VIBRANT

MEscope Application Note 23

Calculating the Forces that Caused Bridge Responses

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** & **VES-4000 Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the **AppNote23** project file. These steps might also require MEscope software with ***a more recent release date***.

APP NOTE 23 PROJECT FILE

- To retrieve the Project file for this App Note, **click here** to download **AppNote23.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click on a* Hotkey** to display its Script window

INTRODUCTION

In the frequency domain, excitation forces and response motions of a vibrating structure are related to one another by and **FRF matrix model**, expressed with the following algebraic equation. The response motions at **N-DOFs** (degrees-of-freedom or points & directions) are related to the forces applied at **M-DOFs** by an equation which includes a **(N by M)** matrix of Frequency Response Functions (**FRFs**).

$$\{\mathbf{X(f)}\}_{\mathbf{Nx1}} = [\mathbf{H(f)}]_{\mathbf{NxM}} \cdot \{\mathbf{F(f)}\}_{\mathbf{Mx1}}$$

{F(f)} is an **M-vector** of the **DFTs** (Digital Fourier Transforms) of multiple excitation force **Inputs** at **M-DOFs**

{X(f)} is an **N-vector** of the **DFTs** of response **Outputs** at **N-DOFs**

[H(f)] is an **(N by M)** ***rectangular matrix*** **of FRFs**

The **FRF matrix model** is also called a **MIMO** (Multiple Input Multiple Output) **model**.

Each **DOF** of the **Input** & **Output** vectors contains a ***point number & direction***. Each FRF defines the dynamic properties of a structure between an **Input DOF** and an **Output DOF**.

If any two elements of the above **MIMO model** are provided, the third element can be calculated using one of the following commands in an MEscope Data Block window.

- **Transform | H1 FRFs ➔** calculates Multiple Reference **H1 FRFs** in the (N by M) **FRF matrix** using an **M-vector** of force **Input DFTs** or **TWFs** and an **N-vector** of response **Output DFTs** or **TWFs**
- **Transform | H2 FRFs ➔** calculates Multiple Reference **H2 FRFs** in the (N by M) **FRF matrix** using an **M-vector** of force **Input DFTs** or **TWFs** and an **N-vector** of response **Output DFTs** or **TWFs**
- **Transform | Outputs ➔** calculates an **N-vector** of response **Output TWFs** using an (N by M) **FRF matrix** and an **M-vector** of force **Input TWFs** (***see* App Note 21**)
- **Transform | Inputs ➔** calculates an **M-vector** of force **Input TWFs** using an (N by M) **FRF matrix** and an **N-vector** of response **Output TWFs**
- **Transform | Sinusoidal ODS ➔** calculates an Operating Deflection Shape (ODS **N-vector**) using an (N by M) **FRF matrix** and an **M-vector** of force **Input TWFs** (***see* App Note 22**)

TWF is an acronym for Time Waveform.

In this App Note, the **Transform | Inputs** command is used to calculate force **Input TWFs** from an **FRF matrix** defining the **Input**-**Output** dynamics of a bridge and a vector of acquired response **Output TWFs**. The calculated forces are then compared with the acquired force **TWFs** to confirm the validity of the **Transform | Inputs** calculations.

Z24 Bridge on the Bern-to-Zurich Swiss Highway A1.

MULTIPLE MEASUREMENT SETS

The data used in this App Note was acquired in **multiple Measurement Sets** from the bridge shown in the figure above.

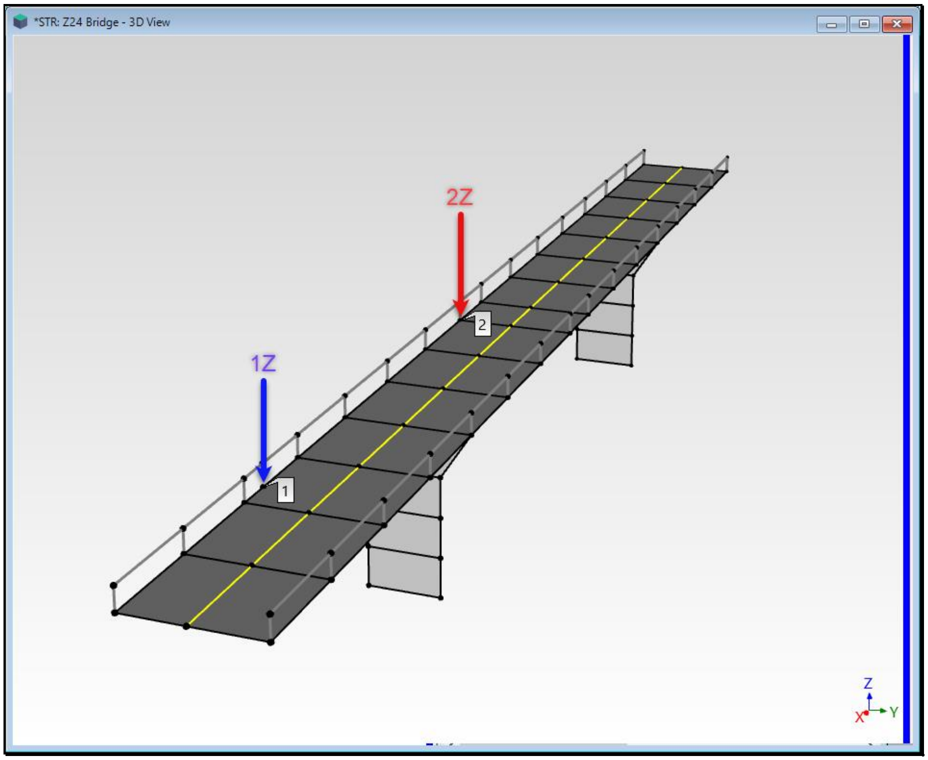
The bridge was tested using two hydraulic shakers with random forcing-functions applied to the shakers. Because of acquisition hardware limitations, **nine Measurement Sets** of force & response **TWFs** were acquired.

Each Measurement Set contains force & response **TWFs** that were ***simultaneously acquired***.

The nine Measurement Sets of force & acceleration response **TWFs** are contained in Data Block **BLK: Z24 Bridge 2 Shaker Test**.

The table below gives details of the **TWFs** in **BLK: Z24 Bridge 2 Shaker Test**. The **Common Response DOFs** are in every Measurement Set. The **Roving DOFs** are unique to each Measurement Set.

Measurement Set	Force DOFs 1Z, 2Z	Common Response DOFs 1X, -2Y, 2Z	Roving Response DOFs	TOTAL M#s
1	**2**	**3**	**12**	**17**
2	**2**	**3**	**6**	**11**
3	**2**	**3**	**9**	**14**
4	**2**	**3**	**6**	**11**
5	**2**	**3**	**6**	**11**
6	**2**	**3**	**12**	**17**
7	**2**	**3**	**3**	**8**
8	**2**	**3**	**12**	**17**
9	**2**	**3**	**6**	**11**
TOTALS	**18**	**27**	**72**	**117**

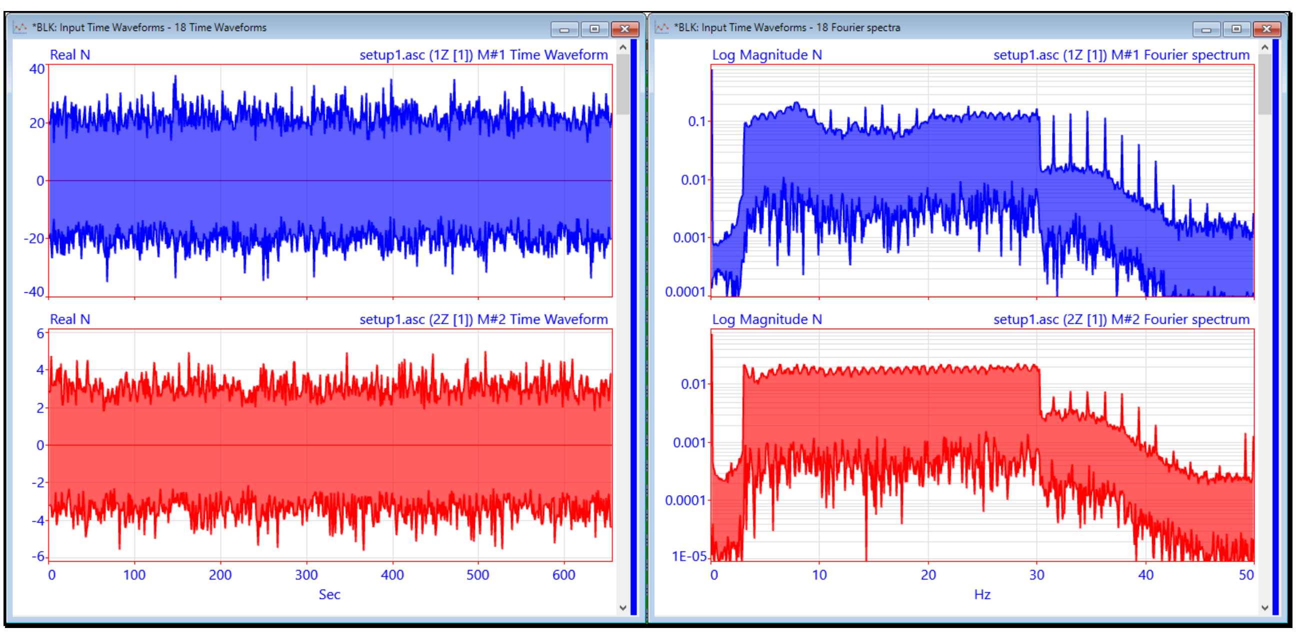


3D Model Showing Two Shaker Locations.

The two shakers had different force capacities. The ***larger shaker*** excited the bridge at **DOF 1Z** and the ***smaller shaker*** excited it at **DOF 2Z**, as shown above. The shakers ***operated simultaneously*** and remained fixed at the same locations throughout the test. We will see from the data that the larger shaker ***had a stronger influence*** on the response of the bridge than the smaller shaker, even though the smaller shaker was located at a more compliant location of the bridge.

SHAKER FORCE SIGNALS

The shakers were driven with computer-generated ***white random noise*** spanning a **3 Hz** to **30 Hz frequency range**. The shaker TWFs and their spectra for Measurement Set [1] are shown in the figure below.



Shaker Force Signals Applied to DOFs 1Z & 2Z During Acquisition of Measurement Set [1].

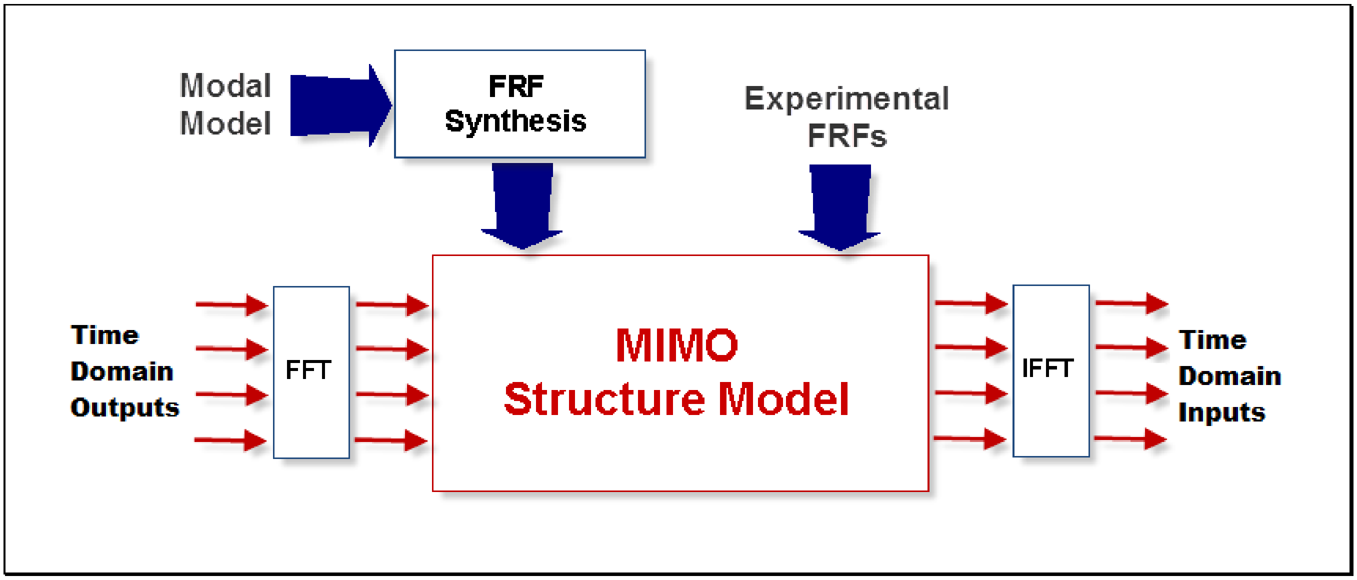
All the structural dynamics between the two force Inputs and 75 response Outputs is *completely captured* by the **TWFs** in **BLK: Z24 Bridge 2 Shaker Test**.

The **Transform | Inputs** command can only be used on ***one Measurement Set at a time***.

For each Measurement Set, the acquired response **Outputs** and the **FRFs** are used to calculate the force Inputs for that Measurement Set.

BLOCK DIAGRAM OF INPUT CALCULATIONS

The block diagram below shows how the two shaker signals (**Inputs**) are calculated with the **Transform | Inputs** command. Acquired response **Output TWFs** of the bridge, are used together with the Experimental **FRFs** for the bridge to calculate the **Input** force **TWFs** for each Measurement Set.



In the **MIMO Structural Model**, **Experimental FRFs** (calculated from the acquired Input and Output **TWFs**) are used together with the **DFTs** of the acquired **Outputs** to calculate the **DFTs** of the **Input** excitation forces. Then the **DFTs** of the **Inputs** are ***Inverse Fourier transformed*** to yield **TWFs** of the excitation forces.

COMPARING DATA BLOCKS USING MAC & SDI

Two commands in MEscope: **Tools | Data Block Correlation** and **Tools | M# Pairs Correlation** are used on Data Blocks of calculated force **TWFs** to compare them with the **TWFs** acquired during the bridge test.

MODAL ASSURANCE CRITERION (MAC)

MAC is a measure of the ***co-linearity*** of two complex shape vectors.

If two shapes ***lie on the same straight line***, they are ***co-linear*** and have a **MAC ➔ 1.0**.

If two shapes ***do not lie on the same straight line***, they are ***linearly independent*** and have a **MAC ➔ *less than* 1.0**.

The following ***rules of thumb*** are used with **MAC**,

MAC values ➔ *between* 0 &1

MAC = 1.0 ➔ two shapes ***are co-linear***

MAC >= 0.9 ➔ two shapes ***are similar***

MAC < 0.9 ➔ two shapes ***are linearly independent***

SHAPE DIFFERENCE INDICATOR (SDI)

SDI is a measure of the ***equality*** of two complex shape vectors.

If two shapes ***have equal components***, they have **SDI → 1.0**.

If two shapes ***do not have equal components***, they have **SDI →** ***less than*** **1.0**.

The following ***rules of thumb*** are used with **SDI**

- **SDI values →** ***between*** **0 &1**
- **SDI = 1.0 →** two shapes ***have equal components***
- **SDI >= 0.9 →** two shapes ***are similar***
- **SDI < 0.9 →** two shapes ***are different*** (***some matching components are not equal***)

TOOLS | DATA BLOCK CORRELATION

The **Data Block Correlation** command calculates **MAC** & **SDI** ***at each sample*** between two Data Blocks with **M#s** having ***the same*** **DOFs** and ***the same*** **X-axis**.

A new Data Block with **two M#s** in it is created with **M#1** as the **MAC** value and **M#2** as the **SDI** value ***at each matching sample*** of data.

TOOLS | M# PAIRS CORRELATION

The **M# Pairs Correlation** command calculates **MAC** & **SDI** between ***all*** **M#** ***pairs*** in two Data Blocks having ***the same*** **DOFs** and ***the same*** **X-axis**.

The **MAC** & **SDI** values are saved as two shapes in a new Shape Table. The **MAC** values are saved as **Shape #1** and the **SDI** values as **Shape #2**.

A Magnitude Ranking plot is then displayed of **MAC** & **SDI** values from largest to smallest for **each DOF pair**.

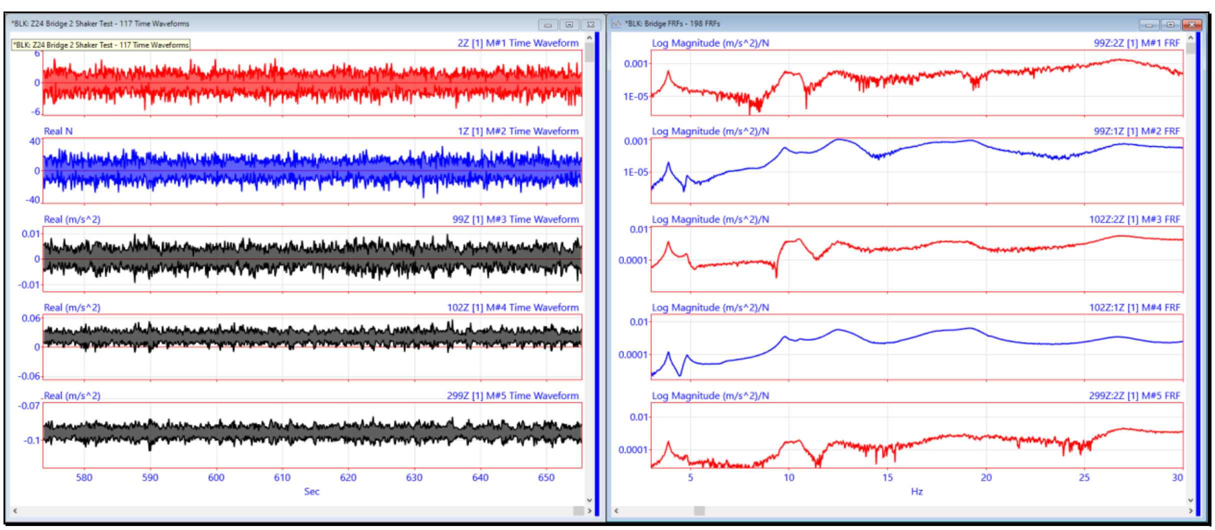
STEP 1 - CALCULATING THE BRIDGE FRFs

- ***Press*** **Hotkey 1 Bridge FRFs**

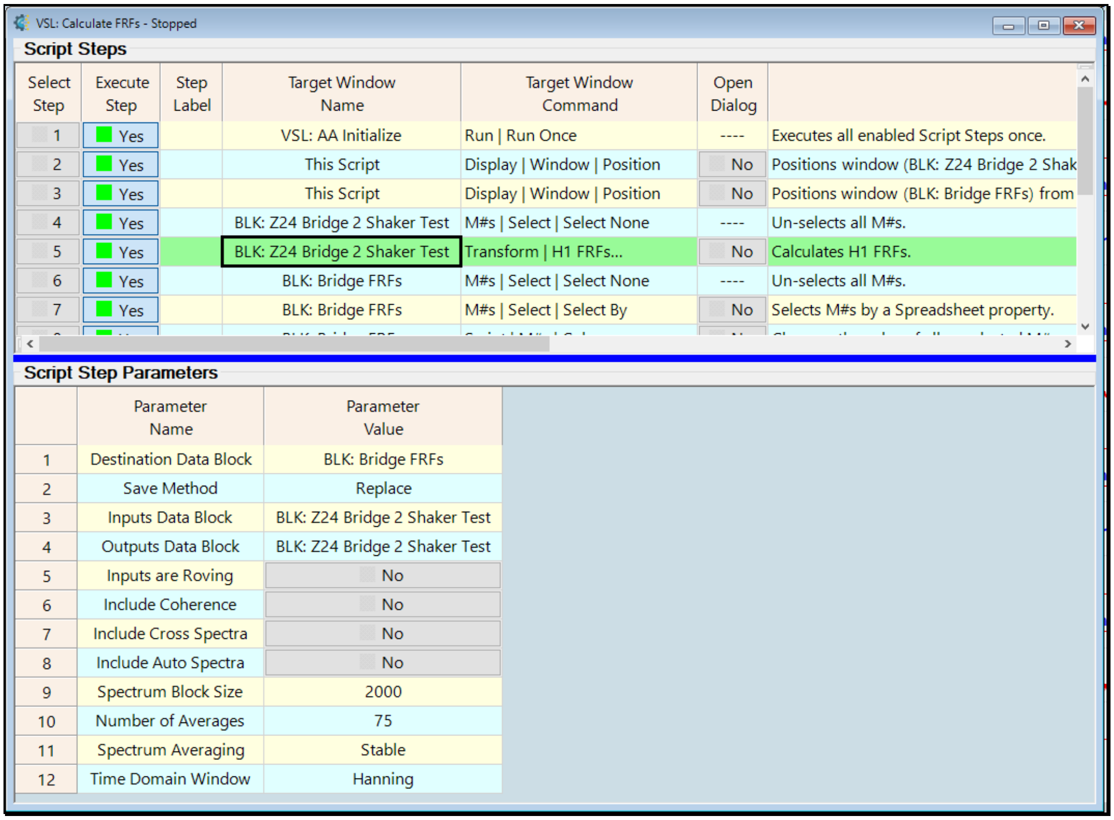
When **Hotkey 1 is** ***pressed***, **FRFs** for all nine Measurement Sets are calculated from the acquired **Input** & **Output TWFs**. The **BLK: Z24 Bridge 2 Shaker Test** window is displayed ***on the left*** and the **BLK: Bridge FRFs** window ***on the right***.

- Use the **Scroll bar** on the right in either window to scroll through the **M#s**

The FRFs for **Input DOF 1Z** are **blue** and the FRFs for Input DOF 2Z are red.



Multi-Reference ***FRFs*** *for 9 measurement Sets.*



Script Window for Hotkey 1 Showing Parameters for FRF Calculation

- ***Hold down* the Ctrl key** and ***click* on Hotkey 1** to open its Script window

With the step with **Transform | H1 FRFs** selected, its **Script Step Parameters** are displayed, as shown above.

- **Spectrum Block Size ➔ 2000 samples**
- **Number of Averages ➔ 75**

The Spectrum Block Size can be changed to any number ***up to one half of the* Block Size** of the **TWFs** in BLK: Z24 Bridge 2 Shaker Test.

A different Spectrum Block Size and Number of Averages will give slightly different, but still accurate, results.

STEP 2 - CALCULATING THE FORCES FOR MEASUREMENT SET [1]

- ***Press* Hotkey 2 Forces for Meas Set [1]**

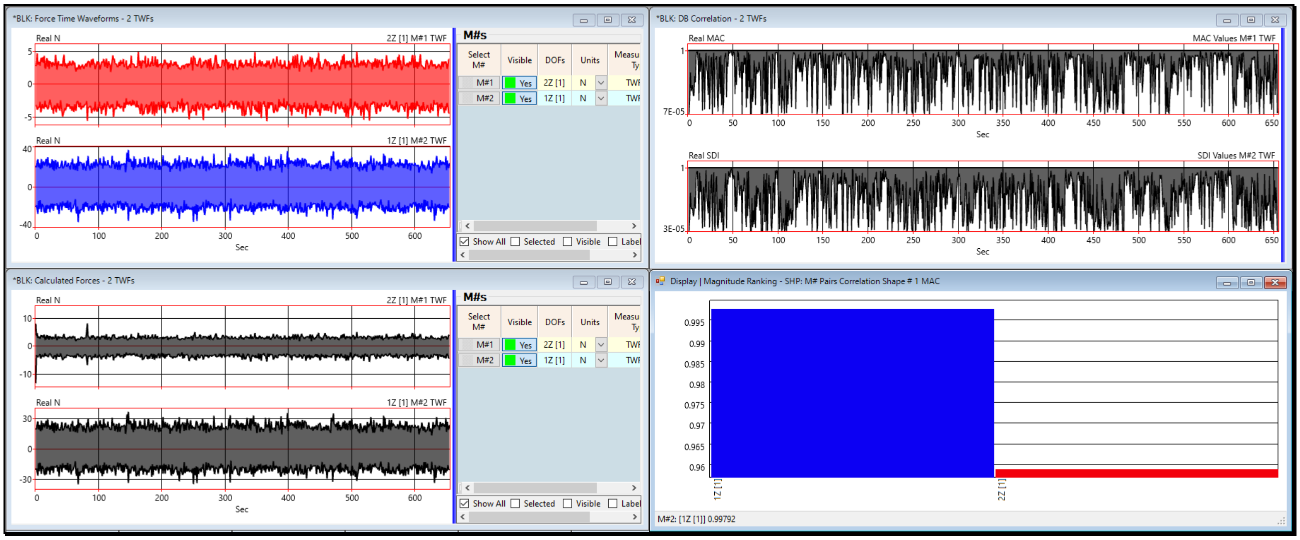
When **Hotkey 2 is *pressed***, the two forces for Measurement Set [1] are calculated using the **FRFs** and acquired **Output TWFs** for Measurement Set [1]. The **acquired force TWFs** are displayed ***on the upper-left side*** and the **calculated force TWFs** are displayed ***on the lower-left***.

- The **Data Block Correlation** between the two Data Blocks on the left is displayed ***on the upper-right***
- The **M# Pairs Correlation** between the two Data Blocks on the left is displayed ***on the lower-right***

The **Data Block Correlation** shows a ***strong correlation*** between the Calculate & Acquired Forces for ***most of the* 65,536 *samples*** in the two Data Blocks.

The **M# Pairs Correlation** also shows a ***strong correlation*** between the two matching pairs of **M#s** in the two Data Blocks, ***again for* 65,536 *samples***.

Even though the two Correlation commands give ***strong correlations*** between the Measured & Calculated **TWFs**, a visual inspection of the **TWFs** indicates that the ***stronger force*** **(at 1Z)** is more accurately calculated than the ***weaker force*** **(at 2Z)**.



DB Correlation & M# Pairs Correlation on the Right for Calculated Force Inputs (Measurement Set [1])

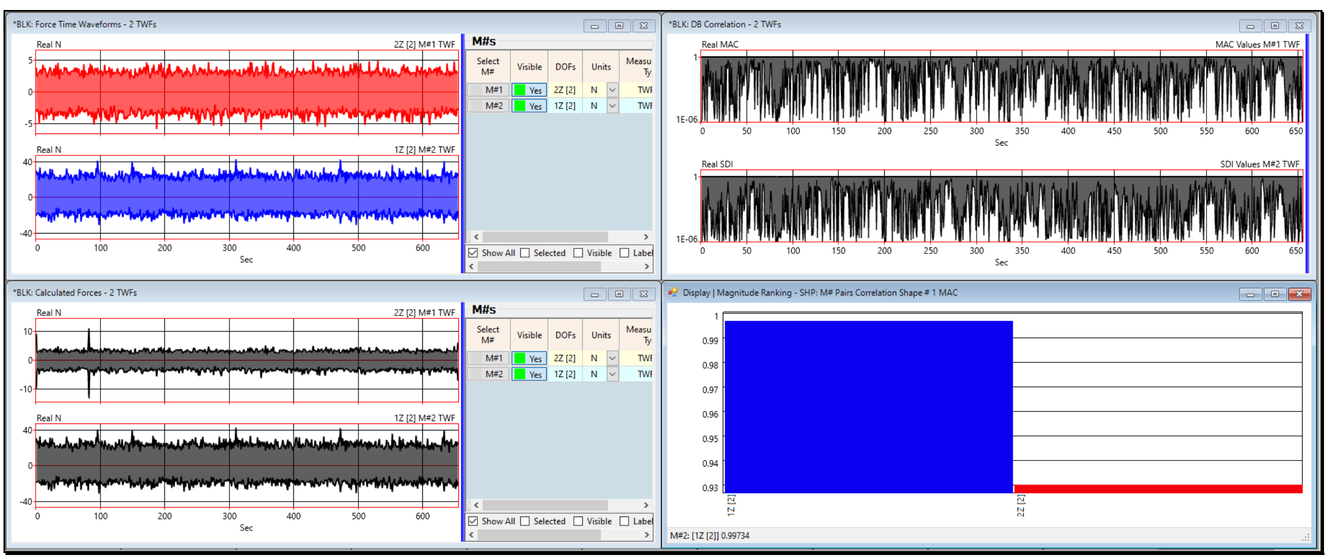
STEP 3 - CALCULATING THE FORCES FOR MEASUREMENT SET [2]

- ***Press* Hotkey 3 Forces for Meas Set [2]**

When **Hotkey 3 is *pressed***, the two forces for Measurement Set [2] are calculated using the **FRFs** and acquired **Outputs** for Measurement Set [2]. The **acquired force TWFs** are displayed ***on the upper-left side*** and the **calculated force TWFs** are displayed ***on the lower-left***.

- The **Data Block Correlation** between the two Data Blocks on the left is displayed ***on the upper-right***
- The **M# Pairs Correlation** between the two Data Blocks on the left is displayed ***on the lower-right***

These results are similar to those obtained when **Hotkey 2 is *pressed***.



DB Correlation & M# Pairs Correlation on the Right for Calculated Force Inputs (Measurement Set [2])

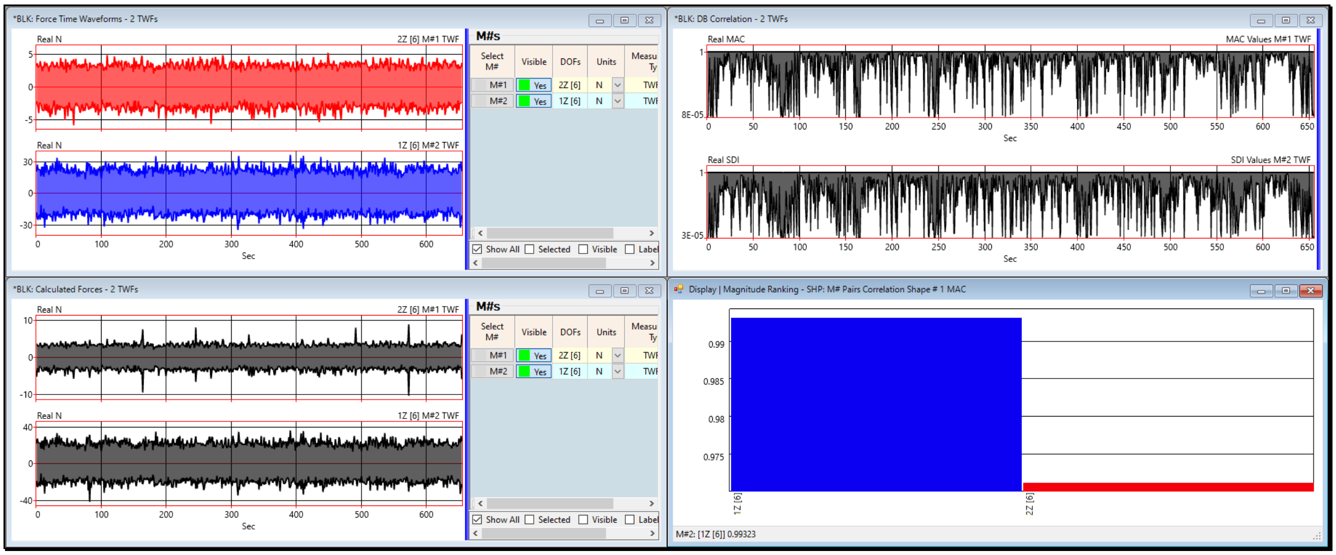
STEP 4 - CALCULATING THE FORCES FOR MEASUREMENT SET [6]

- ***Press* Hotkey 4 Forces for Meas Set [6]**

When **Hotkey 4 is *pressed***, the two forces for Measurement Set [2] are calculated using the **FRFs** and acquired **Outputs** for Measurement Set [2]. The **acquired force** waveforms are displayed ***on the upper-left side*** and the **calculated force** waveforms are displayed ***on the lower-left***.

- The **Data Block Correlation** between the two Data Blocks on the left is displayed ***on the upper-right***
- The **M# Pairs Correlation** between the two Data Blocks on the left is displayed ***on the lower-right***

These results are also similar to those obtained when **Hotkey 2** or **Hotkey 3 is *pressed***.



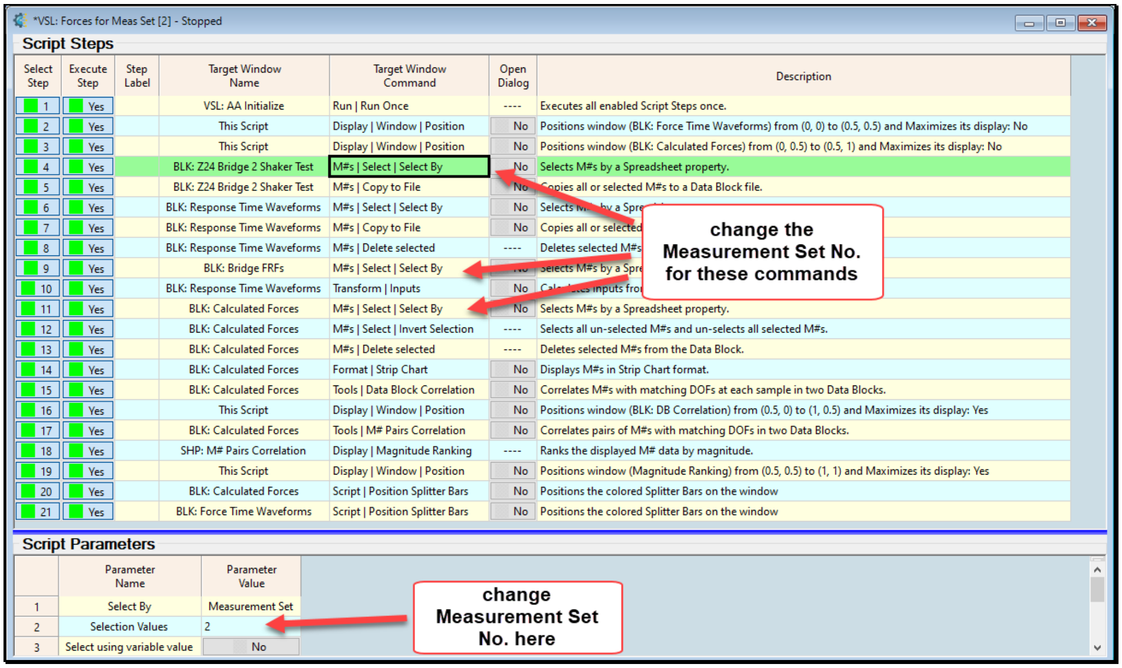
DB Correlation & M# Pairs Correlation on the Right for Calculated Force Inputs (Measurement Set [6])

CALCULATING FORCES FOR ANY MEASUREMENT SET

The force **Input** calculation can be done for any Measurement Set by editing three parameters in the Script window for **Hotkey 2, Hotkey 3, or Hotkey 4**.

- ***Hold down* the Ctrl key** and ***click* on Hotkey 2** to open its Script window

The Script for **Hotkey 2** is shown below, with the **M#s | Select | Select By** command for **BLK: Z24 Bridge 2 Shaker Test** selected.



*Script window for Hotkey 2 Showing **M#s | Select | Select By** Command Parameter.*

The two **M#s | Select | Select By** commands in the Script window above determine which Measurement Set of **Outputs & FRFs** will be used by the **Transform | Inputs** command to calculate Inputs for that Measurement Set.

- Enter a new value (**from 1 to 9**) for the parameter of the two **M#s | Select | Select By** commands
- ***Press* Hotkey 2**

The **Data Block Correlation** and **M# Pairs Correlation** results will be displayed ***on the right side*** of the screen, similar to the results previously shown.

STEP 5 - REVIEW STEPS

To review all the steps of this App Note,

- ***Press* Hotkey 5 Review Steps**

CONCLUSIONS

Two hydraulic shakers were used to excite a highway bridge and nine Measurements Sets of data were acquired during the bridge test. In this App Note, **multi-reference FRFs** were calculated from each Measurement Set of **Input & Output TWFs** acquired from the bridge.

Then using the FRFs and acquired **Output TWFs**, excitation forces were calculated for three different Measurement Sets of data.

Two Data Block commands, **Tools | Data Block Correlation** and **Tools | M# Pairs Correlation** were used to compare the calculated force **TWFs** with the acquired force waveforms. In all cases, the calculated & acquired **TWFs** were closely matched, ***for all*** **65,536** ***time samples*** in each Data Block of excitation force **Inputs**.

It can be concluded from these results that the **multi-reference FRF matrix** model accurately modeled the **Input-Output** dynamics of the bridge, and that the **Transform | Inputs** command accurately calculated the **Input DFTs** and their corresponding **TWFs**.

FRF INTERPOLATION

To calculate force Inputs using the MIMO Model, the **FRFs** ***were interpolated between samples*** to match the frequency-axis parameters of the **DFTs** of the response **Output TWFs**.

The **Block Size** of **BLK: Bridge FRFs** was **2000 samples**.

The **Block Size** of the **DFTs** of the response **Outputs** was **32,768 samples**.

Therefore, interpolation was performed to ***expand each*** **FRF** from ***2000 samples to 32,768 samples*** so that the **DFTs** of the force **Inputs** could be calculated using MIMO Modeling.

The close correlation of the calculated & measured force **Inputs** verifies that ***the linear bridge dynamics was preserved in the interpolated*** **FRFs**.