MEscope Application Note 53

Sinusoidal Response is a Summation of Mode Shapes

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing & VES-4600 Advanced Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the **AppNote53** project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 53 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote53.zip**

This Project contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click* on a Hotkey** to open its Script window

INTRODUCTION

This App Note illustrates a fundamental law of modal analysis,

Fundamental Law of Modal Analysis: *All vibration* is a *summation of mode shapes*.

The modal parameters of a structure can be obtained in two ways,

- **Experimental Modal Analysis (EMA): EMA** mode shapes are extracted by curve fitting a set of experimentally derived time waveforms or frequency spectra that characterize the structural dynamics
- **Finite Element Analysis (FEA): FEA** mode shapes are calculated from a set of differential equations that characterize the structural dynamics

In this App Note analytical **FEA** mode shapes are used to **curve fit and expand** sinusoidal response data to include **DOFs** that were not included in the original data. This approach has two advantages,

Only the **FEA** mode shapes **are required**. Modal frequency & damping **are not required**.

FEA mode shapes derived from an **FEA** model with **free-free boundary conditions** and **no damping** can be used to curve fit experimental data from a real-world structure with **damping and real-world boundary conditions**.

Unlike the **curve fitting** used to extract **EMA** mode shapes from experimental data, in this process the unknown **participation of each mode shape** in the sinusoidal data is determined by a different **least-squared-error curve fitting** process**.** The participation of each mode shape is then used to **expand the sinusoidal data** to include the **DOFs** of the **FEA** mode shapes.

EXPANDING SINUSOIDAL RESPONSES

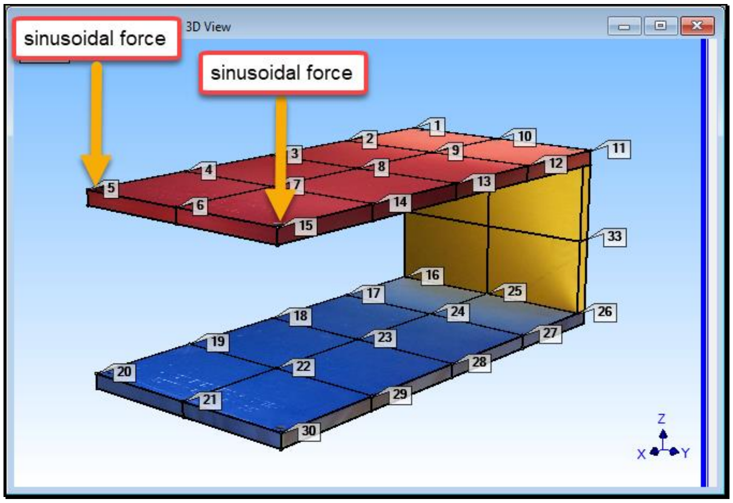
In this App Note, **Multi-Input Multi-Output (MIMO)** modeling is used to calculate responses to two sinusoidal excitation forces applied to the Jim Beam structure shown below. \

Multi-Input Multi-Output (MIMO) modeling is part of the **VES-3600 Advanced Signal Processing** option.

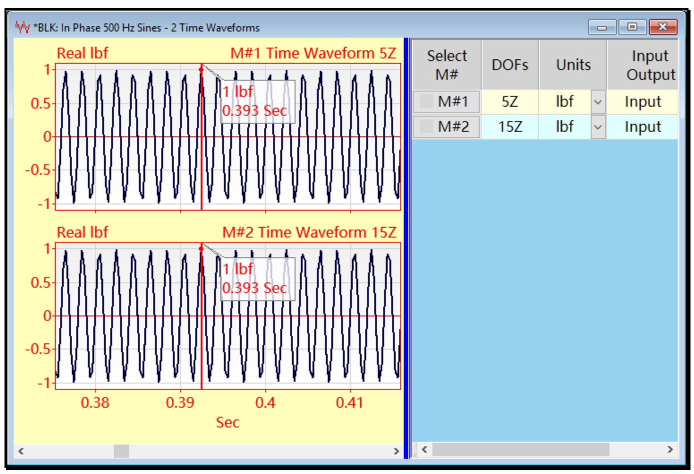
The forces are applied on the top plate of the Jim Beam at points 5 & 15 in the Z-direction, or at **DOFs 5Z & 15 Z**. Two cases are simulated,

- Two **In-Phase 500 Hz** sinusoidal excitation forces
- Two **Out-of-Phase 500 Hz** sinusoidal excitation forces

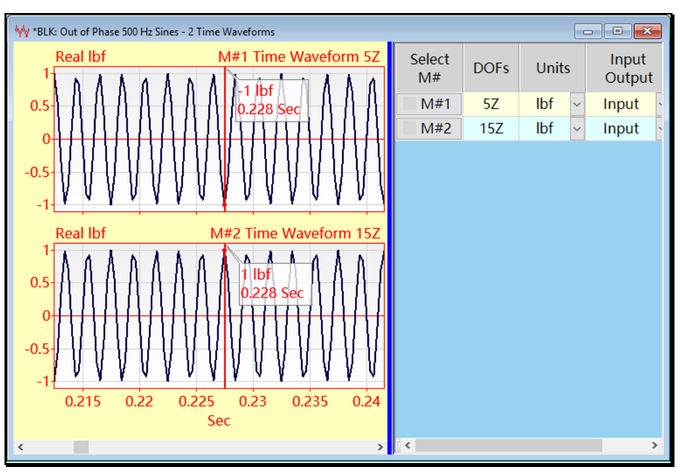
The sinusoidal **In-Phase & Out-of-Phase** force waveforms are also shown below. Each time waveform contains **5000 samples**. The cursor values of the **In-Phase forces** show that those two forces are in-phase with one another. The cursor values of the **Out-of-Phase forces** show that those two forces are out-of-phase with one another. These waveforms were created with the **File | New | Data Block** command.



*500 Hz Sinusoidal Excitation at **DOFs 5Z &15Z**.*



*In-Phase Excitation at **DOFs 5Z & 15Z**.*



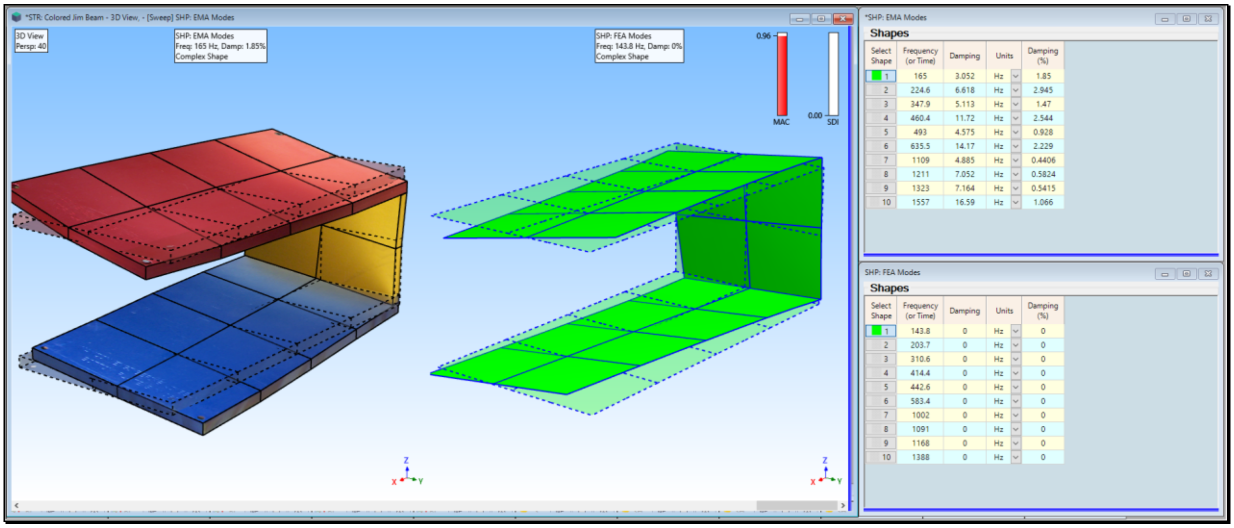
*Out-of-Phase Excitation at **DOFs 5Z & 15Z**.*

STEP 1 - EMA VERSUS FEA MODE SHAPES

- ***Press* Hotkey 1 EMA vs. FEA Mode Shapes**

When **Hotkey 1 is *pressed***, sweep animation through the mode shapes will begin, and each **EMA** mode shape and its ***closest matching*** **FEA** mode shape are displayed together side-by-side. The **MAC** value between each matching pair of mode shapes is also displayed in the ***upper right corner***.

The **Animate | Compare Shapes | Maximum MAC** command has been ***checked*** by the Script so each pair of mode shapes with the **Maximum MAC** value is displayed side-by-side.



Side-by-Side Display of a Matching Pair of EMA & FEA mode shapes.

MODAL ASSURANCE CRITERION (MAC) & SHAPE DIFFERENCE INDICATOR (SDI)

MAC is a measure of the **co-linearity** of two shapes. **SDI** is a measure of the difference between two shapes.

- **MAC & SDI** values range between **0 & 1**
- **MAC = 1** ➔ two shapes are **co-linear**
- **MAC** > **0.9** ➔ two shapes **are similar**
- **SDI = 1** ➔ two shapes **have identical components**
- **SDI** > **0.9** ➔ two shapes **have similar component values**

The **high MAC values** indicate that all pairs shapes **are nearly co-linear**. The **low SDI values** indicate that the FEA & EMA shape values are different for each matching pair.

Two of the **EMA** mode shapes have modal frequencies **close to 500 Hz**. Intuitively speaking, the mode shapes that are **closest to 500 Hz** would be expected **to participate in the response** of a structure to **500 Hz sinusoidal excitation**.

The **493 Hz** mode shape is closest to **500 Hz** so it should **dominate the response** to **500 Hz** sinusoidal excitation.

We will see that the **460 Hz** mode shape **will dominate the response** depending on the phases of the excitation forces.

MODAL PARTICIPATION

Mode shapes participate in the response to sinusoidal excitation depending on,

- The **phases of sinusoidal excitation** forces
- The **phases of the mode shape components** at the excitation **DOFs**

SAME MODE SHAPES BUT DIFFERENT FREQUENCIES

All matching pairs of the **EMA** & **FEA** mode shapes **have high MAC values**, and therefore are **co-linear**. But what about their modal frequencies? The modal frequencies of the **FEA** & **EMA** modes are listed below.

The frequency of each **FEA** mode shape **is much less than** the frequency of its matching **EMA** mode shape.

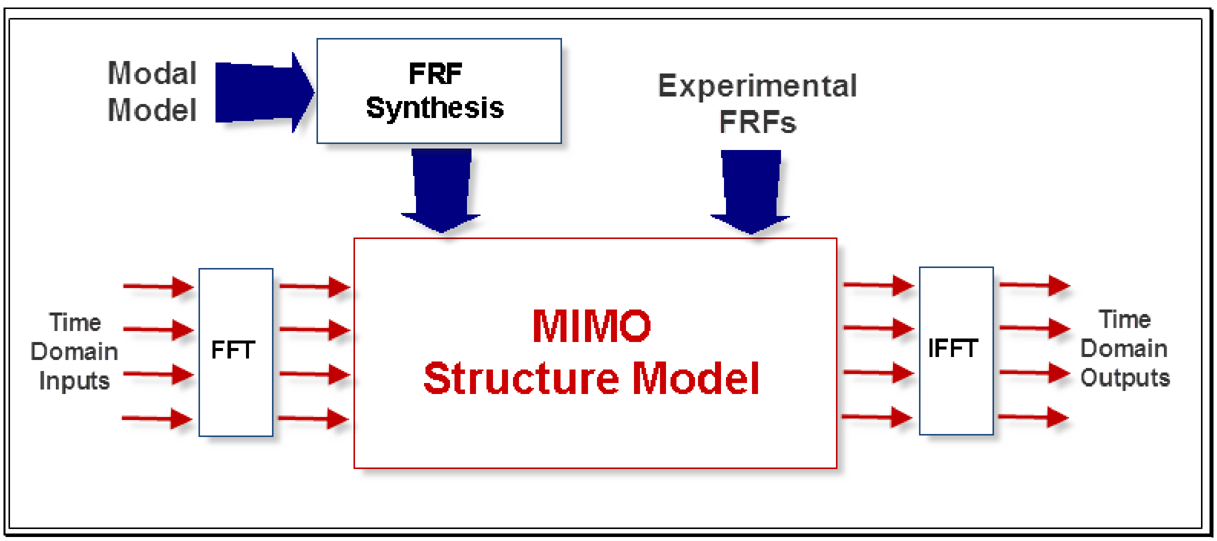
Mode Shape Pair	EMA Frequency (Hz)	EMA Damping (Hz)	FEA Frequency (Hz)	FEA Damping (Hz)	MAC
1	165.0	3.052	143.8	0.0	0.96
2	224.6	6.618	203.7	0.0	0.96
3	347.9	5.113	310.6	0.0	0.95
4	460.4	11.72	414.4	0.0	0.93
5	493.0	4.575	442.6	0.0	0.95
6	635.5	14.17	583.4	0.0	0.93
6	1109.0	4.885	1002.0	0.0	0.90
8	1211.0	7.052	1091.0	0.0	0.89
9	1323.0	7.164	1168.0	0.0	0.86
10	1557.0	16.59	1388.0	0.0	0.83

*Comparison of **EMA & FEA** Mode Shapes.*

STEP 2 - RESPONSE TO IN-PHASE SINUSOIDAL FORCES

- **Press Hotkey 2 In-Phase Sinusoidal Response**

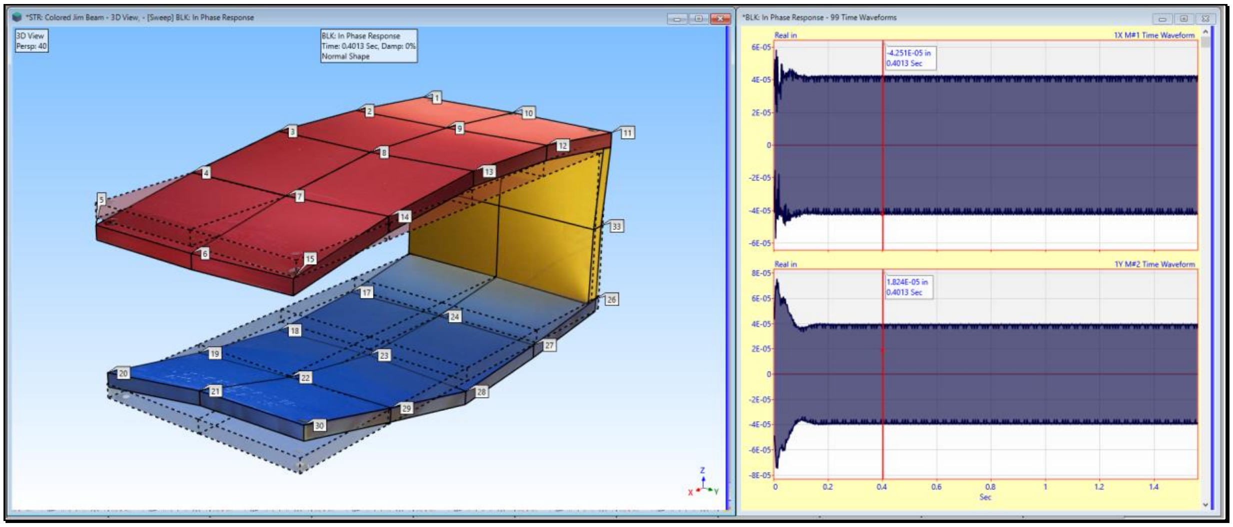
When **Hotkey 2 is *pressed***, the simulated response of the Jim Beam structure to **two In-Phase 500 Hz forces** is calculated and displayed. The MIMO calculation process is depicted below.



Using the MIMO calculations indicated above, the **Transform | Outputs** command carries out the following steps to calculate the sinusoidal response time waveforms,

1. Convert the **EMA UMM** mode shapes to **Residue** mode shapes with **Reference DOFs 5Z & 15Z**
2. Use the **Residue** mode shapes to synthesize **FRFs** with **99 Roving DOFs** and **Reference DOFs 5Z & 15Z**
3. Calculate the **Digital Fourier Transform (DFT)** of each **In-Phase 500 Hz** sine waveform using the **FFT**
4. Multiply the MIMO Structure Model (**FRFs**) by the **DFTs** of the sine wave **Inputs**
5. **Inverse FFT** the **DFTs** of the **Outputs** to obtain the sinusoidal response time waveforms

After the responses are calculated, sweep animation is begun through the Data Block of sinusoidal responses, as shown below. The 3D model of the Jim Beam is deflected using the 3D response data for **99 DOFs**.



*Response to Two 500 Hz In-Phase Sinusoidal Forces Applied at **DOFs 5Z & 15Z**.*

STEP 3 - EXPANDED RESPONSE TO THE IN-PHASE FORCES

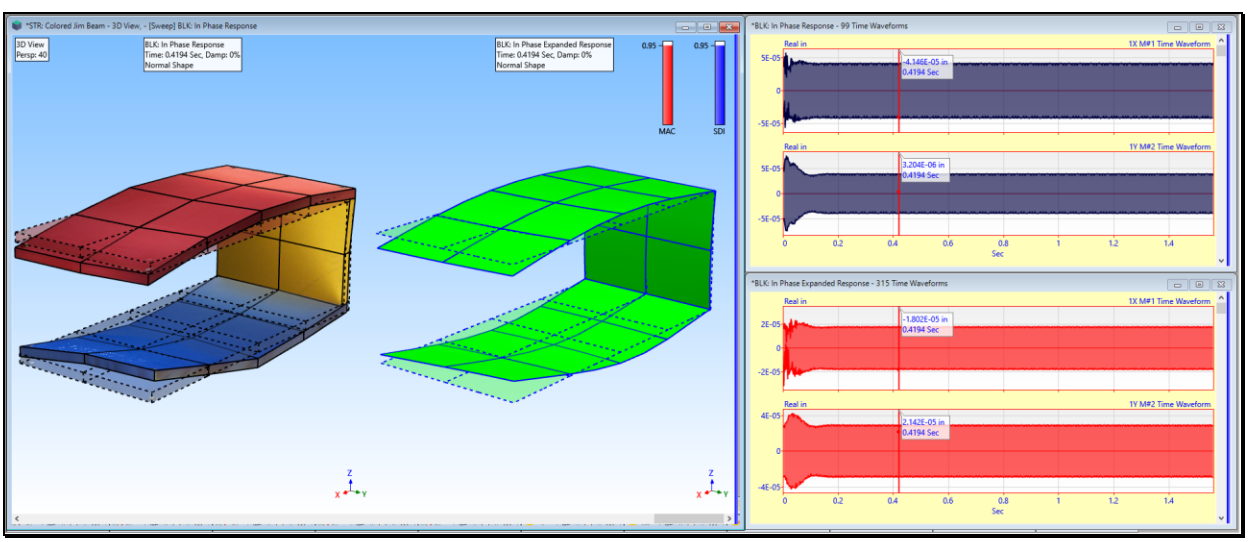
- ***Press* Hotkey 3 In-Phase Expanded Response**

In this step, the **FEA** mode shapes are used to **curve fit** the In-Phase responses that were calculated in the previous step and expand them from **99 to 315 DOFs**. This is done by executing the **Tools | M# Expansion** command in the Data Block **BLK: In-Phase Responses**, the Data Block with the 99 sinusoidal responses calculated in the **Step 2**.

When **Hotkey 3 is *pressed***, a side-by-side comparison of the **time-based ODS** from the **99 DOF** response and the **315 DOF** expanded response are displayed as shown below. The **99 DOF** response time waveforms are displayed on the ***upper right***, and the **315 DOF** expanded response time waveforms are displayed on the ***lower right***.

The time-based **99 DOF ODS** at the **current Line cursor position** is displayed on the ***left-hand*** Jim Beam model, and the expanded **315 DOF ODS** from the **same cursor position** in the expanded responses is displayed on the ***right-hand*** **FEA model**.

- The **MAC** bar between the two time-based ODS's is displayed in the ***upper right corner*** of the display
- **MAC > 0.9** at each time sample (except near zero amplitudes) ➔ the two ODS's are **nearly co-linear**, meaning that they are **essentially the same ODS**



99-DOF vs. 315-DOF Responses to the Two 500 Hz In-Phase Sinusoidal Forces.

STEP 4 - IN-PHASE RESPONSE CORRELATION & MODE SHAPE PARTICIPATION

To review the calculations of the previous two Steps,

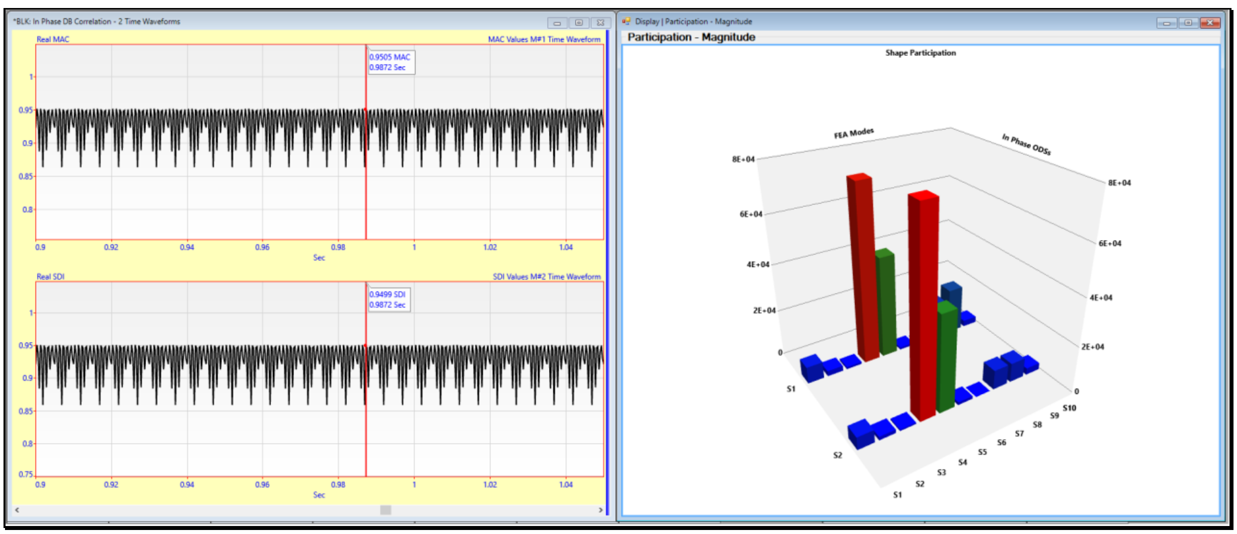
- In **STEP 2** ➔ The **Transform | Outputs** command was used to calculate **99-DOF** responses to two **In-Phase** sinusoidal force Inputs, and the **EMA** mode shapes were used to synthesize **FRF**s for the MIMO Structure Model
- In **STEP 3**, ➔ The **Tools | Shape Expansion** command was used together with the **FEA** mode shapes to expand the **99-DOF** responses from **STEP 2** into **315-DOF** responses
- ***Press* Hotkey 4 In-Phase Response Correlation**

When **Hotkey 4 is *pressed***, the Data Block containing the **99-DOF response time waveforms** and the Data Block containing the **315-DOF response time waveforms** are correlated at each sample using the **Tools | Data Block Correlation** command. A new Data Block of **MAC & SDI** values is shown ***on the right below***.

The **Tools | Data Block Correlation** command calculates a **MAC & SDI** value for each sample of ODS data in two two Data Blocks.

When **Hotkey 4 is *pressed***, the ***right upper graph*** is the **MAC value at each sample** between the **99-DOF time-based ODS** and the **315-DOF time-based ODS**. The ***right lower graph*** is the **SDI value at the same sample** between **99-DOF time-based ODS** and the **315-DOF time-based ODS**.

Both **MAC & SDI** indicate that the time-based ODS's at the same sample **are nearly identica**l for all **5000 samples** in both Data Blocks of time waveforms.



Data Block Correlation & Mode Shape Participation in the Responses to the In-Phase Forces.

MODE SHAPE PARTICIPATION IN THE IN-PHASE RESPONSES

The participation of each **FEA** mode shape in the two time-based ODS's at a single sample is shown on the ***right-hand side*** of the figure above.

- **S1** is the **99-DOF** In-Phase response ODS
- **S2** is the **315-DOF** In-Phase response ODS

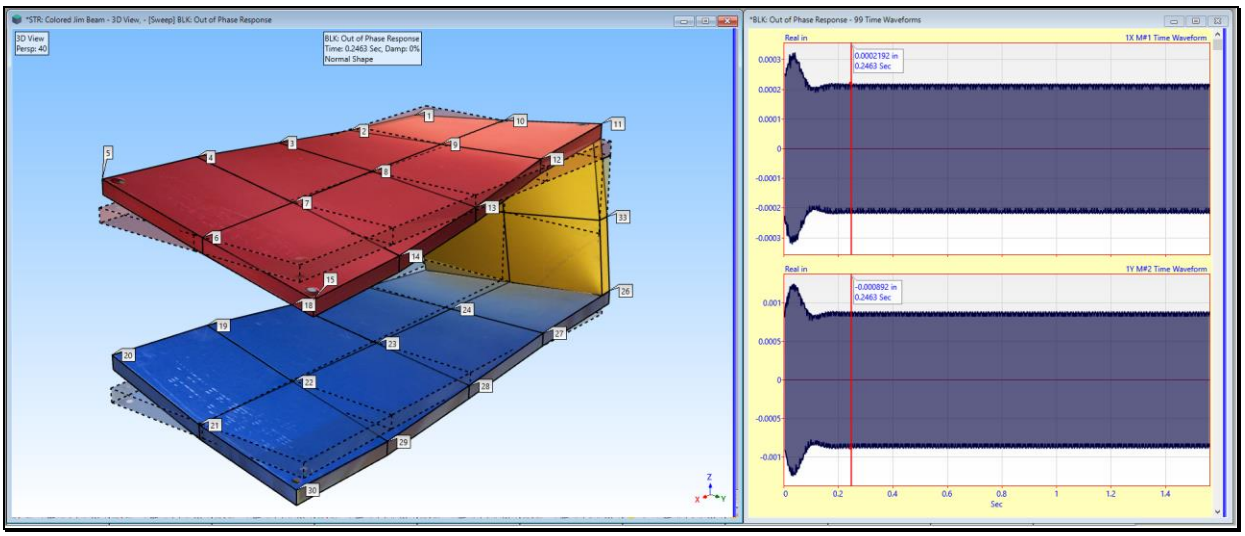
The bar chart shows the following participation of the **FEA** mode shapes in the time-based ODS's

- **FEA** mode shape **#4 dominates** the time-based ODS's because **its mode shape components are in-phase at DOFs 5Z & 15Z** where the **In-Phase forces** are applied
- **FEA** mode shape **#5 participates significantly** in the time-based ODS's because the **493 Hz EMA frequency is close to the 500 Hz excitation frequency**

STEP 5 - RESPONSE TO OUT-OF-PHASE SINUSOIDAL FORCES

- ***Press*** **Hotkey 5 Out-Of-Phase Sinusoidal Response**

When **Hotkey 5 is *pressed***, the **Transform | Outputs** command is used to calculate the response of the Jim Beam structure to **two Out-of-Phase 500 Hz forces**. After the responses are calculated, sweep animation is begun through the Data Block of sinusoidal responses, as shown below. The Jim Beam model is deflected using 3D response data at **99 DOFs**.



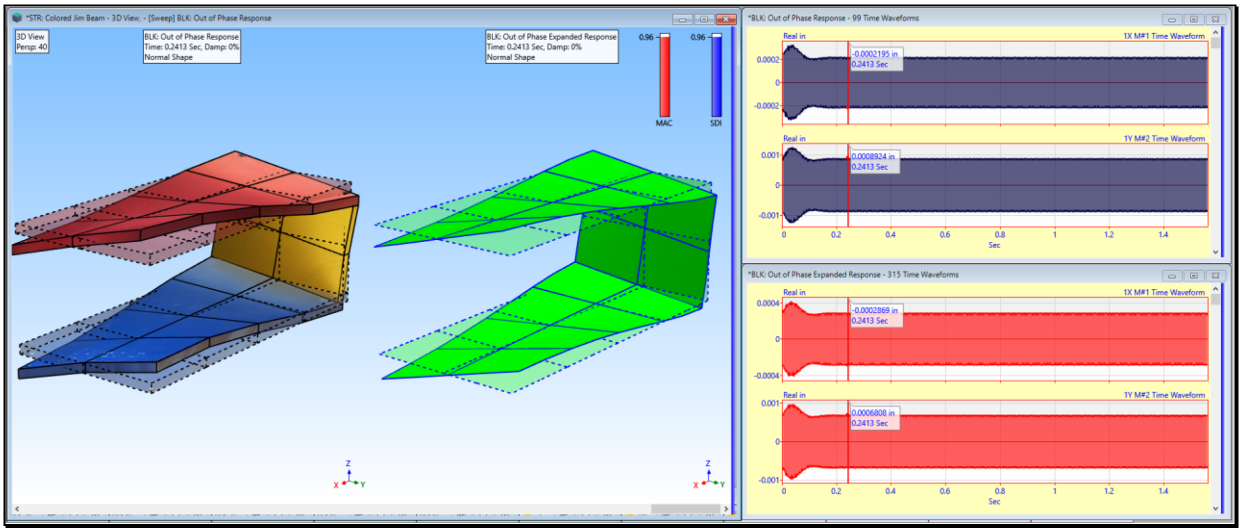
*Response to Two 500 Hz Out-of-Phase Forces Applied at **DOFs 5Z & 15Z**.*

STEP 6 - EXPANDED RESPONSE TO OUT-OF-PHASE FORCES

- ***Press* Hotkey 6 Out-of-Phase Expanded Response**

When **Hotkey 6 is *pressed***, the **FEA** mode shapes are used to **curve fit and expand** the 99-DOF out-of-phase responses from **99 to 315 DOFs**. This is done by executing the **Tools | M# Expansion** command in the Data Block **BLK: Out-of-Phase Responses**.

A side-by -side sweep animation of ODS's from the **99-DOF & 315-DOF** response Data Blocks is begun, as shown below.



99-DOF vs. 315-DOF Responses to the Two 500 Hz Out-of-Phase Sinusoidal Forces.

The **99 DOF** response time waveforms are displayed on the ***upper right***, and the **315 DOF** expanded response time waveforms are displayed on the ***lower right***. The time-based **99 DOF ODS** at the **Line cursor position** is displayed on the ***left-hand* Jim Beam model**, and the expanded **315 DOF ODS** from **the same cursor position** is displayed on the ***right-hand* FEA model**.

The **MAC** bar between the two time-based ODS's is also displayed in the ***upper right corner*** of the display.

- **MAC > 0.9** at each time sample (except near zero amplitude) ➔ the two ODS's are **nearly co-linear**, meaning that they are **essentially the same shape**

STEP 7 – OUT-OF-PHASE RESPONSE CORRELATION & MODE SHAPE PARTICIPATION

- ***Press* Hotkey 7 Out-of-Phase Response Correlation**

In the step, the time-based ODS at each sample in the Data Block containing the **99-DOF** calculated responses is correlated with the time-based ODS at the same sample in the Data Block containing **315-DOF expanded responses**.

The ***upper right graph*** is the **MAC *values*** between each sample of the **99-DOF** time waveforms versus the same sample of the **315-DOF** time waveforms. The ***lower right graph*** is the **SDI** values between the same samples in each Data Block.

Both **MAC & SDI** indicate that the time-based ODS's at the same sample **are nearly identica**l for all **5000 samples** in both Data Blocks of time waveforms.

FEA MODE SHAPE PARTICIPATION IN OUT-OF-PHASE RESPONSES

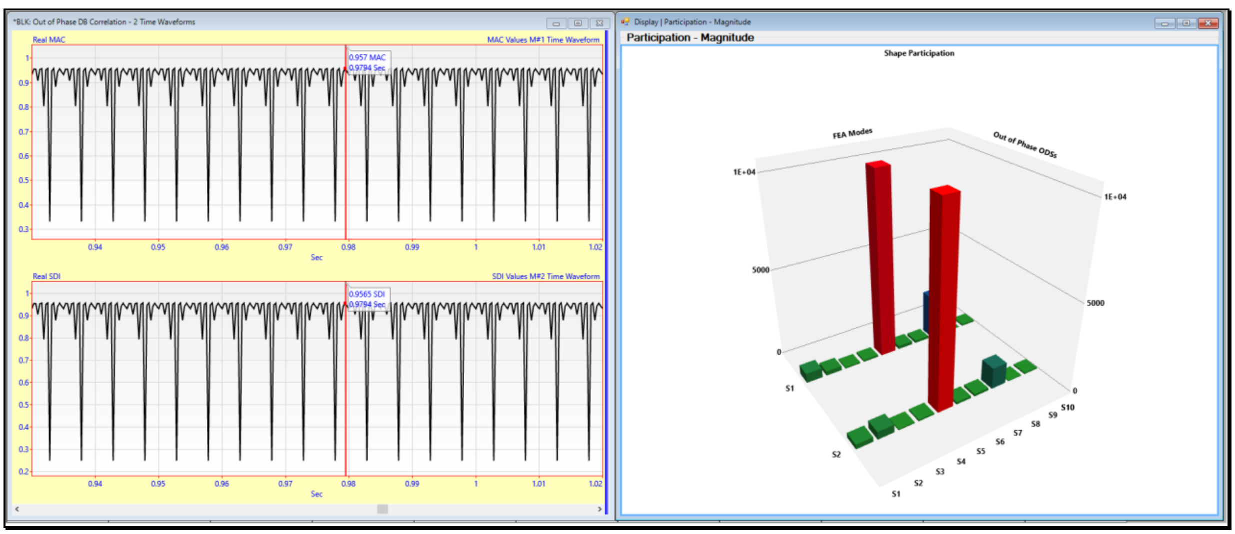
The participation of each **FEA** mode shape in the time-based ODS's **at the same time sample** in both the **99-DOF & 315-DOF** response Data Blocks is shown on the ***right-hand side*** of the figure below.

- **S1** is the **99-DOF** Out-of-Phase Response ODS
- **S2** is the **315-DOF** Out-of-Phase Response ODS

The bar chart shows the participation of the **FEA** mode shapes in the time-based ODS's at **time sample → 0.98 seconds**, where **MAC & SDI** are at maximum values.

Mode shape #5 dominates the participation in both time waveforms.

Mode shape # 5 is expected to dominate the participation because its mode shape components match the two Out-of-Phase excitation forces **at DOFs 5Z & 15Z**.



Data Block Correlation & Mode Shape Participation in the Responses to Two Out-of-Phase Forces.

STEP 8 - REVIEW

To review all the steps of this App Note,

- ***Press* Hotkey 8 Review**

CONCLUSIONS

In this App Note, **10 FEA** mode shapes were used to **curve fit and expand** responses to two **500 Hz** sinusoidal excitation forces applied to corners of the top plate of the Jim Beam test article.

The **FEA** mode shapes were calculated from an **FEA** model of the Jim Beam with **free-free boundary conditions and no damping.**

This App Note confirms the following law when applied to vibration time waveforms,

Fundamental Law of Modal Analysis: *All vibration* is a *summation of mode shapes*.

Expansion of time or frequency waveforms using **FEA** mode shapes offers some important advantages,

1. Real valued time waveforms can be **accurately curve fit and expanded** using **FEA** mode shapes
2. **Normal FEA** mode shapes derived from an **FEA** model with **free-free boundary conditions and no damping** can be used to curve fit real-world vibration data which **always includes real-world boundary conditions & damping**
3. Modal frequencies of the **FEA** mode shapes **are not used** in the calculations

An intuitive belief about modes is that the resonance **with the closest natural frequency** to a sinusoidal excitation force **will always dominate** the vibration response.

But it was shown that the mode shape **that dominates** the response is the one with mode shape components **that more closely align** with the phases of the excitation forces at the **DOFs** where they are applied to the structure.

It was demonstrated that when **two In-Phase 500 Hz sinusoidal forces** are applied to the Jim Beam, the mode shape at **460 Hz dominated the response** because its **mode shape matched the In-Phase forces** better than the **493 Hz** mode shape at, which was **only 7 Hz below the 500 Hz excitation frequency**.

Applying this curve fitting and expansion approach to any time or frequency-based vibration data can provide more complete information for identifying the resonance vibration properties of a machine or structure.

Shape expansion can be used to calculate ODS & mode shape data for ***un-measured*** portions of a machine or structure.