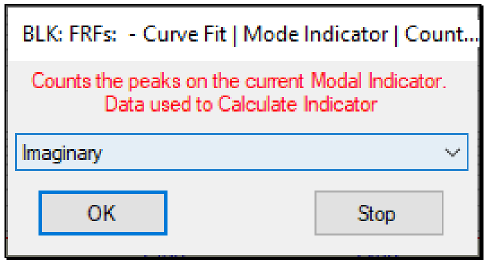
When **Hotkey 2 is *pressed***, a dialog box will open allowing you to choose the type of Mode Indicator function to calculate.



Type of Mode Indicator Dialog Box.

- ***Press*** the **OK** button

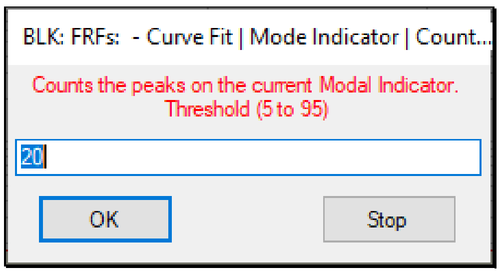
The next dialog box will let you choose which part of the FRF data to use to calculate the Mode Indicator.



The **Real Part**, **Imaginary Part**, or **Magnitude** of ***all*** (or ***selected***) **M#**s of **FRF** data is used to calculate the Mode Indicator.

- ***Press*** the **OK** button

Next, a dialog box will open for entering a percentage for setting the height of the noise threshold bar.



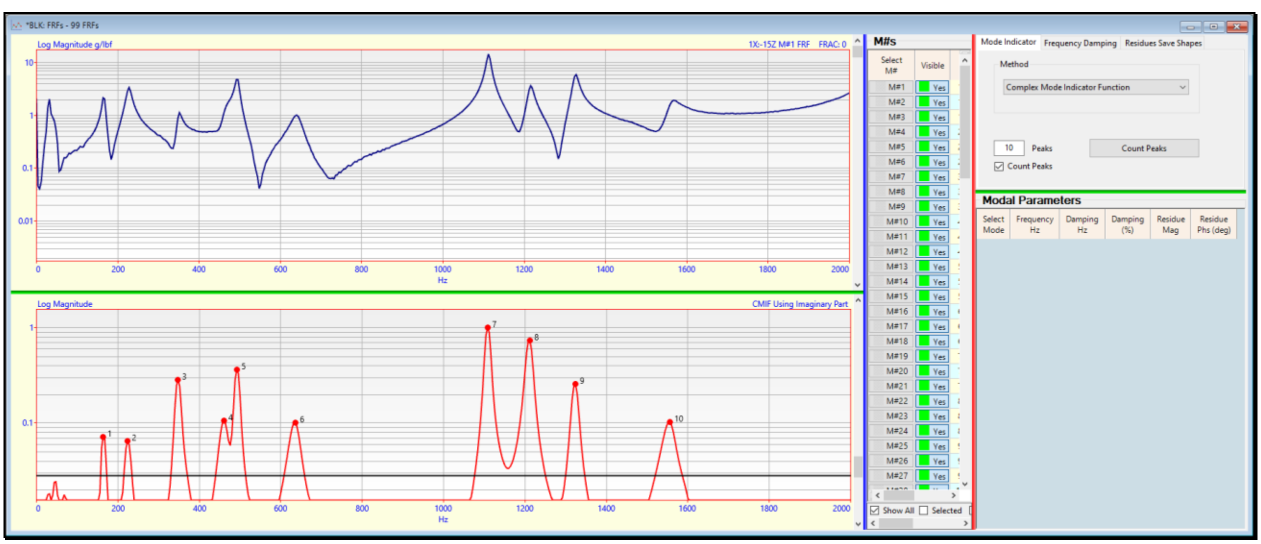
- ***Press*** the **OK** button

After the Mode Indicator has been calculated and its peaks counted, a graph of the Mode Indicator is displayed in the ***lower left corner*** of the window.

The peaks above the **Noise Threshold** (***horizontal line***) on the Mode Indicator graph are counted

Each modal peak counted is numbered and indicated with a **red dot** at the peak

The **Peaks** box on the **Mode Indicator** tab contains the number of peaks counted, in this case **10 peaks**



Mode Indicator Showing 10 Resonance Peaks Counted.

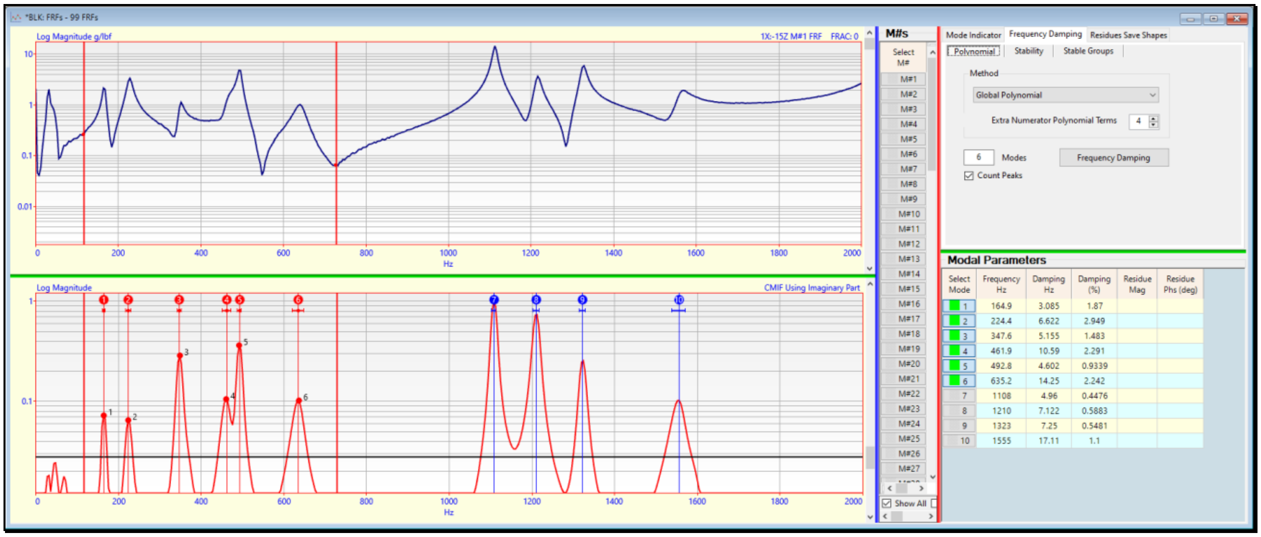
STEP 3 - ESTIMATING MODAL FREQUENCY & DAMPING

- ***Press*** **Hotkey 3 Estimate Frequency & Damping**

In many cases (especially with ***noisy*** data), it is better to ***build up a list*** of modal frequencies & damping in the **Modal Parameters** spreadsheet ***by curve fitting in small cursor bands*** using as few modes as possible.

Since these **FRFs** are relatively noise free, when **Hotkey 3 is *pressed***, **99 FRFs** are curve fit using the **Global Polynomial** method and modal **frequency & damping *for all 10 resonance peaks*** are estimated at once.

When curve fitting is completed, modal frequency & damping for 10 modes is listed in the **Modal Parameters** spreadsheet, as shown below.



Frequency & Damping Estimates for 10 Modes.

VERTICAL FREQUENCY LINES

The ***frequency estimate*** for each mode in the **Modal Parameters** spreadsheet is ***displayed as a vertical line*** on the Mode Indicator graph.

- Display the **Band** cursor and surround several of the vertical lines on the Mode Indicator

The modes within the Cursor band ***are selected*** in the **Modal Parameters** spreadsheet, as shown above.

HORIZONTAL DAMPING LINES

The ***damping estimate*** for each mode is ***displayed as a horizontal line*** crossing the vertical frequency line where,

Length of the horizontal line ➔ **2σ**

σ ➔ half-power-point (or 3-dB-bandwidth) damping (in Hz)

The **half-power-point** damping (**2σ**) is ***approximately equal*** to the width of the resonance peak at **50%** of the **FRF *peak magnitude squared***

The **FRF magnitude squared** is a ***power quantity***, hence the name **half-power-point** damping.

The **half-power-point** damping (**2σ**) is ***approximately equal*** to the width of the resonance peak at **70.7 %** of the **FRF *peak magnitude*** value

Zoomed Display Showing Frequency & Damping Estimates.

- Execute **Display | Zoom In** and zoom the display around several resonance peaks, as shown above

A choice of three different curve fitting methods for estimating frequency & damping is also shown in the figure above. Each method can be used to estimate frequency & damping.

LOCAL POLYNOMIAL METHOD

The **Local Polynomial** method estimates frequency & damping by curve fitting each **FRF**.

Local Polynomial curve fitting is helpful when the ***resonance peaks do not line up*** in a set of **FRFs** due to mass loading during data acquisition.

GLOBAL POLYNOMIAL METHOD

The **Global Polynomial** method estimates frequency & damping by curve fitting ***all*** (or ***selected***) **FRFs**.

Modal **frequency & damping** are ***global properties of most structures***, meaning that they are the same no matter where data is acquired from the structure.

MULTI-REFERENCE POLYNOMIAL METHOD

The **Multi-Reference Polynomial** method is normally used with a set of **multiple-reference FRFs**.

Multi-Reference Polynomial method is used to estimate frequency & damping for ***closely-coupled modes, repeated roots, and local modes***.

For a set of **single-reference FRFs**, the **Multi-Reference Polynomial** method gives the same estimates as the **Global Polynomial** method.

FREQUENCY & DAMPING TERMINOLOGY

Modal frequency & damping are defined in several ways.

$p(k) = -\sigma(k) + j\omega(k)$ ➔ pole location of mode(k) (in Hz)

$\omega(k)$ ➔ damped natural frequency of mode(k) (in Hz)

$\Omega(k)$ ➔ undamped natural frequency of mode(k) (in Hz)

$$\Omega(k) = \sqrt{\omega(k)^2 + \sigma(k)^2}$$

$\sigma(k)$ ➔ damping decay constant (also called the **half-power-point** damping or **3-dB-bandwidth**) of mode(k) (in Hz)

$\zeta(k)$ ➔ damping ratio (in %)

Half-power-point damping **(3 dB bandwidth) damping**	$(\omega_2 - \omega_1) = 2\sigma(k) = \Delta\omega_{3dB}(k) \quad (Hz)$ $\omega_1 = \omega(k) - \sigma(k) \quad \omega_2 = \omega(k) + \sigma(k)$
Damping Ratio **(Percent of critical damping)**	$\zeta(k) = \dfrac{\sigma(k)}{\sqrt{\omega(k)^2 + \sigma(k)^2}} \quad (\%)$
Damping Decay Constant	$\sigma(k) = \dfrac{\omega(k)\,\zeta(k)}{\sqrt{1 - \zeta(k)^2}} \quad (Hz)$
Loss Factor	$\eta(k) = 2\zeta(k)$
Quality Factor	$Q(k) = \dfrac{1}{\eta(k)}$

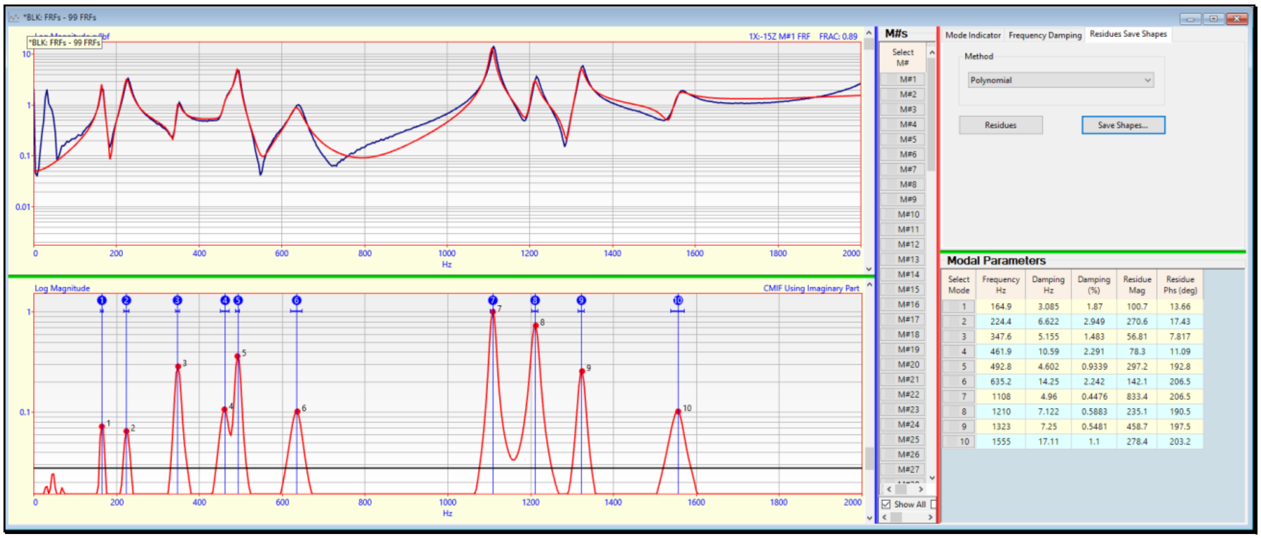
STEP 4 - ESTIMATING RESIDUES

- ***Press* Hotkey 4 Estimate Residues**

After modal frequency & damping have been estimated, modal ***residues*** (mode shape ***components***) are estimated during a ***second curve fitting step***.

When **Hotkey 4 is *pressed***, estimates of residue ***magnitude & phase*** for ***each mode and each* FRF** are added to the **Modal Parameters** spreadsheet.

- ***Scroll*** through the **M#s** and release the mouse button to list the residues for the **M#** being displayed



Residue Curve Fit for 10 Modes and 99 ***FRFs****.*

FIT FUNCTION

After the modal **frequency, damping, & residue** have been estimated for each mode, a **red Fit Function** is overlaid on each **M#**, as shown above.

The **Fit Function** is calculated using the modal parameter estimates

If the modal parameters are accurate, each **Fit Function** will ***closely match its corresponding* FRF**

USING THE BAND CURSOR FOR CURVE FITTING

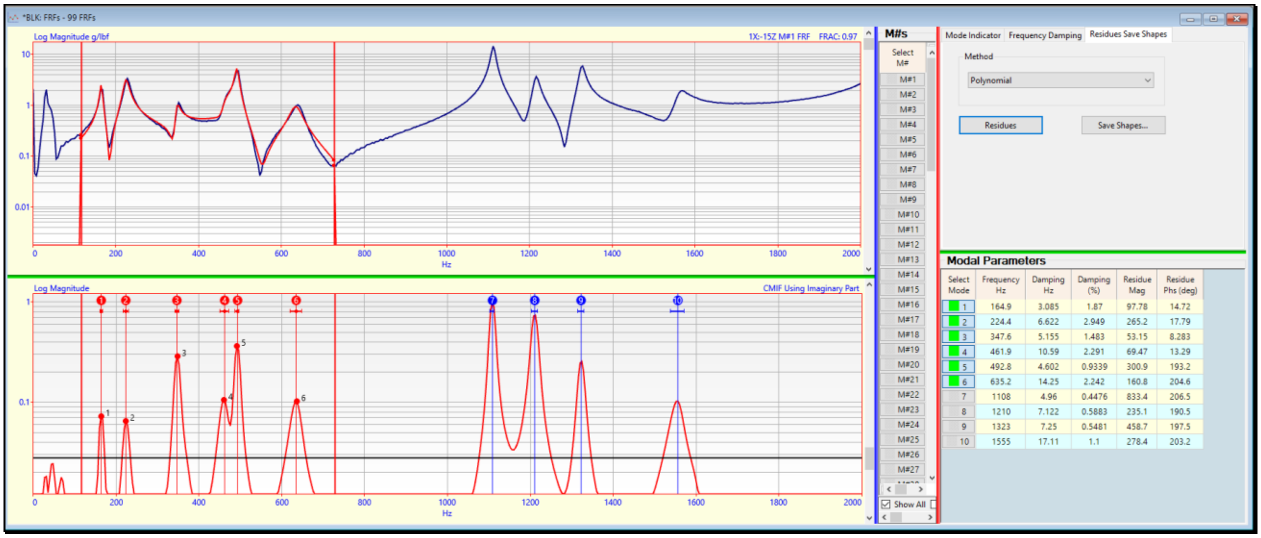
It is often more convenient to curve fit using ***several small cursor bands*** to avoid ***noisy data or non-resonance peaks*** in the **FRFs**.

If the **Band** or **Peak** cursor is displayed, curve fitting is done ***using only the data within the cursor band***.

- Execute **Curve Fit | Fit Function | Clear Fit Functions**
- Execute **Cursor | Band Cursor** and ***drag*** the edges of the band to ***enclose only six resonance peaks***, as shown below
- ***Press*** the **Residues** button on the **Residues Save Shapes** tab

Each **red Fit Function** is only displayed within the cursor band because only that data was used for the Residue curve fitting.

The residue estimates ***might be slightly different*** when residue curve fitting is done ***using several smaller cursor bands*** of data rather than estimating all residues at once using all the **FRF** data.



Residue Curve Fitting in a Cursor Band.

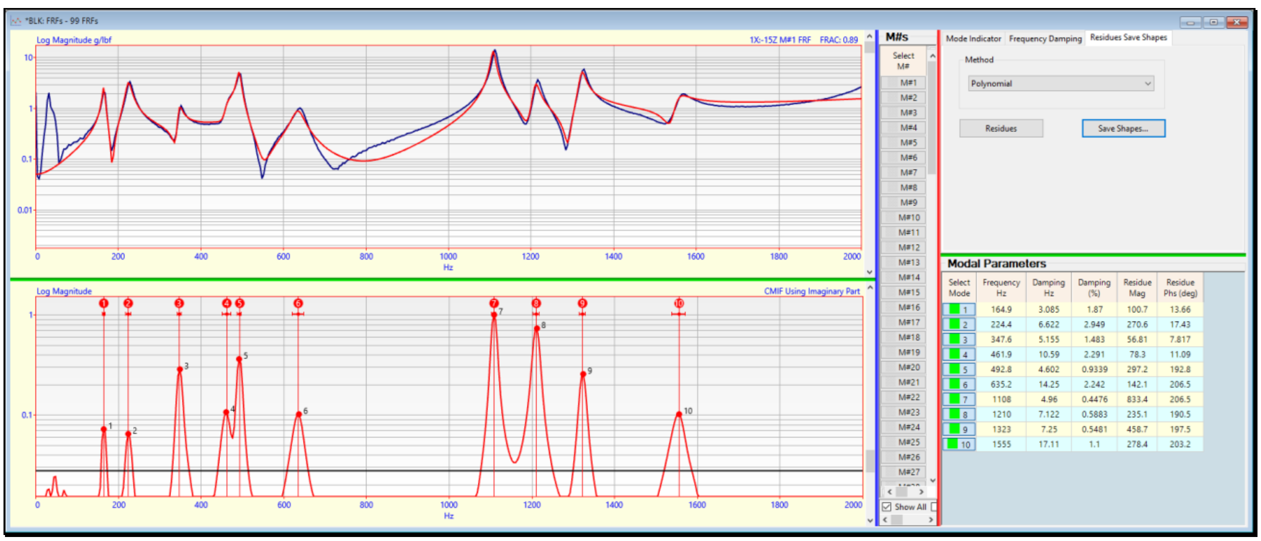
STEP 5 - QUICK FIT

- ***Press* Hotkey 5 Quick Fit**

When all resonance peaks are easily counted in a set of **FRFs**, all the data can be curve fit in one execution of the **Quick Fit** command.

Quick Fit carries out all three curve fitting steps using the methods already chosen on each **Curve Fitting** tab.

When **Hotkey 5 is *pressed***, the **Quick Fit** results are displayed, as shown below.



***Quick Fit** Results from Pressing Hotkey 5.*

DIVIDE AND CONQUER

The quickest way to curve fit a large set of **FRFs** with many resonance peaks is to use **Quick Fit** is several smaller bands of data.

- Display the **Band** cursor to enclose ***several resonance peaks***, and execute **Curve Fit | Quick Fit**
- ***Drag*** the **Band** cursor to enclose ***several more resonance peaks***, and execute **Curve Fit | Quick Fit *again***

Each time the **Quick Fit** command is executed, new modal parameter estimates ***are added*** to the **Modal Parameters** spreadsheet

STEP 6 - COMPARING MODE SHAPES

The best way to validate your curve fitting results is to compare the mode shapes obtained from ***two or more curve fitting methods***.

- ***Press* Hotkey 6 Compare Mode Shapes**

When **Hotkey 6 is *pressed***, the mode shapes obtaining with the **Polynomial** method with be compared with those obtained with the Residue **Peak** method.

RESIDUE POLYNOMIAL CURVE FITTING METHOD

The **Polynomial** method was used by **Quick Fit** when **Hotkey 5 was *pressed***.

Quick Fit uses all the modal frequency & damping estimates together with all the data for each **FRF** (or the data in a cursor band) to ***simultaneously estimate*** the residues for each mode and each **FRF**.

RESIDUE PEAK CURVE FITTING METHOD

The **Peak** method uses the modal frequency & damping estimates for each mode and saves a residue estimate for each mode as the ***peak of the displayed data*** of each **FRF**.

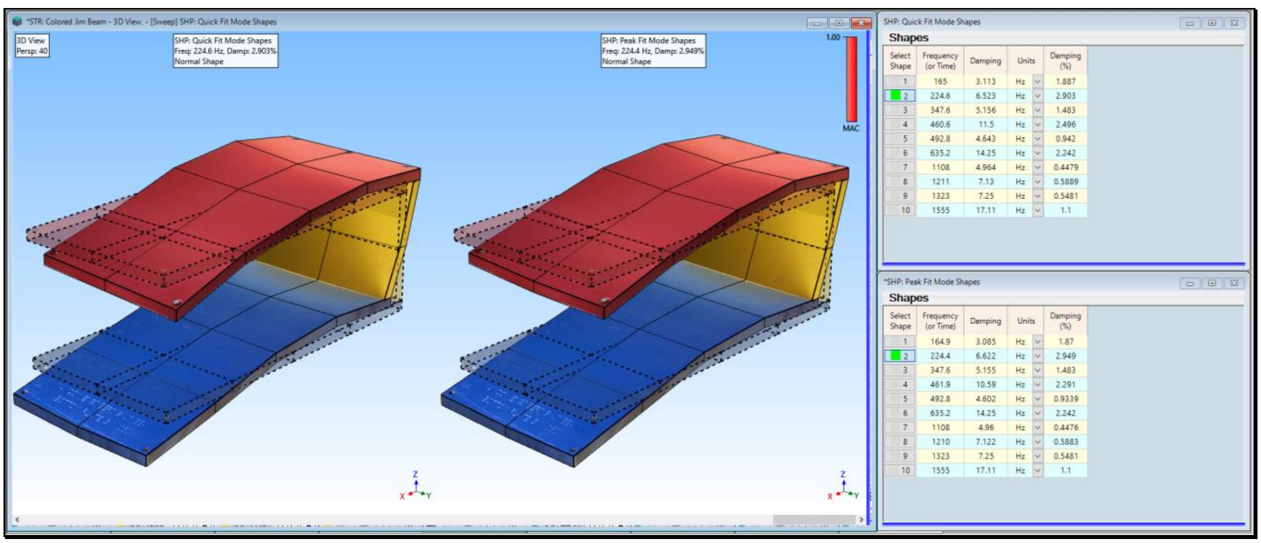
In this case the ***peak value in the imaginary part*** of the FRFs is used.

This simple SDOF method often obtains a ***close approximation of each mode shape***.

When **Hotkey 6 is *pressed*** the **FRFs** are ***curve fit twice*** and the resulting mode shapes are saved in separate Shape Tables. The **Quick Fit** results are saved in the **SHP: Quick Fit Mode Shapes** Shape Table, and the **Peak** method results are saved in the **SHP: Peak Fit Mode Shapes** Shape Table**.**

Then sweep animation is begin through the mode shapes in the **SHP: Quick Fit Mode Shapes** Shape Table.

Each mode shape in the **SHP: Quick Fit Mode Shapes** Shape Table is displayed side-by-side with its ***closest matching* mode shape** from the **SHP: Peak Fit Mode Shapes** Shape Tabl**e.**



*Animation of a **Quick Fit** vs. **Peak Fit** Mode Shape.*

MODE SHAPE PAIR WITH MAXIMUM MAC

The ***closest matching*** mode shape to a shape in **SHP: Quick Fit Mode Shapes** is the mode shape in **SHP: Peak Fit Mode Shapes** that has the **Maximum MAC** value among all the mode shapes in **SHP: Peak Fit Mode Shapes**.

The **Maximum MAC** (**Modal Assurance Criterion**) value is displayed in the ***upper-right corner*** of the mode shape display and indicates ***how closely matched*** each pair of mode shapes is. These Maximum MAC values indicate ***close correlation*** between all pairs of mode shape estimates from the two Residue curve fitting methods.

MAC ➔ ***greater than*** **0.90** ➔ two mode shapes are ***closely matched***

STEP 7 - REVIEW STEPS

To review the steps of this App Note,

- ***Press*** **Hotkey 7 Review Steps**