



MEscope Application Note 22

Responses to Sinusoidal Excitation of a Bridge

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** and **VES-4600 Advanced Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the **AppNote22** project file. These steps might also require MEscope software with *a more recent release date*.

APP NOTE 22 PROJECT FILE

- To retrieve the Project file for this App Note, [click here](#) to download **AppNote22.zip**

This Project file contains *numbered Hotkeys & Scripts* for carrying out the steps of this App Note.

- Hold down the Ctrl key** and *click on a Hotkey* to display its Script window

INTRODUCTION

In the frequency domain, driving forces and response motions of a vibrating structure are related to one another by the following algebraic equation. The response motions at **N-DOFs** (degrees-of-freedom or points & directions) are related to the forces applied at **M-DOFs** by an equation which includes a (**N by M**) matrix of Frequency Response Functions (**FRFs**),

$$\{\mathbf{X}(\mathbf{f})\}_{N \times 1} = [\mathbf{H}(\mathbf{f})]_{N \times M} \cdot \{\mathbf{F}(\mathbf{f})\}_{M \times 1}$$

$\{\mathbf{F}(\mathbf{f})\}$ is an **M-vector** of the Fourier spectra of multiple excitation force **Inputs** at **M-DOFs**

$\{\mathbf{X}(\mathbf{f})\}$ is an **N-vector** of the Fourier spectra of response **Outputs** at **N-DOFs**

$[\mathbf{H}(\mathbf{f})]$ is an (**N by M**) *rectangular matrix* of **FRFs**

Each DOF of the Input & Output vectors contains a *point number & direction*. Each **FRF** defines the dynamic properties of a structure between an Input DOF and an Output DOF.

If any two elements of the above MIMO equation are provided, the third element can be calculated using one of the following Data Block window commands in MEscope.

- Transform | H1 FRFs** → calculates Multiple Reference H1 **FRFs** in the (**N by M**) **FRF** matrix given an M-vector of force **Input** time waveforms and an N-vector of response **Output** waveforms
- Transform | H2 FRFs** → calculates Multiple Reference H2 **FRFs** in the (**N by M**) **FRF** matrix given an M-vector of force **Input** waveforms and an N-vector of response **Output** waveforms
- Transform | Outputs** → calculates an N-vector of response Output time waveforms given an (**N by M**) **FRF** matrix and an M-vector of force **Input** waveforms (*see App Note 21*)
- Transform | Inputs** → calculates an M-vector of force Input waveforms given an (**N by M**) **FRF** matrix and an N-vector of response **Output** waveforms (*see App Note 23*)
- Transform | Sinusoidal ODS** → calculates an Operating Deflection Shape (ODS N-vector) using an (**N by M**) **FRF** matrix and an M-vector of force **Inputs**

In this App Note, the Data Block command **Transform | Sinusoidal ODS** is used to calculate the Operating Deflection Shape (**N-vector**) of a bridge from two sinusoidal force **Inputs** to a matrix of **FRFs** which defines the Input-Output dynamics of the bridge. Also, the Shape Table command **Tools | Sinusoidal ODS** is used to calculate the Operating Deflection Shape of the bridge from two sinusoidal force **Inputs** and a **modal model** that also defines the Input-Output dynamics of the bridge. The ODS's calculated with both commands are then compared to each other using the **MAC** shape metric.



Z24 Bridge on the Bern-to-Zurich highway A1

MULTIPLE MEASUREMENT SETS

The data used in this App Note was acquired in **multiple Measurement Sets** from the bridge shown in the figure above.

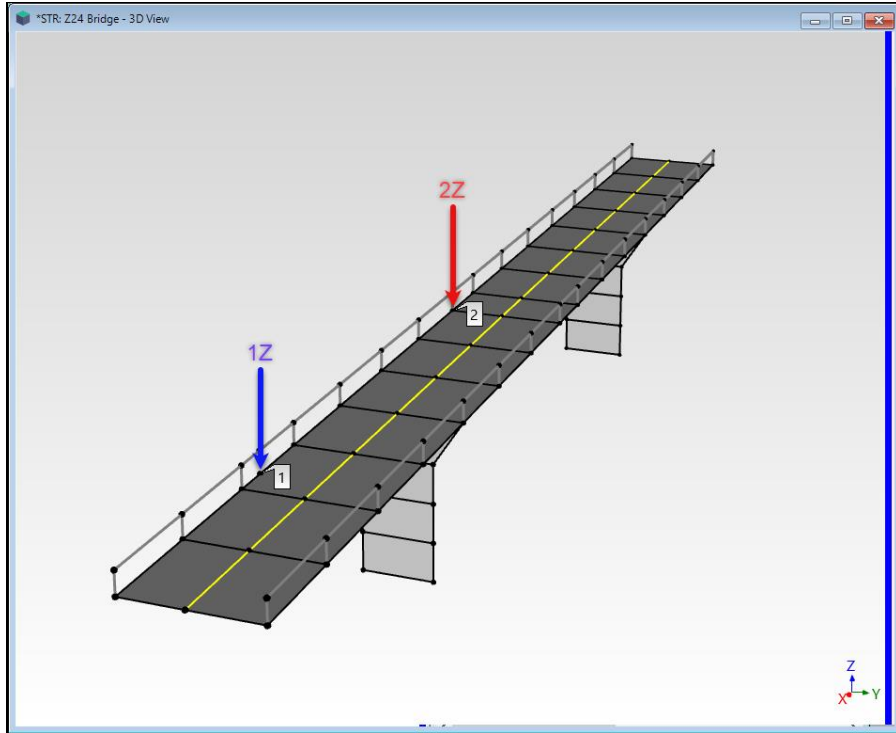
The bridge was excited with two hydraulic shakers. Random forcing-functions were applied to the shakers. Because of acquisition hardware limitations, **nine Measurement Sets** of force & response time waveform data were acquired.

Each Measurement Set contains force & response data that was *simultaneously acquired*.

The nine Measurement Sets of force & acceleration response time waveforms are contained in an MEscape Data Block **BLK: Z24 Bridge 2 Shaker Test**. Multiple Measurement Sets of data are processed by the commands in the Data Block **Transform** menu in MEscape.



Deck of Z24 Bridge During Tests (The Replacement Bridge is Adjacent.)

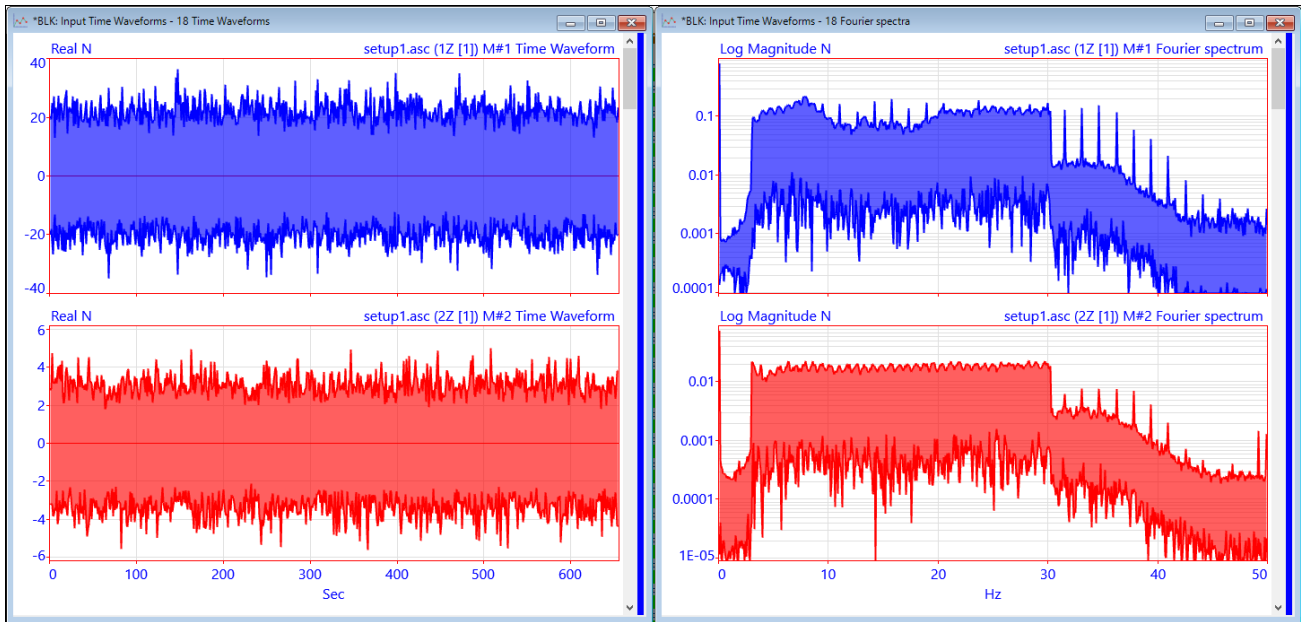


3D Model Showing Two Shaker Locations.

The two shakers had different force capacities. The **larger shaker** excited the bridge at **DOF 1Z** while the **smaller shaker** excited it at **DOF 2Z**, as shown above. The shakers **operated simultaneously** and remained fixed throughout the test. We will see that the larger shaker **had a stronger influence** on the response of the bridge than the smaller shaker, as expected.

SHAKER FORCE SIGNALS

The two shaker time waveforms for Measurement Set [1] and their Fourier spectra (**DFTs**) are shown in the figure below. The shakers were driven with computer-generated **white random noise** spanning a **3 Hz to 30 Hz frequency range**. From the spectra it can be seen that the signals **did excite of a wider 0 Hz to 50 Hz range**.

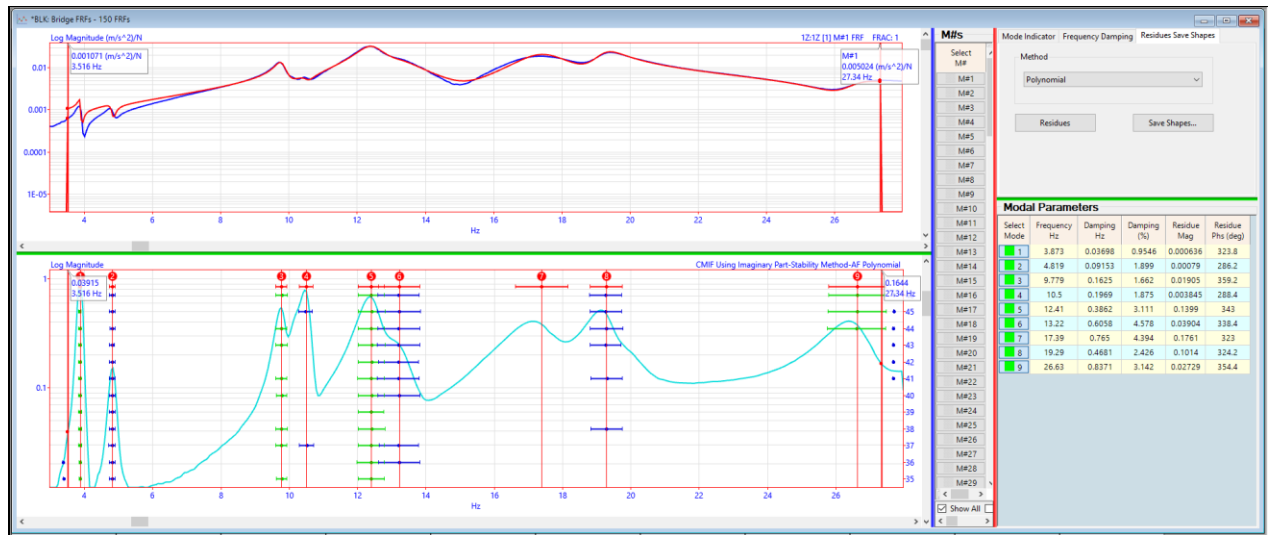


Shaker Force Signals Applied to DOFs 1Z & 2Z During Acquisition of Measurement Set [1].

STEP 1 - CURVE FITTING THE BRIDGE FRFs

In this step, **FRF**-based curve fitting is used to extract mode shapes of the bridge from the **FRFs** in **BLK: Bridge FRFs**.

- **Press Hotkey 1 Curve Fit the FRFs**



Stability Diagram Curve Fit Estimates of Modal Parameters for 9 Modes.

Even though these are multi-reference **FRFs**, they are curve fit using the **single-reference Quick Fit** method.

Multi-reference FRFs can always be curve fit using a *single-reference curve fitting method*.
Multi-reference Residue mode shapes are extracted from **multi-reference FRFs** by *single-reference curve fitting* methods.

The Residue mode shapes extracted from the multi-reference **FRFs** were saved in **SHP: Bridge Mode Shapes**.

STEP 2 - COMPARING MODE SHAPES FROM REFERENCE 1Z VERSUS 2Z

- **Press Hotkey 2 -Compare Multi-Ref Mode Shapes**

To compare the mode shapes excited by the two shakers from locations **1Z** & **2Z** on the bridge, the mode shapes for each reference DOF are saved into separate Shape Tables (**SHP: Reference 1Z Mode Shapes** & **SHP: Reference 2Z Mode Shapes**) so they can be compared side-by-side in animation.

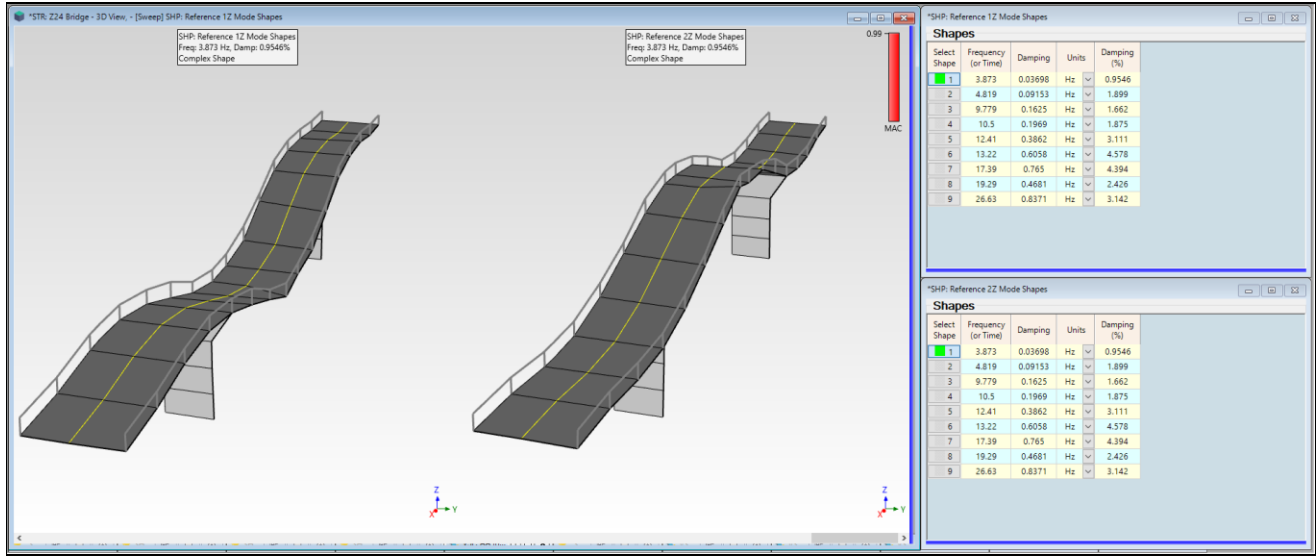
Multi-reference mode shapes can only be displayed in animation from **one Reference DOF at a time**.
 If a **mode shape does not change** when animated from one Reference DOF versus another, a **single resonance** was excited at that frequency of the bridge by the two shakers.

Sweep animation of a selected mode shape from **SHP: Reference 1Z Mode Shapes** is displayed side-by-side with the **closest matching mode shape** from **SHP: Reference 2Z Mode Shapes**.

The closest matching mode shape is the shape with the **Maximum MAC** value compared to all other mode shapes.

MODAL ASSURANCE CRITERION (MAC)

If two shapes lie on the **same straight line**, they are said to be co-linear and their **MAC → 1.0**.
 If two shapes **do not lie on the same straight line**, they are said to be linearly independent and their **MAC → less than 1.0**.
 If **MAC → greater than 0.9**, the same mode was excited by both shakers.
Six of the eight mode shape pairs from the two references (**1Z** & **2Z**) have **MAC → above 0.9**.



Animated Bridge Mode Shapes from References 1Z & 2Z.

Two of the mode shape pairs (**shapes #6 & #7**) have low **MAC** values. This indicates that either the same resonance was not excited from the two references or that the curve fitting estimated a different mode shape from each of the two references of **FRFs**.

STEP 3 - CALCULATING SINUSOIDAL ODS's FROM FRFS

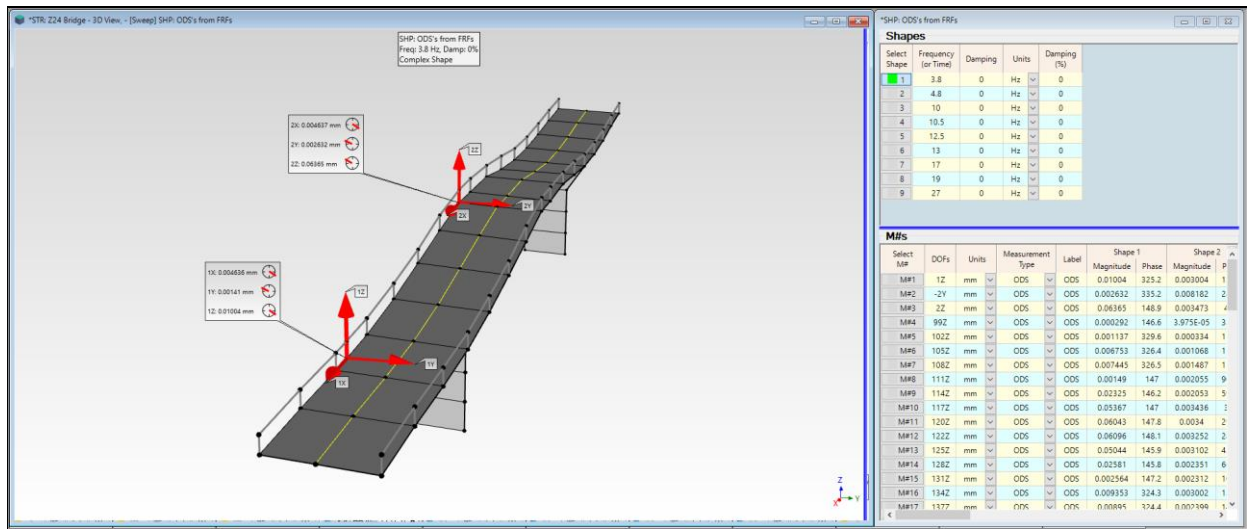
- **Press Hotkey 3 Sinusoidal ODS's from FRFs**

When **Transform | Sinusoidal ODS** is executed, columns of **FRFs** in the **FRF** matrix of the **MIMO** equation are multiplied by force **Inputs** at the frequency of the forces. The resulting **frequency-based ODS** can be saved in a Shape Table or displayed in animation on a model in a Structure (**STR**) window.

If **Transform | Sinusoidal ODS** is executed from a Data Block it must contain **FRFs**.

A sinusoidal ODS can be *calculated at any frequency* for which the **FRFs** are defined.

When **Hotkey 3 is pressed**, sinusoidal ODS's *at eight different frequencies* are calculated and saved in a Shape Table (**SHP: ODS's from FRFs**). These ODS's will be compared with the sinusoidal ODS's calculated at the same frequencies using the modal model of Residue mode shapes estimated from the **FRFs**. After the ODS's have been calculated and saved in **SHP: ODS's from FRFs**, sweep animation will begin through the ODS's, as shown below.



Animation of Sinusoidal ODS's Calculated from FRFs.

MODE SHAPES VERSES FRFS

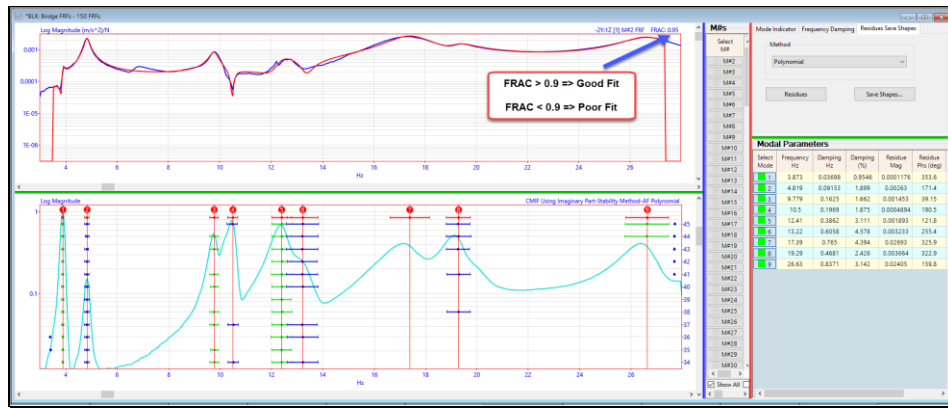
The Residue mode shapes estimated from the **FRFs** are used to synthesize the same **FRFs** that were used in Step 3 to define the Input-Output dynamics between the two sinusoidal force Inputs and the bridge response Outputs. So, how close are the **FRFs** synthesized from the mode shapes to the **FRFs** calculated from the experimental time waveforms?

One way to assess how accurately a **modal model** represents the structural dynamics compared with the **FRFs** is the **FRAC value between each FRF and its overlaid Fit Function**.

FRAC is short for **Frequency Response Assurance Criterion**.

FRAC measures the *co-linearity* between an FRF and its Fit Function during curve fitting.

FRAC has the same properties as **MAC**.



Fit Function Overlaid on FRF with FRAC.

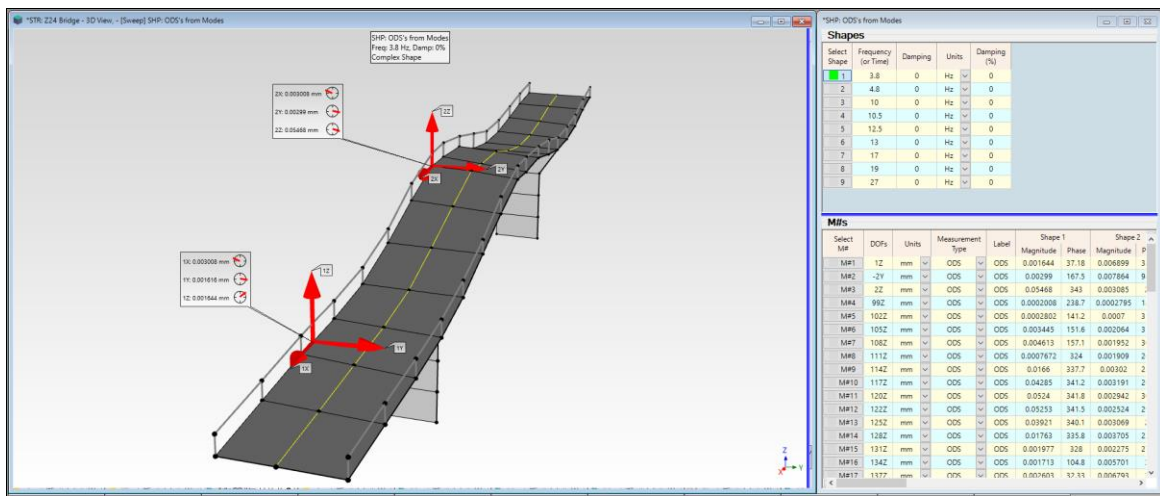
FRAC is displayed *in the upper-right corner* with each **FRF** and overlaid **red Fit Function**, as shown above.

- Press Hotkey 1 Curve Fit the FRFs again, and scroll through the FRFs to examine the **FRAC** values

STEP 4 - CALCULATING SINUSOIDAL ODS'S FROM MODE SHAPES

- Press Hotkey 4 Sinusoidal ODS's from Mode Shapes

In this step, the **Residue** mode shapes estimated from the multi-reference **FRFs** in **Step 1** are used to calculate responses of the bridge to sinusoidal Inputs. The mode shapes are saved in **SHP: Bridge Mode Shapes**. These mode shapes are called a **modal model** because, like the **FRFs**, they also *preserve the Input-Output dynamic properties* of the structure. When **Hotkey 4 is pressed**, sinusoidal ODS's at the *same eight frequencies* as those in **Step 3** are calculated and saved in a Shape Table (**SHP: ODS's from Mode Shapes**), and sweep animation is begun.



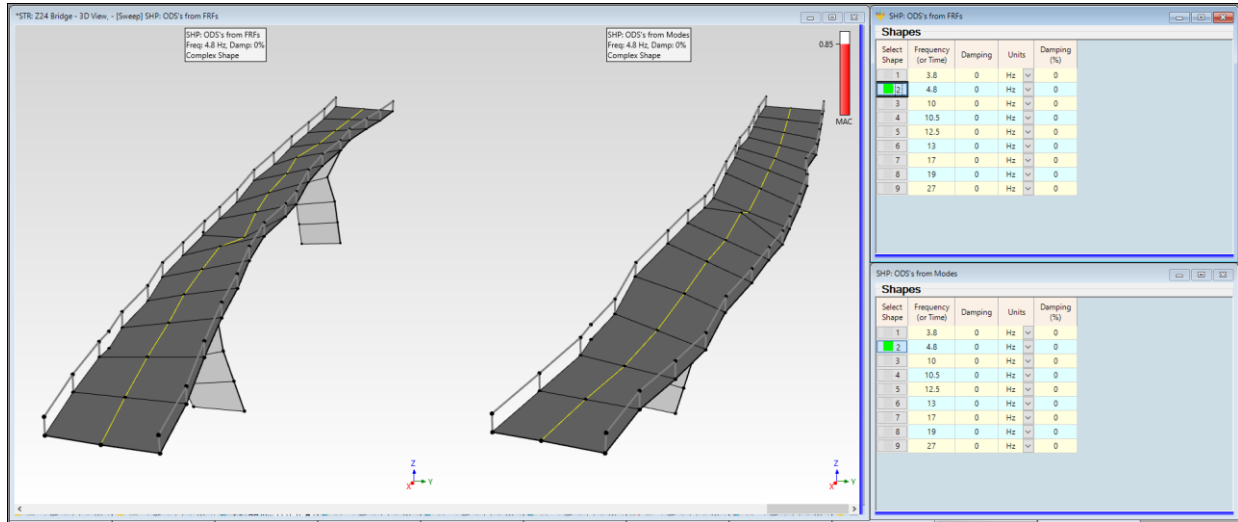
Animation of Sinusoidal ODS's Calculated from Mode Shapes.

STEP 5 - COMPARING ODS's FROM FRFs VERSUS MODE SHAPES

- **Press Hotkey 5 Compare ODS's from FRFs vs. Mode Shapes**

Sweep animation will begin with a selected ODS from **SHP: ODS's From FRFs** displayed side-by-side with the *closest matching ODS* from **SHP: ODS's From Mode Shapes**.

The closest matching ODS's are two shapes with the **Maximum MAC** value among all shape pairs.



Sinusoidal ODS from FRFs Compared with A Mode Shape from Reference 1Z.

Each sinusoidal ODS calculated from the **FRFs** *closely matched* with an ODS calculated from the mode shapes, each pair having **MAC** → *greater than 0.9*, except for the ODS's at 4.8 Hz which had **MAC** → **0.85**.

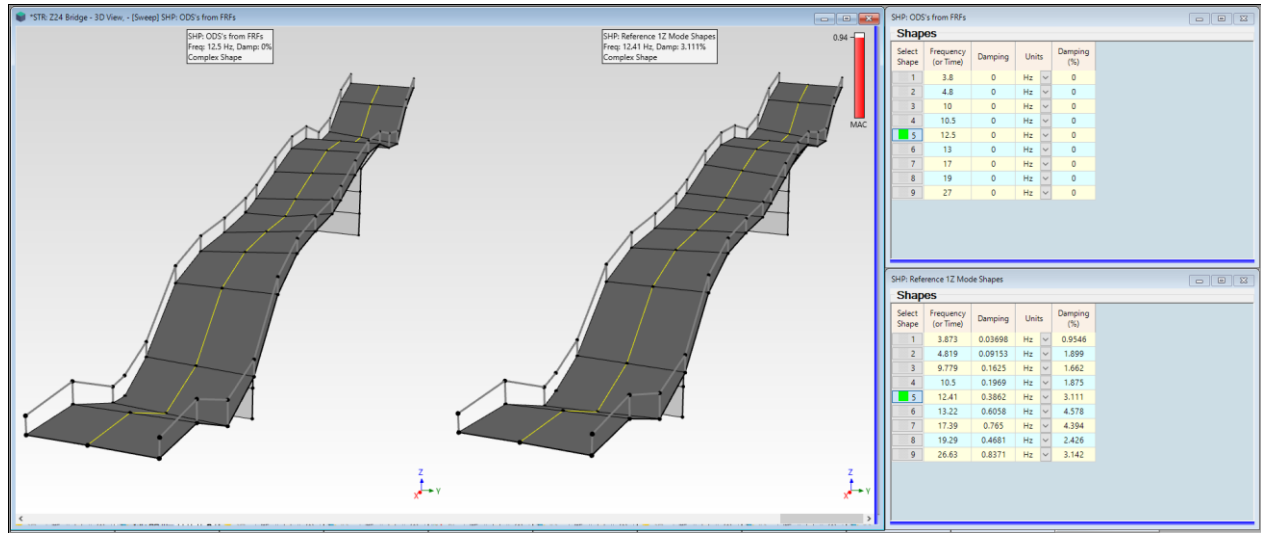
STEP 6 - COMPARING SINUSOIDAL ODS'S WITH REFERENCE 1Z MODE SHAPES

In this step, the mode shapes from **SHP: Reference 1Z Mode Shapes** are compared in animation with ODS's from **SHP: ODS'S From FRFs**.

- **Press Hotkey 6 Compare ODS's with 1Z Mode Shapes**

Sweep animation will begin with a selected ODS from **SHP: ODS's From FRFs** displayed side-by-side with the *closest matching mode shape* in **SHP: Reference 1Z Mode Shapes**.

When the sinusoidal ODS *at or near a modal frequency closely approximates a mode shape*, the ODS is being dominated by the mode shape at that resonant frequency.



Sinusoidal ODS from FRFs Compared with A Mode Shape from Reference 1Z.

Only three of the sinusoidal ODS's calculated from the **FRFs** *closely matched* with a mode shape from reference 1Z, having **MAC** → *greater than 0.9*. There *other six ODS's did not match* with a mode shape from reference 1Z, having **MAC** → *less than 0.9*.

CONCLUSION: The ODS at each resonant frequency had *participation from other modes* of the bridge than simply the mode shape at that frequency.

STEP 7 – REVIEW STEPS

To review all the steps of this App Note,

- **Press Hotkey 7 Review Steps**

SUMMARY

Responses to multiple sinusoidal forces on a bridge were calculated using the **Transform | Sinusoidal ODS** command in a Data Block containing **FRFs**, and the **Tools | Sinusoidal ODS** command in a Shape Table containing a **modal model** of the bridge. *Closest matching pairs* of the sinusoidal ODS's from both commands were displayed side-by-side in animation so they could be compared with one another.

The **MAC** bar chart below verifies that the **Transform | Sinusoidal ODS** command which used an **FRF matrix Input-Output** model *gave closely matched results* to the **Tools | Sinusoidal ODS** command which used a modal model.

All the ODS pairs *along the diagonal* of the bar chart have a **MAC** → *greater than* > 0.9 except the **4.8 Hz ODS** which had **MAC** → 0.85.

