VIBRANT

MEscope Application Note 26

Calculating a 4-Pole Transfer Function Matrix

INTRODUCTION

One way of defining the vibration properties between an input DOF (degree-of-freedom or point & direction) and an output DOF of a structure is to calculate the 4-Pole Transfer Function Matrix between the two DOFs. A 4-Pole Transfer Function Matrix contains the 4 transfer functions that relate two inputs to two outputs.

The block diagram in Figure 1 depicts the 4-Pole Transfer Function matrix describing the dynamic properties of the "black box", with 2 inputs and 2 outputs. The input and outputs can be forces or motions, (acceleration, velocity, or displacement), and they can be either measured or calculated.

A 4-Pole Transfer Function matrix model can be used to characterize the dynamic properties of a vibration isolator, or any vibration path between two DOFs of a structure. It is relatively straightforward to use two impedance heads to measure the input force & acceleration and the output force & acceleration across a vibration isolator when broad-band excitation is applied to it.

In this Application Note synthesized broad-band random signals will be applied to two DOFs of a real structure, and its corresponding acceleration responses at the same two DOFs will be calculated. Then the force & acceleration signals at one DOF will be chosen as the inputs and other force & acceleration as outputs of the ***simulated*** vibration isolator. These signal pairs will then be used to calculate the 4-Pole Transfer function matrix between the inputs and outputs.

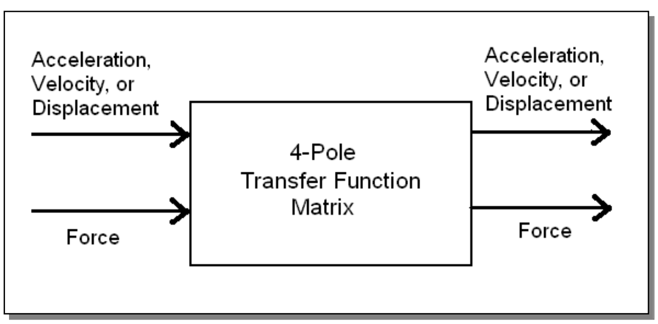


Figure 1. Block Diagram of 4-Pole Transfer Function Matrix.

An example 4-Pole Transfer Function matrix is shown in Figure 2.

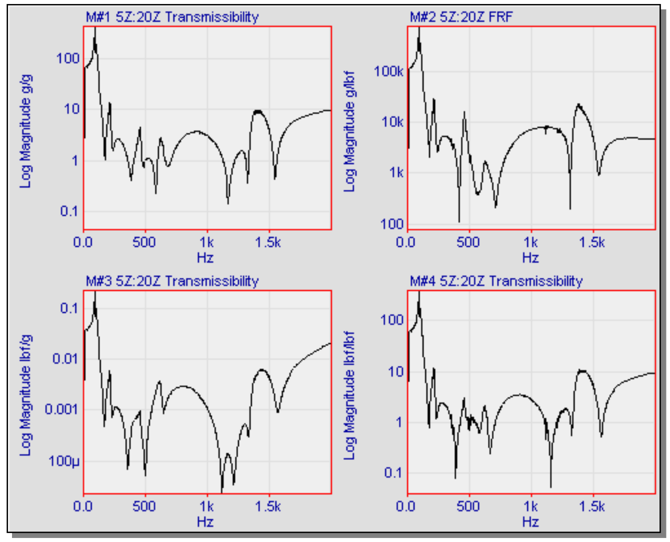


Figure 2. 4-Pole Transfer Function Matrix.

Each transfer function in the matrix defines the dynamic characteristics between an input and an output, as a function of frequency. Notice that the units of each transfer function indicate that it defines a different input-output relationship;

M#1, with units of **g/g,** is the **transmissibility** between the input acceleration (denominator) and the output acceleration (numerator)

M#2, with units of **g/lbf**, is an **FRF** relating the input force to the output acceleration. (This is also called the **inertance**.)

M#3, with units of **lbf/g**, is the **transmissibility** relating the input acceleration to the output force. (This is also called the **dynamic mass**.)

M#4, with units of **lbf/lbf**, is the **transmissibility** between the input force (denominator) and the output force (numerator).

MEASURING FORCE & ACCELERATION.

One advantage of the 4-Pole Transfer Function matrix is that it can be easily measured using two Impedance Heads, one at the input DOF (point & direction) and one at the output. An Impedance Head contains both an accelerometer and a force gage, or load cell. Therefore, it simultaneously measures both force & acceleration at the DOF where it is located.

A 4-Pole Transfer Function matrix can be used to specify how much force & motion is transmitted through a vibration isolator, In other words, this matrix defines how well force & motion are suppressed by the isolator, as a function of frequency.

The 4-Pole Transfer Function matrix is a special case of a Multi-Input Multi-Output (MIMO) dynamic model that relates the two inputs to the two outputs. It is calculated using the MIMO calculation capability in MEscope.

In general, to model the dynamics between **M** Inputs and **N** Outputs, the MIMO model is expressed mathematically as,

$$\{\text{Outputs}\} = [\text{Transfer Matrix}]\{\text{Inputs}\}$$

Where:

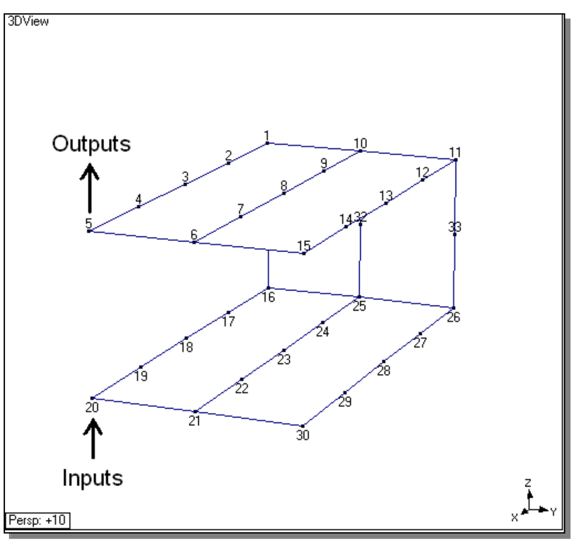
$$\{\text{Outputs}\} = \text{N} - \text{Vector of Outputs}$$
$$\{\text{Inputs}\} = \text{M} - \text{Vector of Inputs}$$
$$[\text{Transfer Matrix}] = (\text{N by M}) \text{ matrix of transfer functions}$$

SIMULATED FORCED RESPONSE

To generate some input and output force & motion signals for a structure, the **Transform | MIMO | Responses** command will be used to calculate 2 acceleration responses due to two forces. The accelerations will then be integrated to velocities. Then, the DOFs of the signals will be edited to create an input force & velocity pair and an output force & velocity pair. Finally, these input/output pairs of signals will be used to calculate a 4-Pole Transfer Function matrix between the top plate (DOF **5Z**) and bottom plate (DOF **20Z**) of the Jim Beam structure.

- Click on the **Mode Shapes** Demo button on the MEscope window Toolbar.

The **Jim BeamS.PRJ** Project will open, showing one of the mode shapes in the **Jim Beam.SHP** file in animation.



Jim Beam Model Showing Input & Output DOFs.

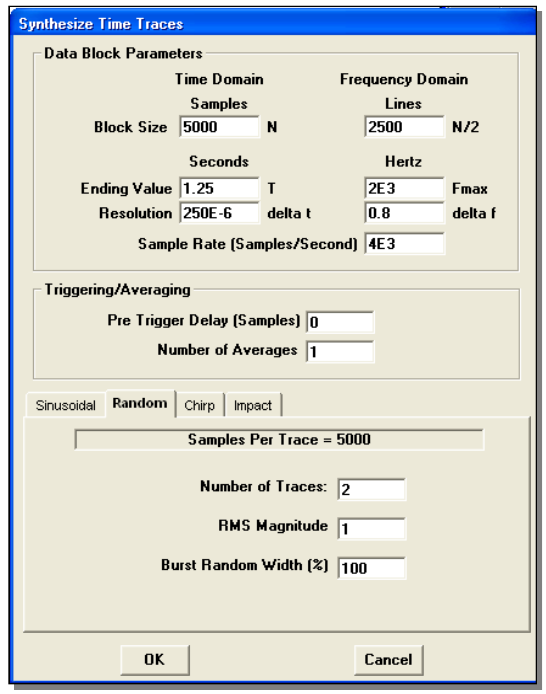
- Execute **Animate | Draw** in the Structure window to stop the animation.
- Execute **Display | Point Labels** to display the Point numbers on the model.

We will calculate a 4-Pole Transfer Function matrix between the force & velocity at DOF **20Z**, assumed to be inputs, and the force & velocity at DOF **5Z**, assumed to be outputs. In other words the transfer functions will be calculated which relate force & motion at DOF 20Z of the bottom plate to force & motion at DOF 5Z of the top plate.

Creating Random Forces

To create two broad-band random force signals for exciting the structure,

- Execute **File | New | Data Block** in the ME"scopeVES window. The Synthesize Time Traces window will open.



Synthesize Time Traces Dialog Box.

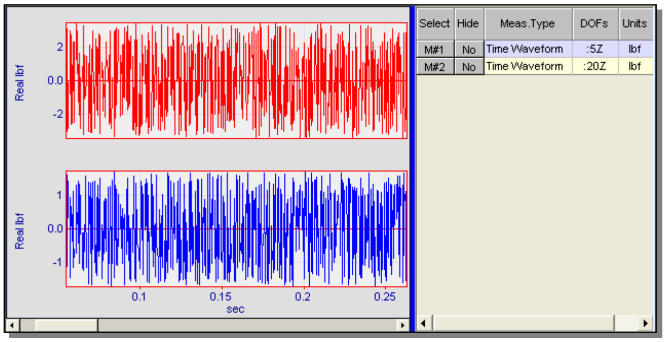
- Enter parameters into the dialog box as shown above, and press the **OK** button.

A new Data Block window will open with 2 random time domain signals in it. Each of these signals will excite the structure over a frequency range of (0 to 2000) Hz.

Force DOFs and Units

To characterize the 2 random signals as forces to be applied to the structure at DOFs **5Z** & **20Z**,

- Double click on the **Units** column heading in the **Traces** spreadsheet.



Synthesize Random Time Traces.

- Select **lbf** from units list, and click on **OK**. Click on **OK** again to change the Trace units to force units (**lbf**).
- In the Traces spreadsheet, edit the DOF of **M#1** to **:5Z**, and the DOF of **M#2** to **:20Z**. (Make sure both DOFs have a **colon (:)** in front of them to designate them as references, or input forces in this case.)

Now the Data Block is ready to calculate acceleration responses at DOFs **5Z** & **20Z** of the Jim Beam structure.

The mode shapes in the **Jim Beam.SHP** file will be used to synthesize 4 FRFs (two driving point FRFs and 2 cross FRFs) between DOFs **20Z** & **5Z** of the structure. To create a modal model for this purpose, the mode shapes must first be scaled to Unit Modal Masses (**UMM**).

Scaling Modes Shapes to UMM

Notice in Shapes (lower) spreadsheet in the **Jim Beam.SHP** window that the mode shapes are **Residue mode shapes**, and that all of the shape DOFs have the same **reference** DOF (**:-15Z)**. These residue mode shapes could only be used to calculate forced responses due to forces applied at DOF **15Z**. However, if the mode shapes are re-scaled to UMM shapes, they can be used to synthesize FRFs between any pair of DOFs in the mode shapes. To re-scale the shapes to UMM shapes,

- Execute **Tools | Scaling | Unit Modal Mass** in the Shape Table window.

Notice that the mode shapes in the lower spreadsheet are now **UMM mode shapes** with only Roving DOFs.

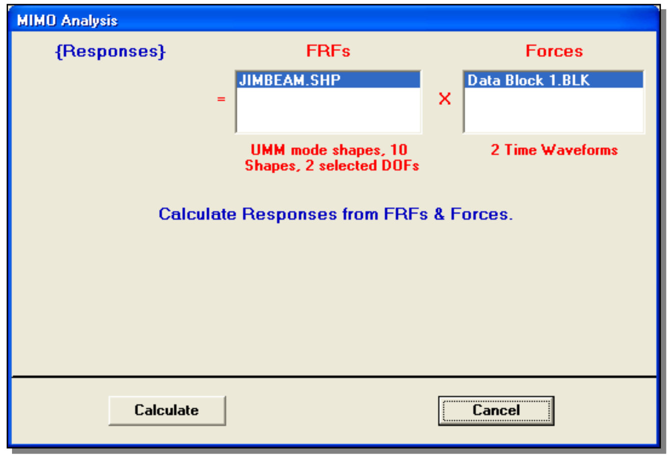
Calculating Forced Responses

To calculate the forced responses at DOFs **5Z** & **20Z** due to two random forces applied at the same DOFs,

- Click on the **M#15** (DOF **5Z**) and **M#45** (DOF **20Z**) **Select** buttons in the shape spreadsheet to **Select** only these two DOFs of the shapes.
- Execute **Transform | MIMO | Responses** in the Data Block window.

The MIMO calculation dialog box will open, showing the 2 Forces Data Block files in the **Forces** list, and the Jim Beam.SHP file in the **FRFs** list, as shown below.

- Click on **Calculate** and **OK** to calculate acceleration Responses due to the two random forces, using the mode shapes to synthesize the appropriate FRFs for the MIMO model.



MIMO Analysis Dialog Ready for Response Calculation.

When the calculation is completed, the Data Block file selection box will open, allowing you to choose a Data Block file for saving the Response signals.

- Select the Data Block with the two random forces in it, and click on the **Add To** button.

Now, the Data Block window will contain the two random force plus the two random response signals at DOFs **5Z** & **20Z**.

Creating Force & Acceleration Pairs.

To calculate the 4-Pole Transfer function matrix, the DOFs of the two of the four signals in the Data Block window must be edited to designate one force & acceleration pair and inputs and the other pair as outputs.

- Add a **colon (:)** in front of the acceleration signal at DOF 20Z (**M# 4**).
- Remove the **colon (:)** from the force signal at 5 Z (**M# 1**).

Integrating Acceleration to Velocity Responses

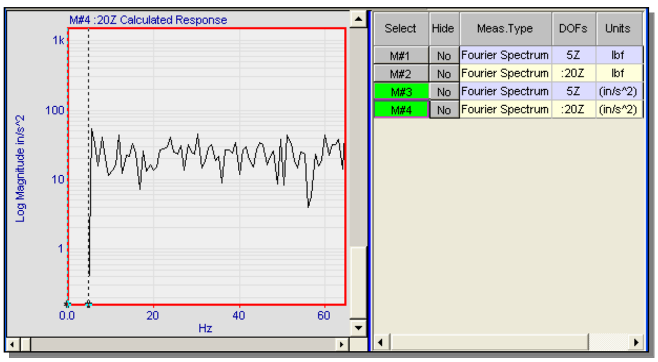
Even though acceleration responses are usually measured (in this case they were simulated), it is often more desirable to express the responses as velocities or displacements. MEscope can be used to integrate the acceleration signals to velocities prior to calculating the 4-Pole Transfer functions.

Integration can be done in either the time or frequency domain. Prior to performing the integration, the DC portion of the acceleration signals should be removed.

Removing DC before Integrating

To remove DC from the acceleration response signals,

- Execute **Transform | FFT** to transform the Traces in the Data Block to the frequency domain.
- In the Traces spreadsheet, **select** the two Traces with acceleration units.
- Turn on the **Band** Cursor and surround the data from DC (0 Hz) to about 5 Hz, as shown below.
- Execute **Transform | Window Data | Notch**. This will zero the data in the cursor band for the **selected** Traces, as shown below.



Acceleration Spectra with DC Removed.

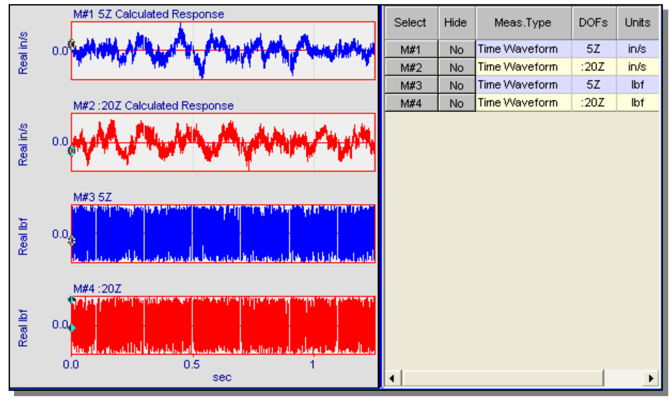
- Execute **Tools | Integrate** to integrate the **selected** Traces.
- Execute **Transform | Inverse FFT** to return the signals to the time domain.

Sorting the Traces

To sort the Traces so that the forces follow the velocity Traces,

- Execute **Edit | Sort Traces | Manual**, and click on **OK** in the dialog box that opens.
- Click on each of the velocity Traces to select it, and execute **Edit | Sort Traces | Manual** again to sort the Traces.

Now the four signals shown below are ready for the 4-Pole Transfer function matrix calculation.

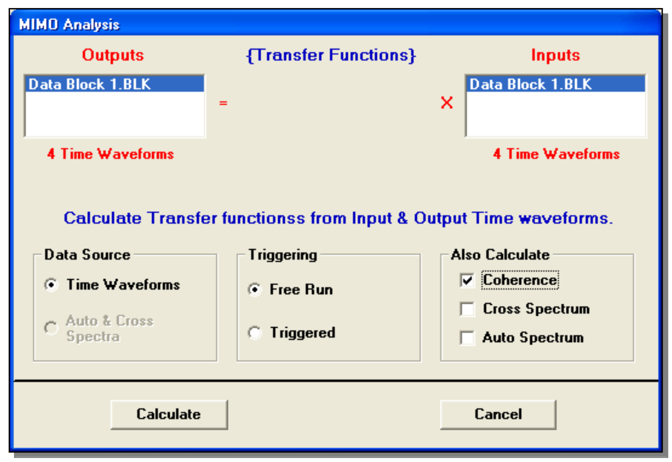


Force & Velocity Input-Output Signal Pairs.

CALCULATING THE 4-POLE TRANSFER MATRIX

To calculate the 4-Pole Transfer matrix,

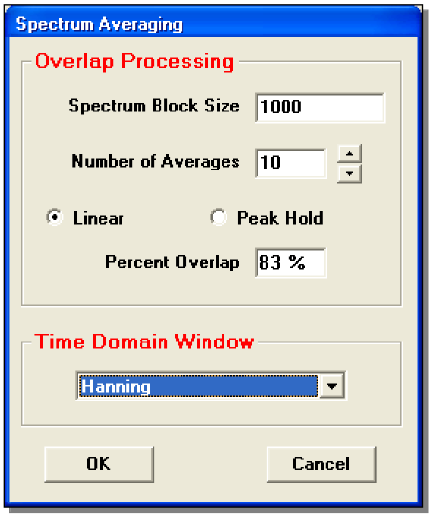
- Execute **Transform | MIMO | Transfer Matrix** in the Data Block window. The MIMO Analysis dialog box will open, as shown below.



- Check **Coherence** in the dialog box, and click on the **Calculate** button.

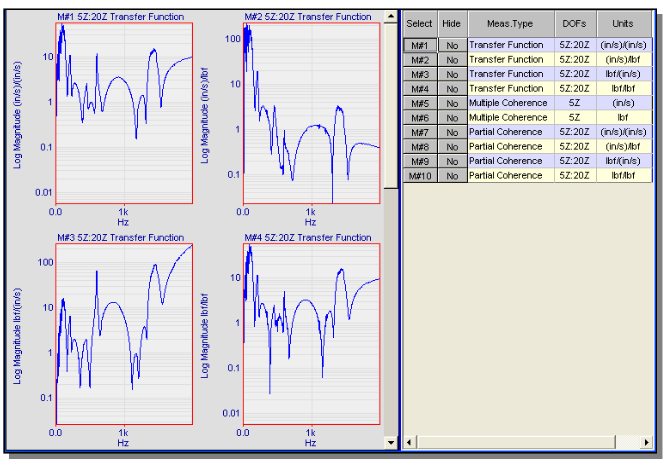
Next the Spectrum Averaging dialog box will open.

- Enter the parameters shown below, and click on **OK**.
- When the 4-Pole Transfer matrix has been calculated, click on **OK**, **New File**, and **OK** in the dialog boxes that open to save the results into a new Data Block file.



To arrange the transfer functions in a (2 by 2) matrix format,

- In the new Data Block window, execute **Format | Rows/Columns**, and select **(2, 2)** from the drop down list.

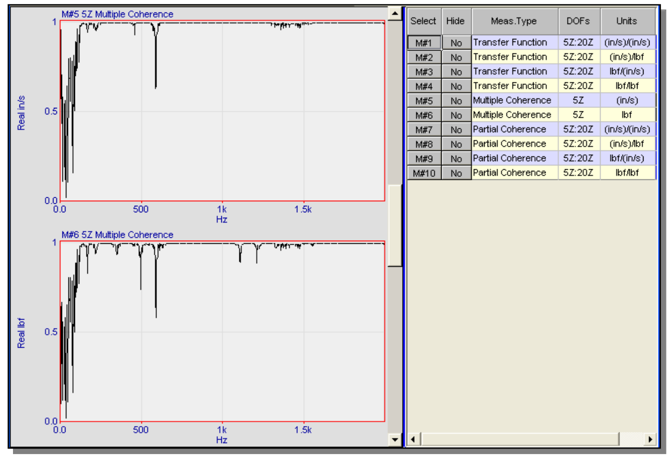


Data Block of 4-Pole Transfer Functions & Coherences.

- Scroll thru the Traces to examine the Transfer functions and Coherences.

MULTIPLE COHERENCES

The two multiple coherences (Traces **M#5** & **M#6**) indicate how much of each output is linearly related to (or caused by) the two inputs. A value of "1" indicates that an output is being caused by the two inputs. Values less than "1" indicate that some other (unmeasured) inputs are causing the output, of that the structure is behaving in a non-linear manner.

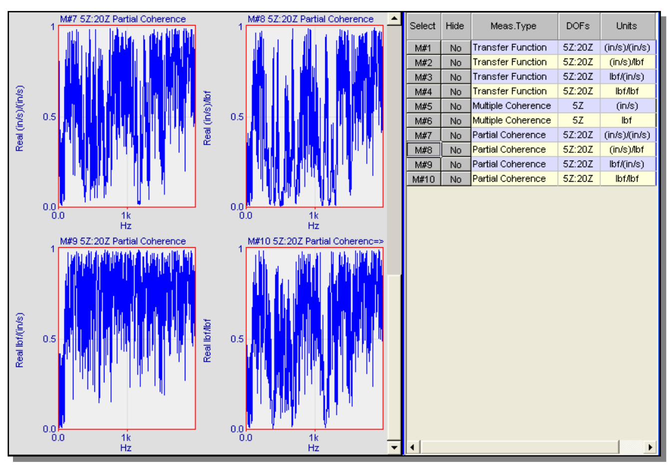


Mulitple Coherences.

Both Multiple Coherences indicate that, in fact, the outputs are related to the inputs, as expected.

PARTIAL COHERENCES

Partial Coherence indicates how much of each output is due to each input. The results shown below indicate that each output is quite strongly linearly related to each input, except in a few small frequency bands.



Partial Coherences.

CONCLUSIONS

We have calculated a 4-Pole Transfer function matrix using simulated random force & acceleration input and output pairs. Before the Transfer functions were calculated, the accelerations were integrated to velocities however.

The Transfer function calculation used the MIMO signal processing in MEscope to calculate the 4 transfer functions relating the force & velocity inputs to the force & velocity outputs. This calculation also yielded the Multiple and Partial Coherences that define the degree of linearity between the inputs and outputs.