MEscope Application Note 38

Digital Signal Processing

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** option. Without this option, you can still carry out the steps in this App Note using the **AppNote38** project file. These steps might also require a ***more recent release date*** of MEscope.

APP NOTE 38 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote38.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down*** **the Ctrl key** and ***click on a*** **Hotkey** to open its Script window

THE FFT AND THE DFT

The **FFT** is a ***computer algorithm*** that calculates the **Digital Fourier Transform *(*DFT**) of a ***uniformly sampled*** time waveform. ***Three equations*** govern the **FFT** algorithm.

1. SAMPLED TIME WAVEFORM EQUATION

The **FFT** ***assumes*** that the time waveform contains **N** ***uniformly spaced samples***

The ***spacing*** (or ***resolution***) between time samples is denoted as Δt (in seconds)

The sampling time period (also called the ***sampling window***), spans the time period **(t ➔ 0 to T)** (in seconds)

The time waveform parameters are related by the equation,

$$T = N (\Delta t)$$ (in seconds)

2. DIGITAL FOURIER TRANSFORM (DFT) EQUATION

The **DFT** contains (**N/2**) ***uniformly spaced samples*** of ***complex*** (***magnitude & phase***) data

The ***spacing*** (or ***resolution***) between frequency samples is denoted as Δf (in Hz)

The **DFT** is calculated over a frequency span **(f ➔ 0** to **Fmax)** (in Hz)

The **DFT** parameters are related by the equation,

$$F_{max} = (N/2) \Delta f$$ (in Hz)

3. SHANNON'S (NYQUIST) SAMPLING CRITERION

Shannon's Sampling Criterion says that ***to calculate an accurate*** **DFT** over the span **(f ➔ 0 to Fmax)**,

The time waveform ***must be sampled at no less than twice*** the frequency **Fmax**

The minimum sampling rate is called the ***Nyquist sampling rate***

The sampling criterion relates **Fmax** and the ***Nyquist sampling rate*** by the equation,

Nyquist sampling rate ➔ $1/\Delta t = 2 F_{max}$ (in Hz)

This formula states that to obtain a valid **DFT**, a digital time waveform ***must be sampled at twice*** the expected value of **Fmax**.

FUNDAMENTAL SAMPLING RULE

Another important equation is derived from the three equations above.

$$\Delta f = 1/T \quad \text{(in Hz)}$$

This equation says that the ***frequency resolution*** (**Δf**) of the **DFT** is the ***inverse of the time length*** **(T)** of the time domain sampling window.

SAMPLING RATE VERSUS FREQUENCY RESOLUTION

To ***increase the frequency resolution*** (***reduce*** **Δf**) of a **DFT,** the time domain signal ***must be sampled over a longer time period*** (**T**).

Increasing the sampling rate (**1/Δt**) of the time waveform ***does not increase the frequency resolution*** (***reduce*** **Δf**) of its **DFT**.

ANTI-ALIASING FILTERS

When a continuous analog time domain signal is sampled, ***frequencies higher than*** **Fmax** in the signal ***will fold back*** and appear as lower frequencies in the **DFT**.

These ***aliased high frequency components*** are not part of the **DFT** at ***frequencies below*** **Fmax** in the original signal.

To ensure that no frequencies higher than **Fmax** are contained in a **DFT**, higher frequencies must be removed from the analog time waveform ***before it is sampled.***

Frequencies higher than **Fmax** are removed using an ***analog low pass filter called an*** **anti-aliasing filter**.

Passing a time waveform through an anti-aliasing filter ***before sampling it*** ensures that all frequency components higher than **Fmax** are removed from the **frequency span ➔ 0 to Fmax** of the **DFT**.

All anti-aliasing filters have a ***finite roll off frequency band***.

If the ***cutoff frequency*** (start of the filter roll off) is set to **80%** of **Fmax**, or **40%** of the ***sampling frequency***, then **80% of a frequency span ➔ 0 to Fmax** will be ***alias-free***.

Most **FFT** analyzers have anti-aliasing filters with a ***cutoff frequency*** set to **80% of Fmax**, or **40%** of the ***sampling frequency***.

FOURIER SPECTRUM (DFT)

Several types of frequency spectra can be calculated in MEscope.

The Fourier spectrum is the **DFT** of a time waveform

The **FFT** algorithm is used to calculate the **DFT** of a time waveform

The **DFT** is ***complex valued***, with **Real & Imaginary** parts, or **Magnitude & Phase**

AUTO SPECTRUM

Each Auto spectrum estimate is calculated by ***multiplying a* DFT *by its own complex conjugate***

An ***average* Auto spectrum** is calculated by averaging together multiple Auto spectra

An Auto spectrum is ***real-valued, with* Magnitude *only***

CROSS SPECTRUM

Each Cross spectrum estimate is calculated by multiplying the **DFT** of one signal by the ***complex conjugate*** of the **DFT** of a different signal

An ***average*** **Cross spectrum** is calculated by averaging together multiple Cross spectra

The Cross spectrum is ***complex valued***, with **Real & Imaginary** parts, or **Magnitude & Phase**

POWER SPECTRAL DENSITY (PSD)

A **PSD** is an Auto spectrum ***divided by the frequency resolution*** of the Auto spectrum

If the units of an Auto spectrum are **(g^2)**, the units of its corresponding **PSD** are **(g^2 / Hz)**

ENERGY SPECTRAL DENSITY (ESD)

An **ESD** is a **PSD** ***multiplied by the time length*** (**T**) of the time waveform used to create the spectrum

If the units of a **PSD** are **(g^2 / Hz)**, the units of its corresponding **ESD** are **(g^2 - sec / Hz)**

ESDs are used mostly to characterize transient signals

TIME DOMAIN WINDOWS

The **FFT** algorithm assumes that the time waveform to be transformed is ***periodic in its sampling window***.

A signal is ***periodic in its sampling window*** if it satisfies one of the following criteria,

1. An ***integer number of cycles*** of the signal are contained within its sampling window
2. The signal has ***no discontinuity*** between its beginning & end in its sampling window
3. The signal is ***completely contained*** within its sampling window

NON-PERIODIC SIGNAL

Many signals are ***non- periodic*** in their sampling window.

A ***purely*** **random signal** is is ***non-periodic*** (***never completely contained***) within a finite length sampling window.

WHAT IS LEAKAGE?

If a time waveform is ***non-periodic in its sampling window***, a ***smearing of its spectrum*** (called ***leakage***) will occur when it is transformed to the frequency domain as a **DFT**.

Leakage ***distorts*** the spectrum, especially around resonance peaks.

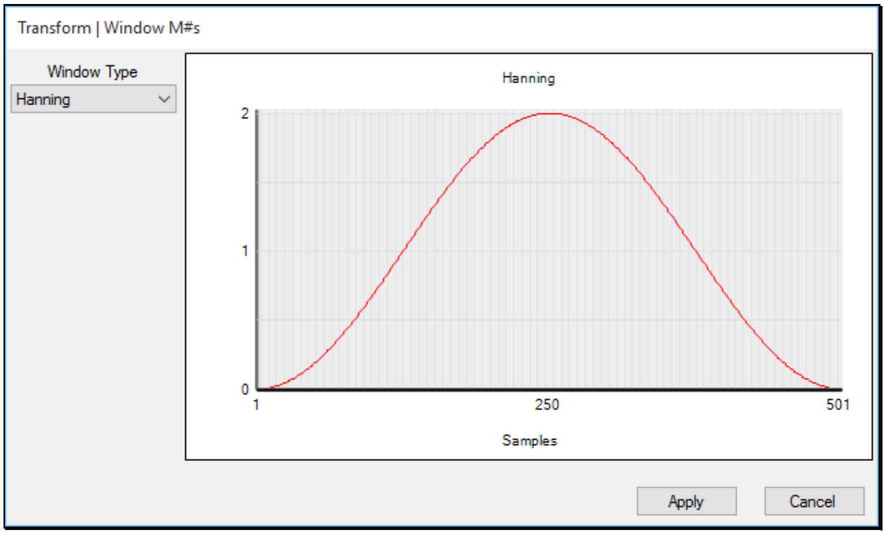
Leakage ***spreads*** the spectrum surrounding resonance peaks, which is detrimental for modal parameter estimation (curve fitting).

Leakage ***is reduced*** by multiplying the sampled time waveform by a ***special weighting function*** (called a **time domain window**), ***before*** the **FFT** is applied to the time waveform.

HANNING WINDOW FOR BROADBAND SIGNALS

If a time waveform is ***non-periodic in its sampling window***, leakage ***cannot be eliminated***, but it ***can be reduced***.

A Hanning window ***reduces the leakage*** in the spectrum of a ***broad band signal*** such as a random signal.



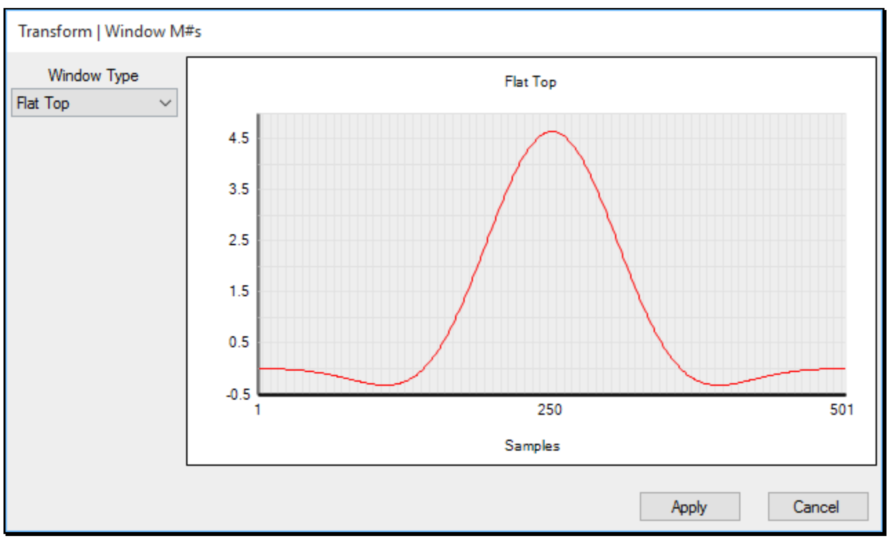
Hanning Window.

FLAT TOP WINDOW FOR NARROW BAND SIGNALS

A Flat Top window makes the ***magnitudes of peaks more accurate*** in the **DFT** of a ***narrow band signal*** such as a sinusoidal signal.

A Flat Top window also ***reduces leakage*** in the spectrum of a ***narrow band signal***.

A Flat Top window also ***makes the peaks wider*** in the spectrum of a ***narrow band signal***.



Flat Top Window.

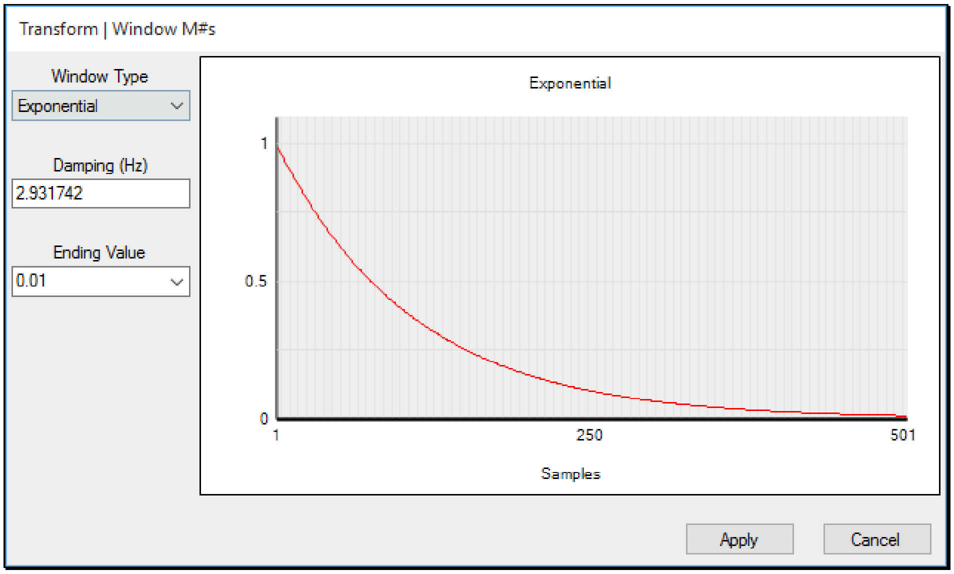
EXPONENTIAL WINDOW FOR TRANSIENT SIGNALS

A ***decreasing*** **Exponential window** should be applied to transient (or impulse response) signals that ***do not decay completely*** within their sampling window

A ***decreasing*** Exponential window ***artificially damps*** the signal toward zero before the end of its sampling window, thus making it ***nearly periodic in its sampling window***.

An Exponential window ***adds a fixed amount of damping*** to all the decay waveforms in an impulse response.

Following curve fitting in MEscope, the artificial damping added by an Exponential window ***is subtracted from the damping estimates*** of all modes.



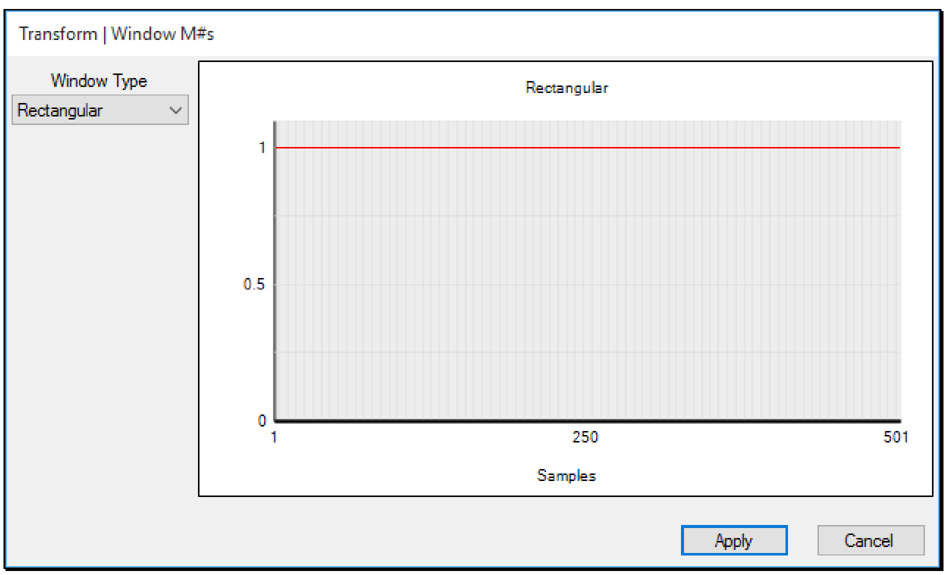
Exponential Window.

RECTANGULAR WINDOW FOR PERIODIC SIGNALS

A Rectangular window is used on a signal that is ***periodic (or nearly periodic)***, in its sampling window

All values of a rectangular window ➔ **"1"**

This window is also called a **Box Car** window or **No** window



Rectangular Window.

SPECTRUM AVERAGING

Spectrum averaging is used for two important reasons,

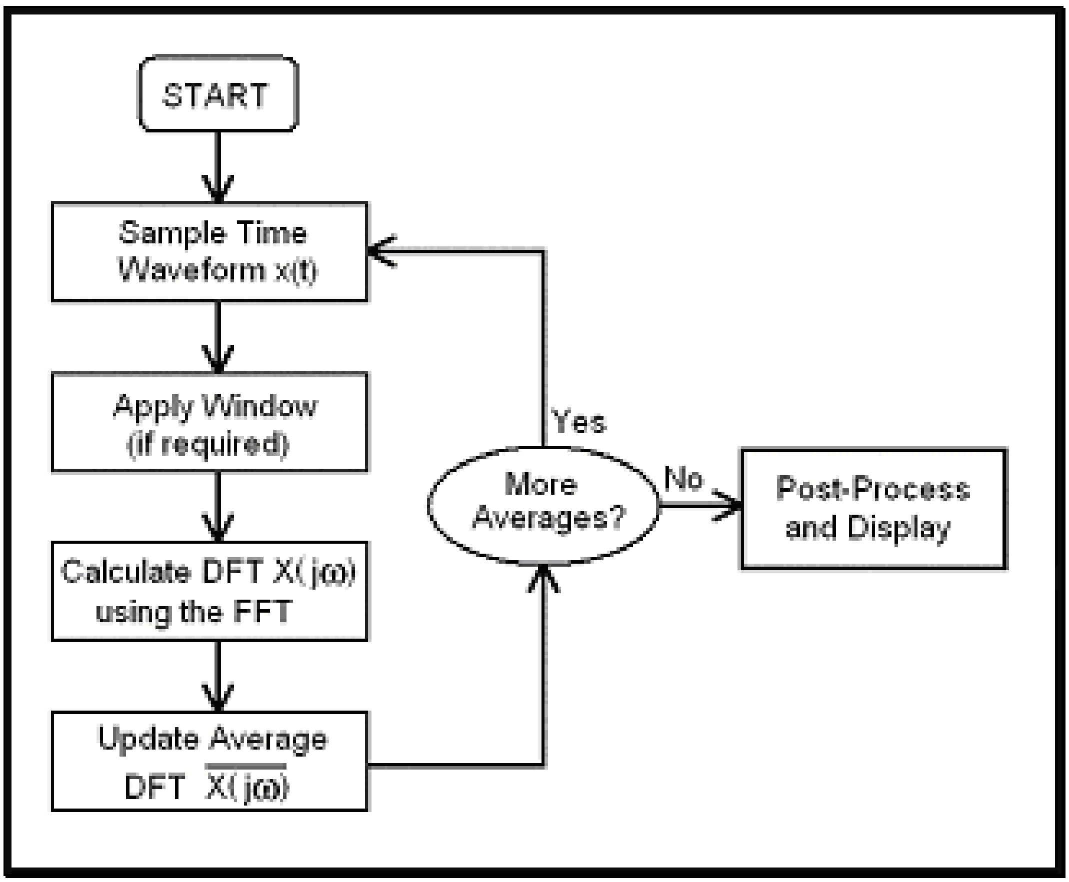
1. ***To remove extraneous random noise*** from the **DFT** of a signal
2. ***To remove randomly excited non-linearities***, which appear as random noise in the **DFT**

The time waveform **Block Size (number of samples)** is twice the **Block Size (number of samples)** in its corresponding DFT

Time Waveform Block Size = 2 DFT Block Size.

The following steps are carried out during spectrum averaging by the **Transform | Spectra** command in the Data Block window.

1. Each time waveform is divided into ***several smaller sampling windows***
2. Each sampling window is ***windowed*** (***multiplied by a time domain window***) to reduce leakage in its spectral estimate
3. Each ***windowed time waveform*** is transformed into its **Digital Fourier Transform (DFT)** using the **FFT**
4. An Auto spectrum estimate is calculated from ***each*** **DFT**
5. Multiple Auto spectrum estimates are ***averaged*** together to yield a single Auto spectrum for each **M#** in the original Data Block



Spectrum Averaging Calculation Loop

NUMBER OF AVERAGES

Depending on the Block Size of the time waveforms in a time domain Data Block, two cases can occur,

DFT Block Size ➔ 1/2 (Time Waveform Block Size)

In this case, only one spectrum estimate can be calculated using ***all the time waveform samples***.

DFT Block Size ➔ less than 1/2 (Time Waveform Block Size)

In this case, a large time domain waveform can be divided into many smaller sampling windows, and spectrum averaging can be performed.

OVERLAP PROCESSING

Overlap processing divides each time waveform into a series of smaller ***overlapping sampling windows***.

The percentage of overlap of the sampling windows depends on three parameters,

1. The **time waveform Block Size** (the total number of time waveform samples)
2. The **spectrum Block Size**
3. The **Number of Spectrum Averages**

Increasing the Number of Spectrum Averages ***increases the percentage*** of overlap processing

50 % Overlap means that ***half of the time waveform samples are used over again*** in each successive sampling window

0% Overlap means that ***unique time waveform samples*** are used for each new sampling window

LINEAR (OR STABLE) AVERAGING

Linear averaging is the same as ***summing together*** all the spectral estimates and ***dividing by the number of averages***.

A ***stable averaging*** formula is used for linear spectrum averaging. Each stable average is calculated using a weighted sum of the current spectrum estimate and the preceding stable average.

The $\mathbf{N^{th}}$ ***stable average*** is calculated with the following formula,

Stable Average (N) = (1/N) Spectrum (N) + (1 - (1/N)) Stable Average(N-1)

PEAK HOLD AVERAGING

Peak Hold averaging ***retains the maximum value*** at each sample from all spectral estimates.

The $\mathbf{I^{th}}$ sample of the $\mathbf{N^{th}}$ average is determined with the formula,

Peak Average (N,I) =Maximum (Spectrum (I) , Peak Average(N-1,I))

STEP 1 - FOURIER SPECTRUM OF PERIODIC SINE WAVES

- ***Press*** **Hotkey 1 Fourier Spectrum of Periodic Sine Waves**

To illustrate the calculation of a Fourier spectrum, a Data Block file with a time waveform containing ***three periodic sine waves*** was created using the **File | New | Data Block** command. The waveform was saved in **BLK: 20 30 50 Hz Sine Waves**.

When **Hotkey 1 is *pressed***, two Data Block windows will open. The Data Block **BLK: 20 30 50 Hz Sine Waves** is displayed ***on the left*** and contains a time waveform with **20,000 samples** of sinusoidal data in it.

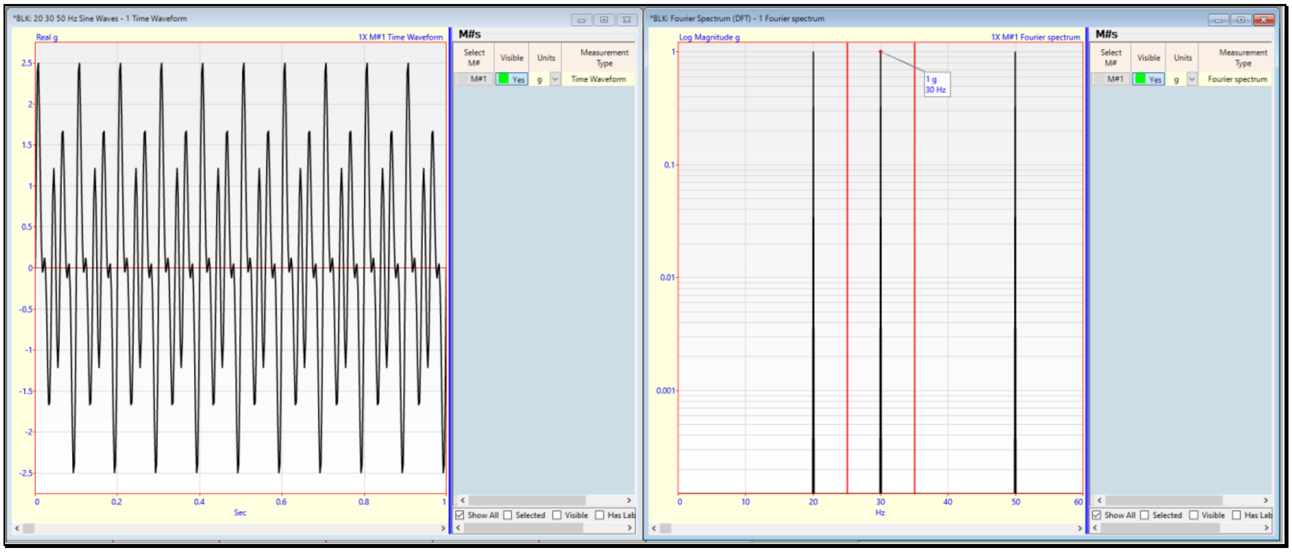
BLK: 20 30 50 Hz Sine Waves contains data for a **T ➔ 50** seconds, but ***only* 0 to 1 seconds** of data is displayed

The Data Block **BLK: Fourier Spectrum (DFT)** is displayed ***on the right*** and contains the Fourier spectrum (**DFT**) of the time waveform in **BLK: 20 30 50 Hz Sine Waves**. The **DFT** has three peaks at **20, 30** & **50 Hz** with **magnitude** = **1** and **phase** = **0**.

BLK: Fourier Spectrum (DFT) has a frequency span ➔ **0** to **200** Hz but ***only* 0 to 60 Hz** is displayed

The ***Peak cursor*** in the Fourier spectrum shows a **magnitude = 1g** for the **30 Hz peak**

The frequencies of the three sine waves (**20, 30, 50**) **Hz** divide evenly into **200 Hz**, hence they are periodic in the 20,000 sample time domain window and there is no leakage in their spectrum



*Fourier Spectrum (**DFT**) of a Periodic Signal Containing Three Sine Waves.*

ONE-SIDED VERSUS TWO-SIDED FFT

The Fourier Transform is defined as an integral over ***all frequencies, positive & negative***.

The **DFT** is also defined over ***all frequencies, positive & negative***.

The spectrum for the negative frequencies has the same information in it as the spectrum for the positive frequencies.

Therefore, ***only the* DFT *for positive frequencies*** is displayed in MEscope.

A **One-Sided FFT** assigns ***all the energy*** from the time waveform to the ***positive frequencies*** of its **DFT** (the part that is displayed)

A Two-Sided **FFT** assigns ***half of the energy*** to the ***positive frequencies*** and half of the energy to the ***negative frequencies*** of its **DFT**

DFT values from a **One-Sided FFT** are ***twice as large*** as the **DFT** values from the **Two-Sided FFT**

STEP 2 - SPECTRUM AVERAGING USING A FLAT TOP WINDOW

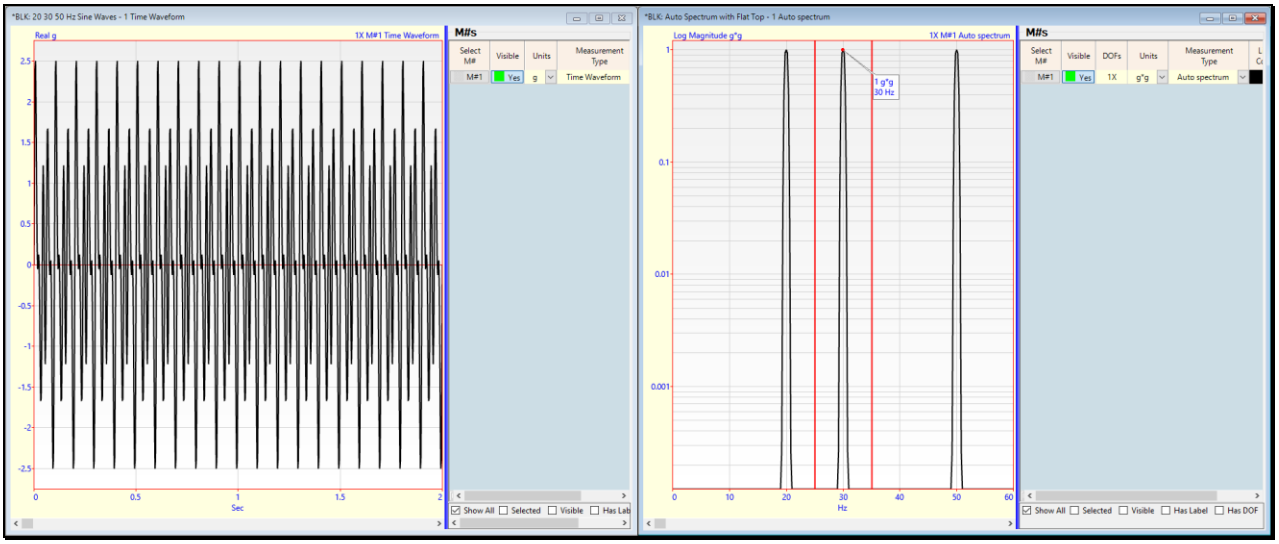
- ***Press* Hotkey 2 Auto Spectrum with Flat Top**

When **Hotkey 2 is *pressed***, two Data Block windows will open. The Data Block **BLK: 20 30 50 Hz Sine Waves** is displayed ***on the left*** and contains a time waveform with **20,000 samples** of sinusoidal data in it.

BLK: 20 30 50 Hz Sine Waves contains data for a **T ➔ 50** seconds, but ***only* 0 to 1 seconds** of data is displayed

The Data Block **BLK: Auto Spectrum** is displayed ***on the right*** contain the Auto spectrum which has three peaks at **20, 30** & **50 Hz** with **magnitude** = **1** & **phase** = **0**.

BLK: Auto Spectrum has a frequency span ➔ **0** to **200** Hz but ***only* 0 to 60 Hz** is displayed



Auto Spectrum of a Periodic Signal Containing Three Sine Waves With a Flat Top Window Applied.

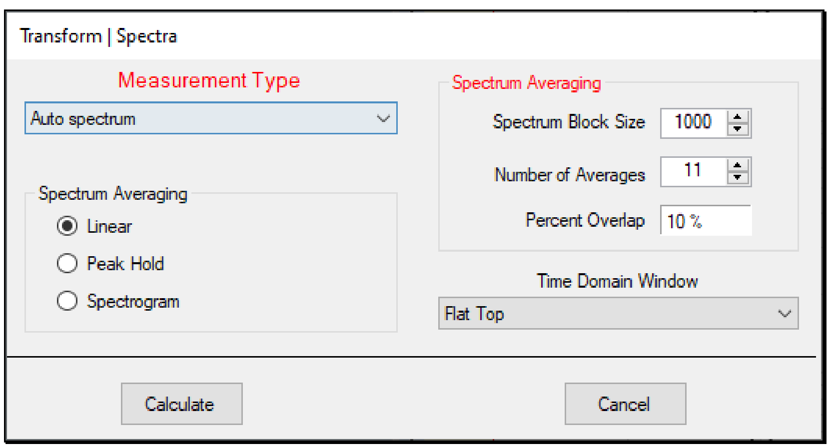
The ***Peak cursor*** value in the Auto spectrum shows the **magnitude of 1 g*g** for the **30 Hz sine wave**.

OVERLAP PROCESSING

Now the calculations done when **Hotkey 2 was *pressed*** will be done manually

- ***Right click*** in Data Block **BLK: 20 30 50 Hz Sine Waves** and execute **Transform | Spectra**

The following dialog box will open.



Transform | Spectra Dialog Box.

- Verify that **Spectrum Block Size ➔ 1000**, **Number of Averages ➔ 11**, and **Percent Overlap ➔ 10%**

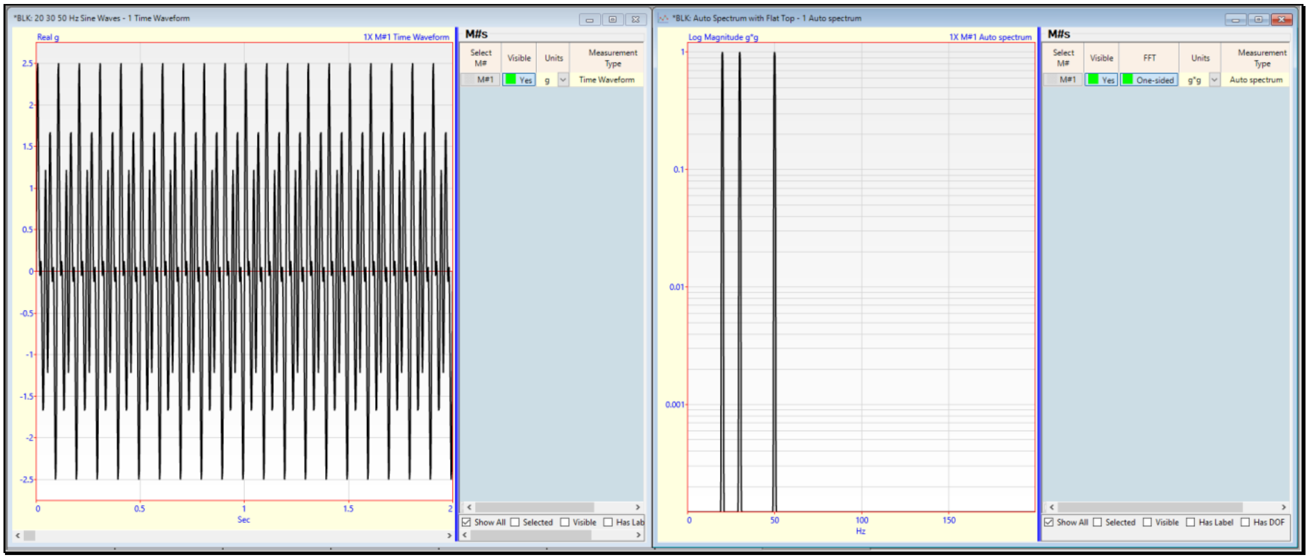
This means that to calculate **11 Auto spectra** and average them together**, *10 percent*** of the time waveform samples will be used over again in ***each successive* sampling window**.

TIME DOMAIN WINDOW

When spectrum averaging is used, data that is ***periodic for all samples*** in a time waveform window ***might not be periodic in each* sampling window**.

Therefore, to preserve the magnitudes of the three sine waves in the Auto Spectrum, a **Flat Top** window will be applied to each sampling window before the **FFT** is used to calculate its **DFT**.

Time Domain Window ➔ Flat Top is also listed in the dialog box above



Auto Spectrum With a Flat Top Window Applied and 11 Spectrum Averages & Overlap Processing.

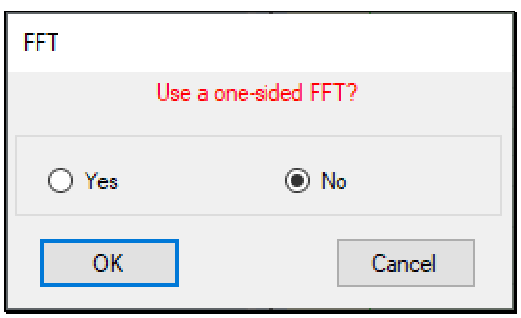
- ***Press* Calculate** in the **Transform | Spectra** dialog box
- Save the Auto Spectrum in **BLK: Auto Spectrum with Flat Top**

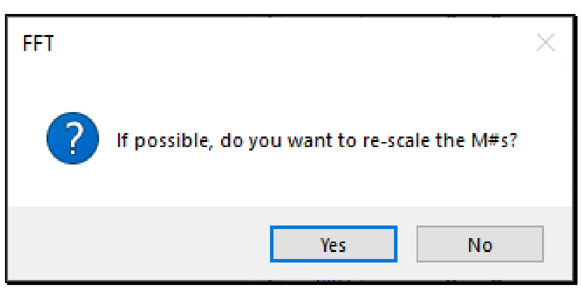
Because the Flat Top window was used, the three sine wave peaks appear at their respective frequencies (**20, 30, 50**) Hz, with magnitudes ➔ **"1 g*g"** in the Auto spectrum. However, compared to the Fourier spectrum, the sine wave peaks ***now have "width" to them***.

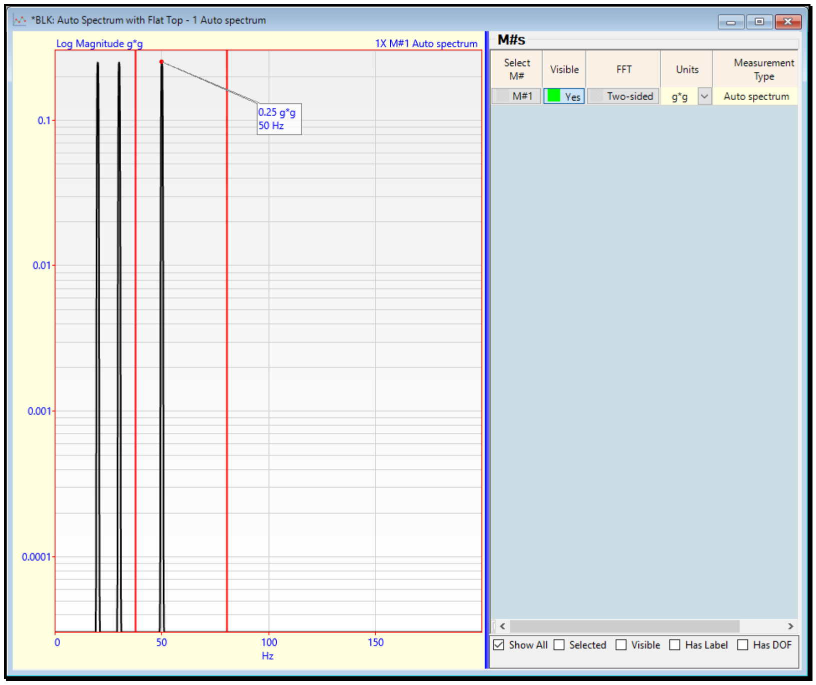
A **Flat Top** window ***preserves peak magnitudes*** but ***increases peak widths***.

TWO-SIDED FFT

- ***Double click*** on the **FFT column** in the **M#s** spreadsheet in Data Block **BLK: Auto Spectrum with Flat Top**
- In the dialog box that opens, choose **No**, ***click*** on **OK**
- Choose **Yes** in the next dialog box to ***re-scale the data***







*Auto Spectrum from Two-Sided **FFT**.*

The magnitude of each spectrum peak in the Auto spectrum is now **0.25 g*g** because half of the sine wave energy has been assigned to the ***negative frequency peaks***.

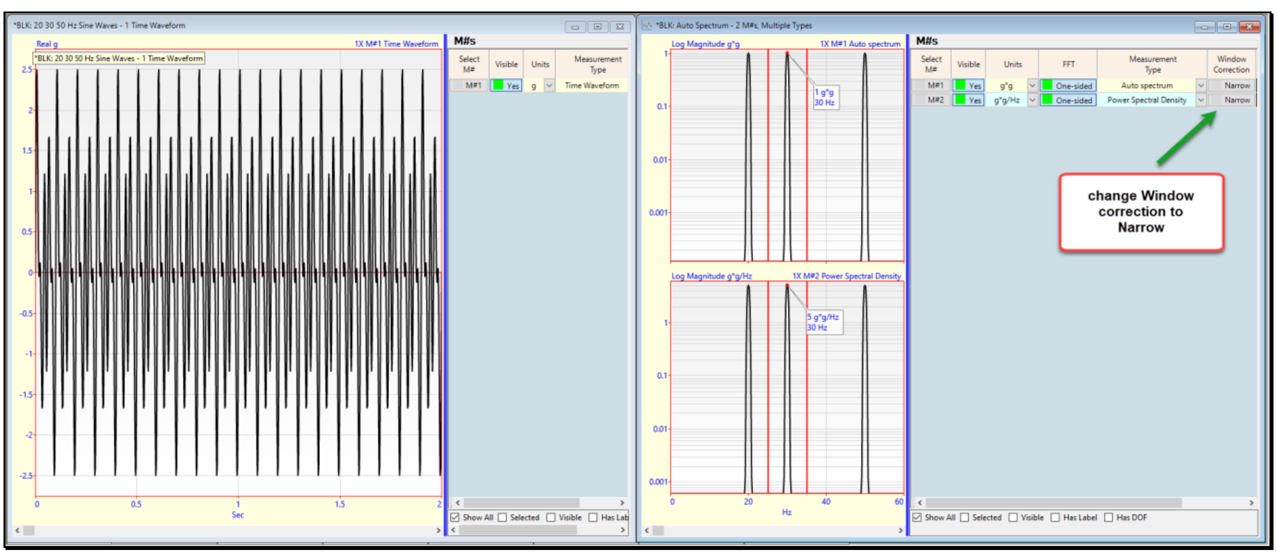
STEP 3 - PSD USING A FLAT TOP WINDOW

- ***Press* Hotkey 3 PSD with Flat Top**

When **Hotkey 3 is *pressed***, the same sinusoidal signal that was used to calculate the Auto spectrum will now be used to calculate a **PSD** and add it to the **BLK: Auto Spectrum with Flat Top** Data Block.

Two Data Block windows will open. The Data Block ***on the left*** shows the same time waveform with **20,000 samples** of sinusoidal time waveform data in it.

The Data Block ***on the right*** shows both the **Auto spectrum** & **PSD** of the waveform on the left, with three peaks at (**20, 30**, **50) Hz**.



*Auto spectrum and **PSD** of the Sinusoidal Waveform.*

The **Peak cursor** in the Auto spectrum shows a magnitude of **1 g*g** for the **30 H**z sine wave

The units of the **PSD** are (**g*g/Hz**) which is a ***power*** (mean squared) quantity.

In the **Window Correction** column of **BLK: Auto Spectrum with Flat Top**, choose **Narrow** for **M#** 2

Answer **Yes** in the dialog that opens to rescale the **PSD**

The **Peak cursor** in the PDS now shows a magnitude of **5 g*g/Hz** for the **30 Hz** sine wave

The magnitudes of the three sine waves are **1 g** of the original time waveform**.** Therefore, the three Auto spectrum peaks have magnitude ➔ **1 g*g**.

A **PSD** is an Auto spectrum ***"normalized by"*** (divided by) the **frequency resolution (Δf)** of the spectrum.

- Execute **File | Properties** in the **BLK: Auto Spectrum with Flat Top** window

Frequency Resolution ➔ 0.2 Hz

Therefore, the **PSD** peaks should be **5** (**g*g/Hz**), which is confirmed by the cursor value on the **PSD** (**M#2**) in the Data Block window shown above.

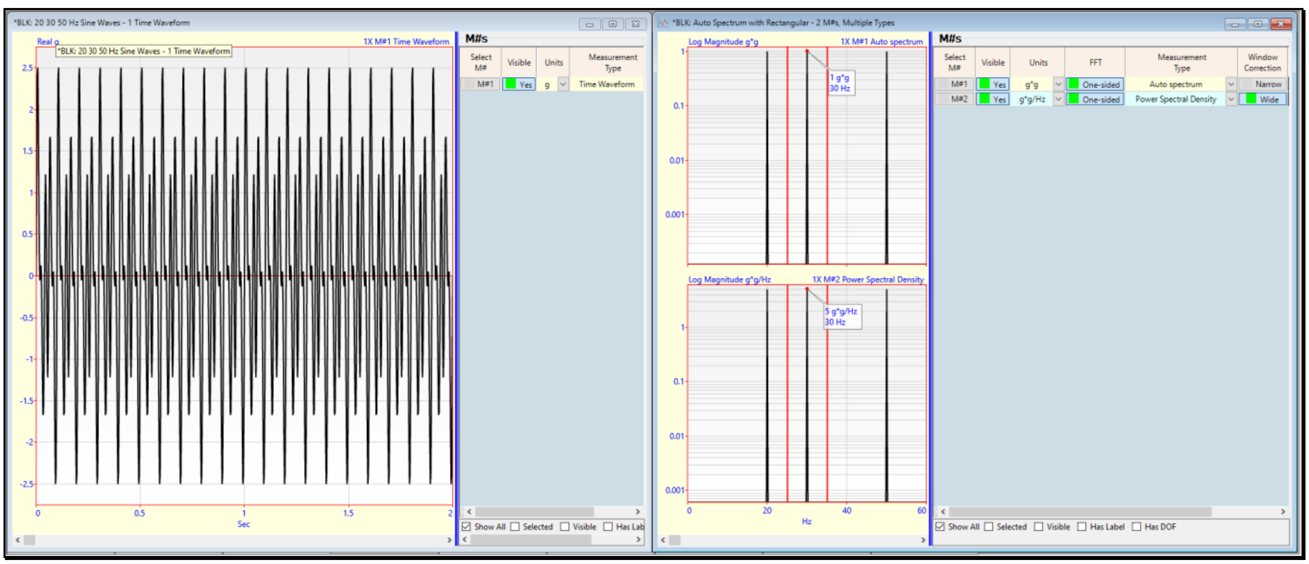
STEP 4 - SPECTRUM AVERAGING USING A RECTANGULAR WINDOW

- **Press Hotkey 4 Auto Spectrum with Rectangular**

When **Hotkey 4 is *pressed***, the same sinusoidal signal that was used to calculate the Auto spectrum will now be used to calculate a **PSD** and add it to the **BLK: Auto Spectrum with Flat Top** Data Block.

Two Data Block windows will open. The Data Block ***on the left*** shows the same time waveform with **20,000 samples** of sinusoidal time waveform data in it.

The Data Block ***on the right*** again shows both the **Auto spectrum** & **PSD** of the time waveform ***on the left***, with three peaks at (**20, 30**, **50) Hz**.



*Auto spectrum & **PSD** of a Periodic Signal Using a Rectangular Window.*

The narrow peaks at the three frequencies at (**20, 30**, **50) Hz** in both the **Auto spectrum** & **PSD** verify that the time waveform remained ***periodic in each*** **sampling window** when **11 averages** were calculated with **10% overlap processing**.

CONCLUSION

In **Steps 2 &** 3, a **Flat Top** window was applied to the time waveforms in each sampling window to obtain ***accurate magnitudes*** in each spectral estimate**.** An Auto spectrum and **PSD** were calculated from the time waveform in **BLK: 20 30 50 Hz Sine Waves** using the following parameters

Spectrum Block Size ➔ 1000 samples

Number of Averages ➔ 11

Overlap processing ➔ 10%

Time Domain Window ➔ Flat Top

The time waveform in **BLK: 20 30 50 Hz Sine Waves** contains **20000 samples** over a time of **T ➔ 50 seconds**.

With a **Spectrum Block Size ➔ 1000** samples, each time domain ***sampling window contains 2000 samples*** over a time of **T ➔ 5 seconds**.

With these sampling parameters, each sinusoidal waveform ***is still periodic in its*** **sampling window**.

- The three sine waves ***complete exactly*** **100 (20 Hz), 150 (30 Hz), 250 (50 Hz**) cycles in **T ➔ 5 seconds**
- With overlap processing, the next sampling window starts after **10 (20 Hz), 15 (30 Hz), 25 (50 Hz)** cycles of each sine wave
- ***No leakage*** will occur in the calculated **DFT**, and therefore the **Auto spectrum** & **PSD** ***have no leakage***

Because each sinusoidal waveform ***is periodic in its*** **sampling window** a **Rectangular** window can also be used instead of a **Flat Top** window.

STEP 5 - REVIEW STEPS

To review the steps of this App Note,

- ***Press*** **Hotkey 5 Review Steps**