MEscope Application Note 21

Calculating Bridge Responses to Multiple Shaker Excitation

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** and **VES-4000 Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the **AppNote21** project file. These steps might also require ***a more recent release date*** of MEscope.

APP NOTE 21 PROJECT FILE

- To retrieve the Project file for this App Note, **click here** to download **AppNote21.zip**

This Project file contains numbered Hotkeys & Scripts for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click on a* Hotkey** to display its Script window

INTRODUCTION

In the frequency domain, driving forces and response motions of a vibrating structure are related to one another by the following algebraic equation. The response motions at **N-DOFs** (degrees-of-freedom or points & directions) are related to the forces applied at **M-DOFs** by an equation which includes a **(N by M)** matrix of Frequency Response Functions (**FRFs**),

$$\{\mathbf{X(f)}\}_{\mathbf{Nx1}} = [\mathbf{H(f)}]_{\mathbf{NxM}} \cdot \{\mathbf{F(f)}\}_{\mathbf{Mx1}}$$

{F(f)} is an **M-*vector*** of the Fourier spectra (**DFTs**) of multiple excitation force **Inputs** at **M-DOFs**

{X(f)} is an **N-*vector*** of the Fourier spectra (**DFTs**) of response **Outputs** at **N-DOFs**

[H(f)] is an **(N by M) *rectangular matrix* of FRFs**

Each DOF of the **Input** & **Output** vectors contains a ***point number & direction***. Each **FRF** defines the dynamic properties of a structure between an **Input** DOF and an **Output** DOF.

If any two elements of the above **MIMO** equation are provided, the third element can be calculated using one of the following Data Block window commands in MEscope.

- **Transform | H1 FRFs ➔** calculates **Multiple Reference H1 FRFs** in the (N by M) **FRF** matrix given an **M-*vector*** of force **Input** time waveforms and an N-*vector* of response **Output** waveforms
- **Transform | H2 FRFs ➔** calculates **Multiple Reference H2 FRFs** in the (N by M) **FRF** matrix given an M-*vector* of force **Input** waveforms and an **N-*vector*** of response **Output** waveforms
- **Transform | Outputs ➔** calculates an **N-*vector*** of response **Output** time waveforms given an **(N by M) FRF matrix** and an **M-*vector*** of force **Input** waveforms (***see* App Note 21**)
- **Transform | Inputs ➔** calculates an **M-*vector*** of force **Input** waveforms given an **(N by M) FRF matrix** and an **N-*vector*** of response **Output** waveforms (***see* App Note 23**)
- **Transform | Sinusoidal ODS ➔** calculates an **ODS N-*vector*** given an (N by M) **FRF** matrix and an **M-*vector*** of sinusoidal force **Inputs** (*see* App Note 22)

In this App Note, the **Transform | Outputs** command is used to calculate response time waveforms given an **FRF** matrix and a vector of force **Input** waveforms. Calculated responses will be compared with measured responses to confirm the validity of the results.

Responses will be calculated using both **FRFs** and mode shapes. Both represent the dynamics of the bridge structure.

Elements of the **FRF** matrix can either be calculated from experimental data or synthesized from mode shapes of the structure. Both dynamic models are used in this App Note.

Z24 Bridge viewed from Bern-to-Zurich highway A.

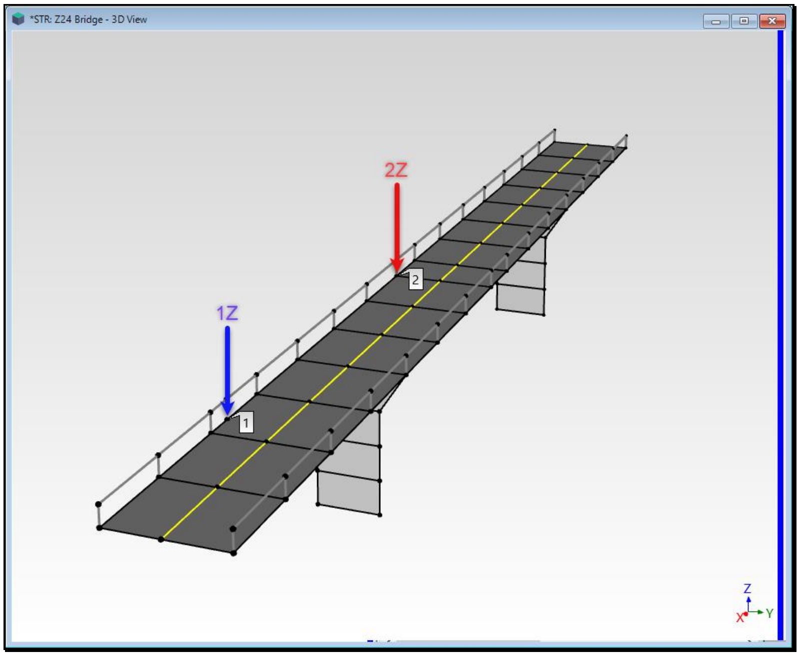
MULTIPLE MEASUREMENT SETS

The data used in this App Note was acquired in ***multiple* Measurement Sets** from the bridge shown in the figure above.

The bridge was tested using two hydraulic shakers with random forcing-functions applied to the shakers. Because of acquisition hardware limitations, **nine Measurement Sets** of force & response data were acquired. Each Measurement Set contains force & response data that was ***simultaneously acquired***.

The nine Measurement Sets of force & acceleration response time waveforms are contained in an MEscope Data Block **BLK: Z24 Bridge 2 Shaker Test**. Multiple Measurement Sets of data are automatically post-processed by the **FRF**, Outputs, & Inputs commands in the Transform menu in MEscope.

Deck of Z24 Bridge During Tests (The Replacement Bridge is Adjacent.)

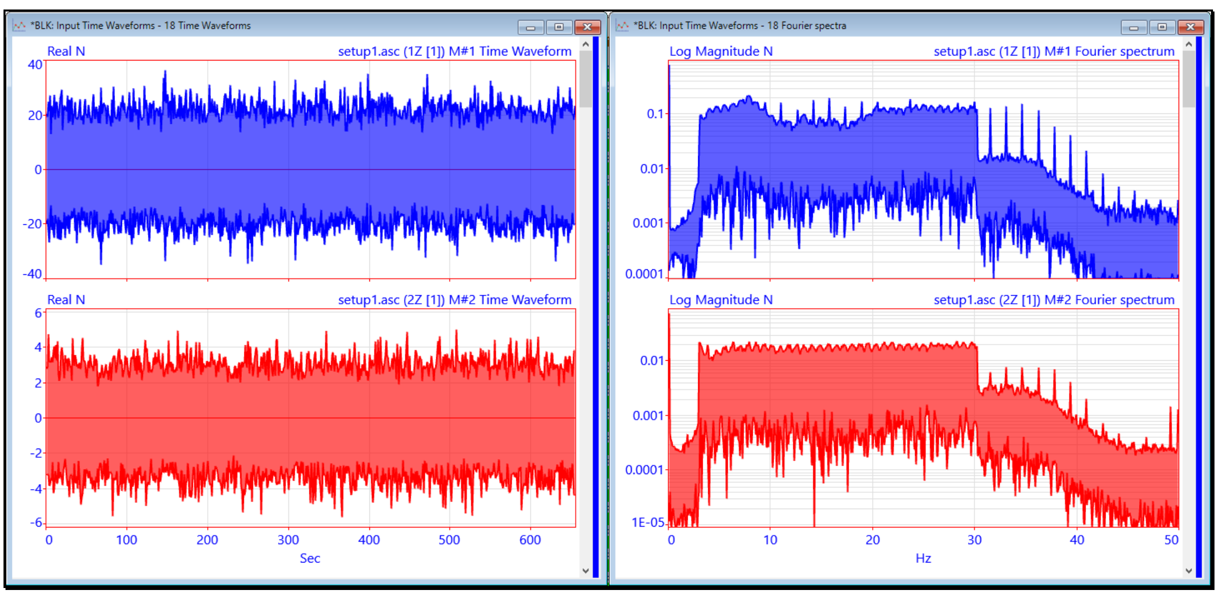


3D Model Showing Two Shaker Locations.

The two shakers had different force capacities. The ***larger shaker*** excited the bridge at **DOF 1Z** while the ***smaller shaker*** excited it at **DOF 2Z** as shown above. The shakers ***operated simultaneously*** and remained fixed throughout the test. We will see from the data that the larger shaker ***had a stronger influence*** on the response of the bridge than the smaller shaker.

SHAKER FORCE SIGNALS

The shakers were driven with computer-generated ***white random noise*** spanning a **3 Hz** to **30 Hz** frequency range. The shaker time waveforms and their spectra **for Measurement Set [1]** are shown in the figure below.

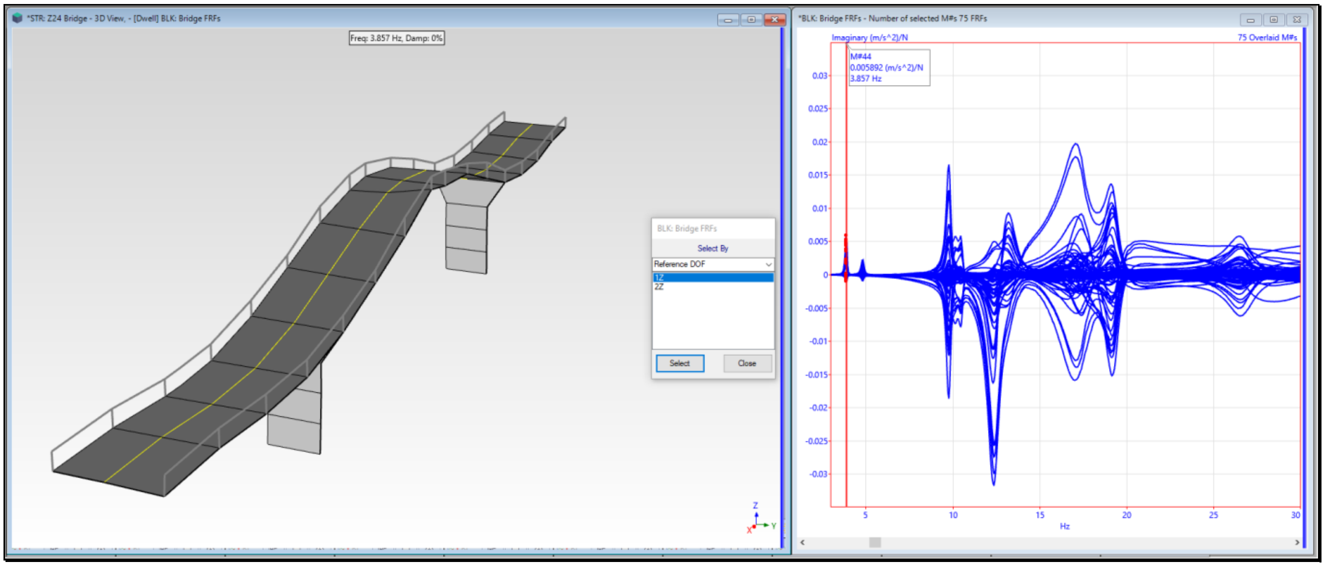


*Shaker Force Signals Applied to DOFs **1Z** & **2Z** During Acquisition of* Measurement Set [1].

STEP 1 – DISPLAYING ODS's FROM MULTIPLE REFERENCE FRFs

- ***Press* Hotkey 1 Bridge ODS's from Multi-Ref FRFs**

Animation of the **ODS** at **3.9 Hz** will begin, as shown below.



ODS at 3.9 Hz Displayed from DOF 1Z.

When distinct resonance peaks are clearly visible in a set of multi-reference **FRFs**, the **ODS** at the frequency of a resonance peak can only be displayed by selecting **FRFs** from ***one reference DOF at a time***.

Displaying a frequency-based **ODS** from different reference DOFs of multi-reference **FRF** data is a good way to determine whether an ***ODS is dominated by a single mode shape***.

If a frequency-based **ODS** displayed from one reference DOF ***is the same as*** the **ODS** displayed from another reference DOF, a ***single mode shape is dominating the ODS*** at that frequency.

Since **BLK: Bridge FRFs** contains multi-reference **FRFs**, **ODS**'s can only be displayed ***from one reference DOF at a time***.

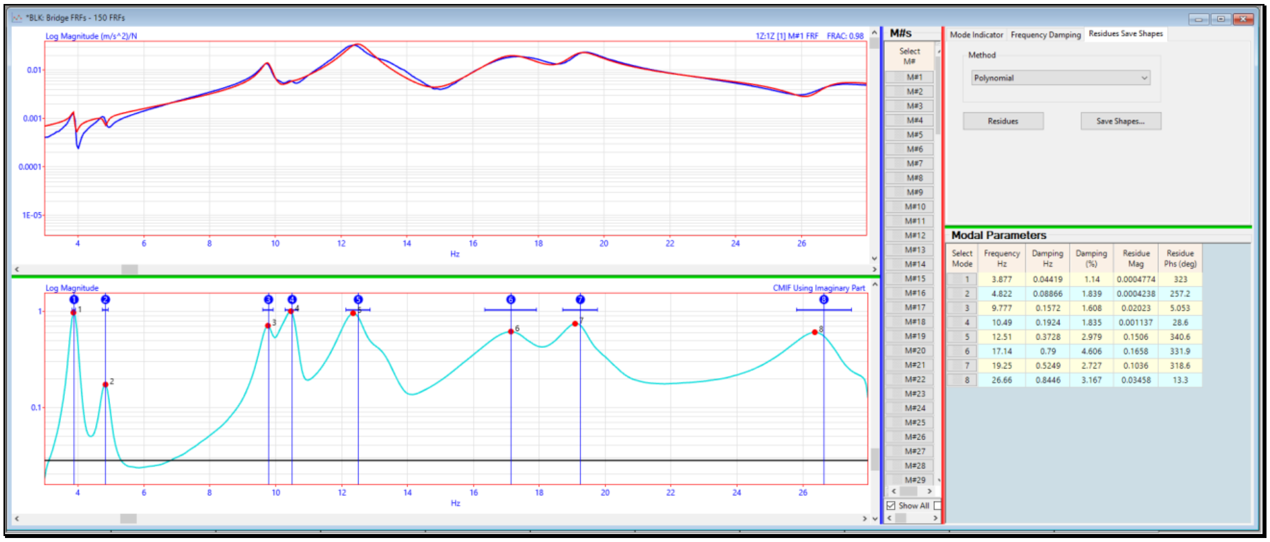
- ***Select* a Reference DOF** (**1Z** or **2Z)** in the **M# Select By** dialog box displayed in front of **BLK: Bridge FRFs**
- ***Drag* the Line cursor** to display the **ODS** at another resonance peak in **BLK: Bridge FRFs**

Displaying an **ODS** at a resonance peak in **BLK: Bridge FRFs** and selecting one Reference DOF and then another will show whether that **ODS** is ***dominated by one of the mode shapes*** of the bridge or not.

STEP 2 - CURVE FITTING MULTIPLE REFERENCE FRFs

In this step, **FRF**-based curve fitting is used to extract mode shapes from the **FRFs** in **BLK: Bridge FRFs**.

- ***Press* Hotkey 2 Curve Fit Multi-Ref FRFs**



Quick Fit Estimation of Modal Parameters for 8 Modes.

Even though these are multi-reference **FRFs**, they are curve fit using the **single-reference Quick Fit method**.

Multi-reference FRFs can always be curve fit using a ***single-reference curve fitting method***.

The **Residue** mode shapes extracted from **multi-reference FRFs** by a single-reference curve fitting method are **multi-reference Residue** mode shapes.

The multi-reference **Residue** mode shapes extracted from the multi-reference **FRFs** were saved in **SHP: Bridge Mode Shapes.**

STEP 3 – COMPARING MULTIPLE REFERENCE MODE SHAPES

- ***Press* Hotkey 2 -Compare Multi-Ref Mode Shapes**

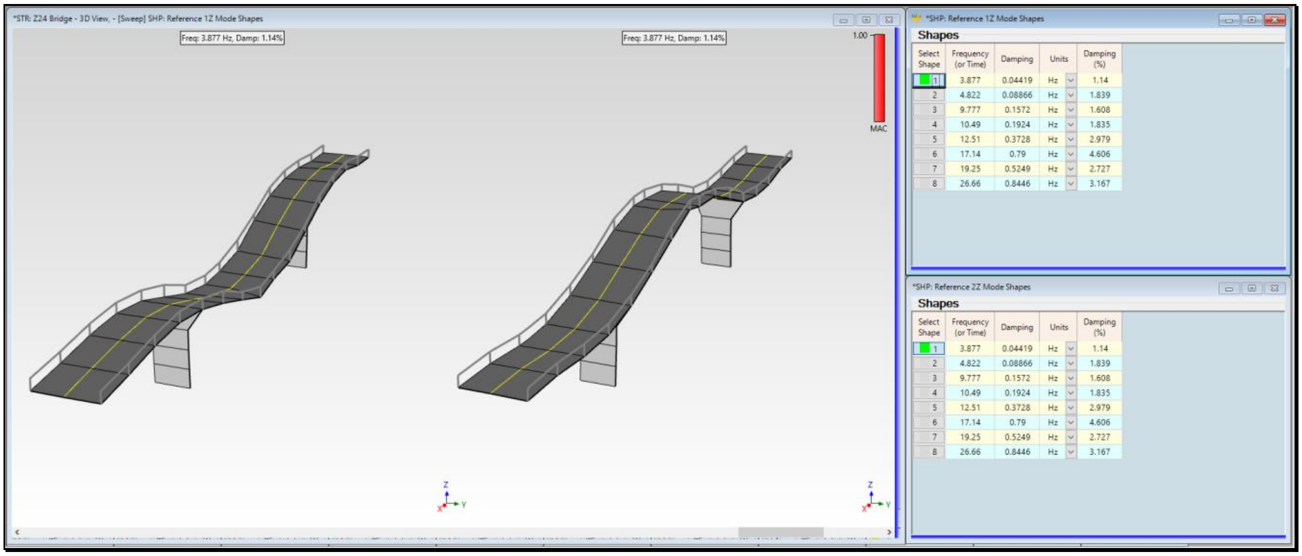
Sweep animation will begin with a selected mode shape from **SHP: Reference 1Z Mode Shapes** displayed side-by-side with the ***closest matching mode shape*** from **SHP: Reference 2Z Mode Shapes**.

Closest matching mode shapes are two mode shapes with the **Maximum MAC** value among all mode shape pairs.

To compare the mode shapes excited by the two shakers from locations **1Z** & **2Z** on the bridge, the mode shapes for each reference DOF were saved in separate Shape Tables (**SHP: Reference 1Z Mode Shapes** & **SHP: Reference 2Z Mode Shapes**) so they can be compared side-by-side in animation.

Multi-reference Residue mode shapes can only be displayed in animation from ***one* Reference DOF *at a time***.

If a ***mode shape does not change*** when animated from ***one* Reference DOF *versus another***, ***a single resonance*** was excited at the references, (in this case by two shakers), on the bridge structure.



Animated Bridge Mode Shapes from References 1Z & 2Z.

MODAL ASSURANCE CRITERION (MAC)

MAC is a measure of the ***co-linearity*** of two shape vectors. If two shapes ***lie on the same straight line***, they are said to be ***co-linear*** and their **MAC ➔** ***1.0***. If two shapes ***do not lie on the same straight line***, they are said to be ***linearly independent*** and their **MAC ➔** ***less than 1.0***. In MEscope, the following ***rules of thumb*** are used with **MAC**,

MAC values **➔** ***between 0 & 1***

MAC = 1.0 ➔ two shapes are ***co-linear***

MAC >= 0.9 ➔ two shapes are ***similar***

MAC < 0.9 ➔ two shapes are ***different*** (or ***linearly independent***)

In the side-by-side animation of mode shapes, ***six of the eight mode shape pairs*** from the two references (**1Z & 2Z**) have **MAC ➔** ***above 0.9***.

MAC ➔ ***above 0.9*** means that the ***same mode*** was excited by both shakers.

Two mode shape pairs (**shapes #6 & #7**) ***have low*** **MAC** ***values***.

The shape animation shows that the ***mode shapes from reference*** **1Z** ***appear to be more accurate***.

RESPONSE TIME WAVEFORMS

In the next two steps, the response time waveforms due to the two force waveforms applied at DOFs **1Z** & **2Z** in Measurement Set [1] are calculated. The responses will be calculated in two ways,

1. Using the **FRFs** in **BLK: Bridge FRFs**
2. Using the mode shapes in **SHP: Bridge Mode Shapes**

Since ***different random forces*** were applied to the bridge when each Measurement Set was acquired, the forced responses must be calculated separately for each Measurement Set.

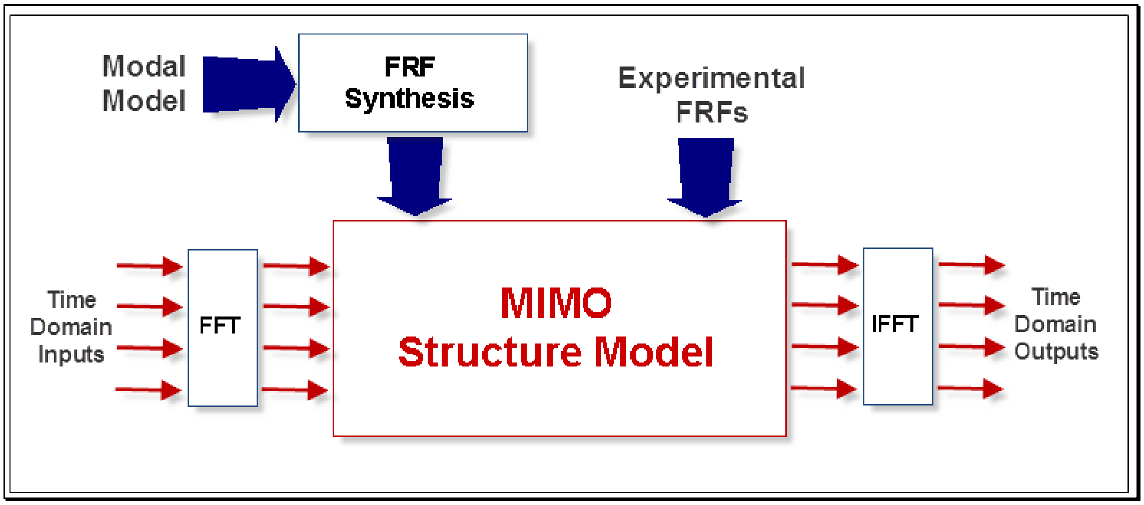
The calculated responses are then compared with the actual measured response time waveforms of Measurement Set [1].

BLOCK DIAGRAM OF OUTPUT CALCULATIONS

The block diagram below shows how responses due to the two shaker (**Input**) signals are calculated with the **Transform | Outputs** command. The two forces, together with either **experimental FRFs** (used in **Step 4**) or a **modal model** (used in the **Step 5**), are used to calculate the responses of the bridge acquired in Measurement Set [1].

The **FRF** matrix is called the **MIMO) Structure Model** in the figure below.

The **FRFs** (either experimental or synthesized from mode shapes) are multiplied by the Fourier spectra (**DFTs**) of the two excitation forces (**Inputs**) to obtain the Fourier spectra of the responses (**Outputs**). Then the Fourier spectra of the responses are ***Inverse Fourier transformed*** to yield the response time waveforms of the bridge.

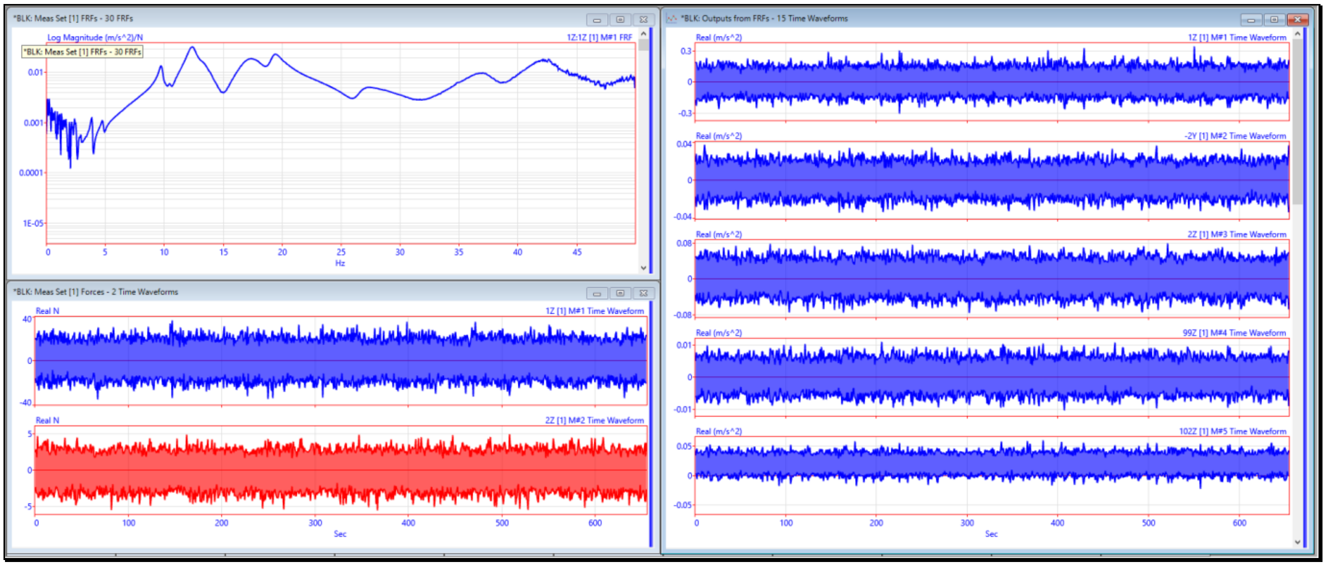


STEP 4 - CALCULATING THE BRIDGE RESPONSE USING FRFs

- ***Press* Hotkey 4 Bridge Response Using FRFs**

Only the forces and **FRFs** for **Measurement Set [1]** are used in this Step.

There are 30 **FRFs** for Measurement Set [1], ***15 between* Reference DOF 1Z *and 15 bridge responses***, and ***15 between* Reference DOF 2Z *and the same 15 bridge responses***.



Bridge Response Time Waveforms Calculated with FRFs.

The **FRFs** originally calculated from Measurement Set [1] are displayed ***on the upper-left*** and the two force time waveforms from Measurement Set [1] are ***on the lower-left***. The calculated responses (**Outputs**) for Measurement Set [1] are displayed ***on the right***.

Each of the **Output** time waveforms **on the right** has ***65,536 samples*** in it.

COMPARISON WITH MEASURED RESPONSES

Two different Data Block commands in MEscope are used to compare the calculated Responses with the measured Responses of the bridge for Measurement Set [1].

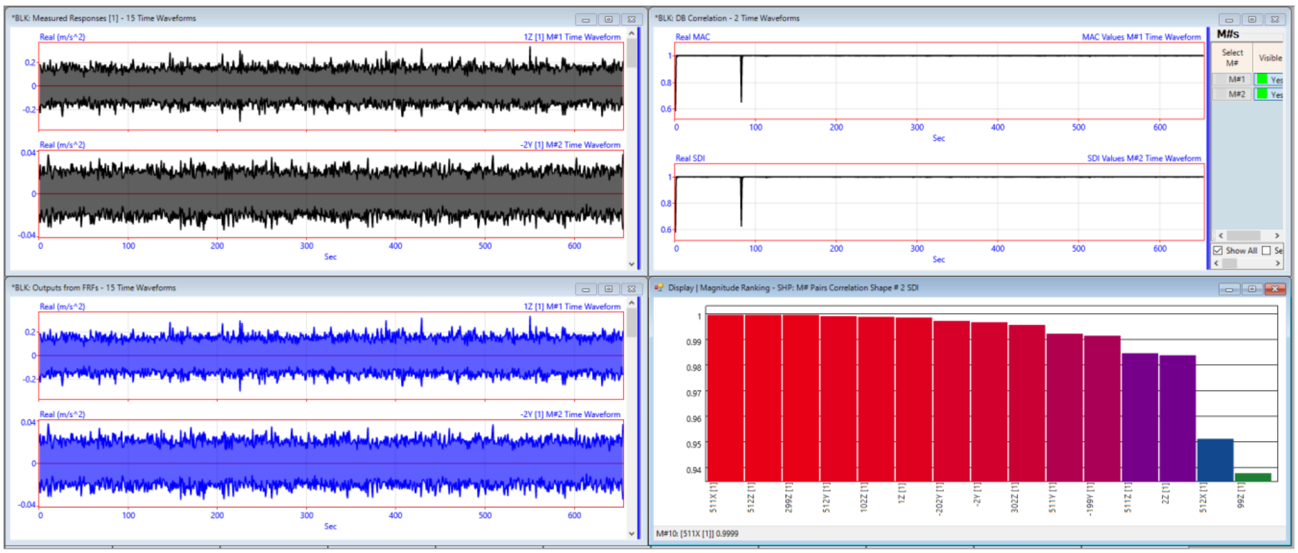
Tools | Data Block Correlation compares two Data Blocks at each sample.

Tools | M# Pairs Correlation compares pairs of ***M#s with the same*** **DOFs** between two Data Blocks.

STEP 5 – COMPARING MEASURED WITH CALCULATED RESPONSE FROM FRFs

- ***Press*** **Hotkey 5 Measured vs. Calculated Response from FRFs**

The **Measured responses** for Measurement Set [1] are displayed ***on the upper-left side***. The **Calculated responses** for Measurement Set [1] are displaced ***on the lower-left side***. The **Data Block Correlation** is displayed ***on the upper-right*** side, and the **M# Pairs Correlation** is displayed ***on the lower-right side***.



Response Time Waveforms Compared Using Data Block Correlation & M# Pairs Correlation.

DATA BLOCK CORRELATION

The Data Block Correlation on the upper-right side contains two **M#s**. **M#1** is the **MAC** value between each matching sample of data in the Measured & Calculated response Data Blocks ***on the left side***. **M#2** is the **SDI** value between each matching sample of data. **MAC** & **SDI** calculations were done for all samples of data between the two Data Blocks.

Both **MAC** & **SDI** have values **between 0 & 1.0**

MAC ***measures the co-linearity*** of two vectors of data

SDI ***measures the difference*** between two vectors of data

The **Block Size** of the Measured & Calculated Data Blocks is **65,536**.

With a couple exceptions, **MAC ➔ 1.0** *&* **SDI ➔ 1.0** for ***all 65,536 samples***

The Data Block Correlation confirms that the Calculated response matched the Measured response, sample by sample.

M# PAIRS CORRELATION

The **M# Pairs Correlation** command calculates the **MAC** & **SDI** between ***M#s with matching*** **DOFs** in the two Data Blocks ***on the left***.

These **MAC** & **SDI** values are saved as two shapes in a Shape Table.

Each shape component is the **MAC** or **SDI** value between ***all samples of the two*** **M#s** ***with matching*** **DOFs** in the two Data Blocks.

The Magnitude Ranking plot ***on the lower-right*** ranks the **SDI** values of each pair of **M#s** with matching DOFS in the Data Blocks of Measured & Calculated responses.

The ***lowest*** **SDI ➔ 0.9377** is between the two response time waveforms for **DOF 99Z**.

The M# Pairs Correlation also confirms that the **Calculated response** ***matched the*** **Measured response**, **M#** ***by*** **M#** ***and sample by sampl***e.

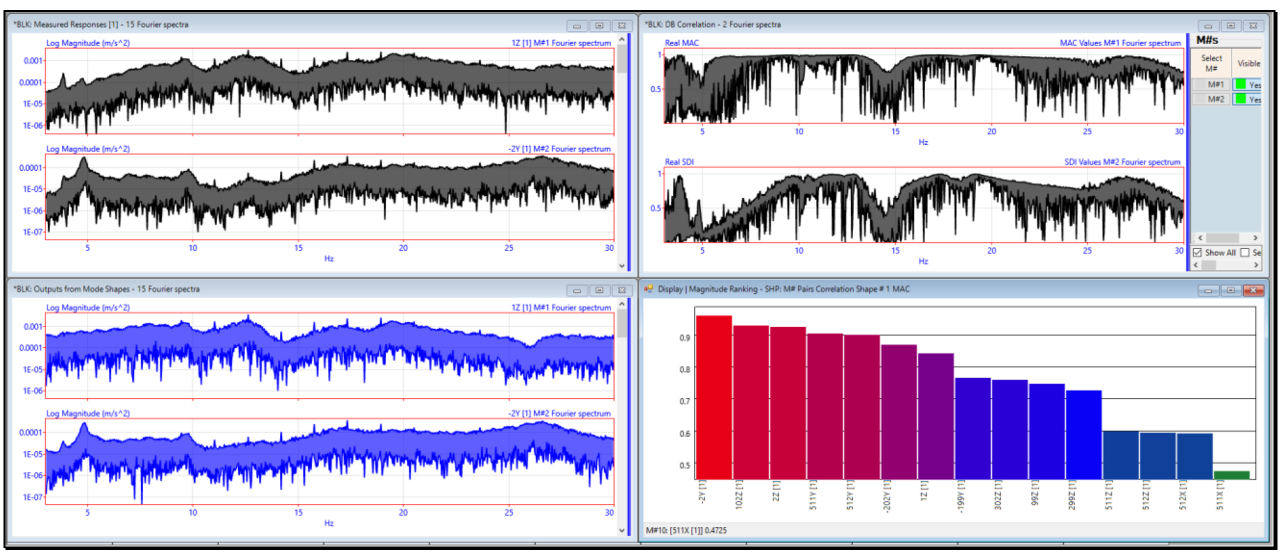
STEP 6 – COMPARING MEASURED WITH CALCULATED RESPONSE FROM MODE SHAPES

- ***Press*** **Hotkey 6 Compare Measured with Calculated Response from Mode Shapes**

Pressing **Hotkey 6** carries out the same calculations as ***pressing*** **Hotkey 5**, but instead of using the experimental **FRFs**, the multi-reference mode shapes estimated by curve fitting the experimental **FRFs** are used to synthesize multi-reference **FRFs**.

Since the mode shapes were only obtained by curve fitting over the **frequency span 3 to 30 Hz**, the Measured & Calculated bridge response spectra are compared over the ***same*** **3 to 30 Hz span**.

The Fourier spectra (**DFTs**) of the **Measured responses** for Measurement Set [1] are displayed ***on the upper-left side***. The Fourier spectra of the **Calculated responses** for Measurement Set [1] are displaced ***on the lower-left side***. The **Data Block Correlation** between the two Data Blocks on the left is displayed ***on the upper-right side***, and the **M# Pairs Correlation** between the two Data Blocks is displayed ***on the lower-right side***.



Response Fourier Spectra Compared Using Data Block Correlation & M# Pairs Correlation.

DATA BLOCK CORRELATION

The Data Block Correlation ***on the upper-right*** shows **MAC** & **SDI** values ***nearly equal to*** *1.0* at or near the modal frequencies.

This indicates that the **FRFs** synthesized from the modal model by the **Transform | Outputs** command were accurate ***near the frequencies of the mode shapes***.

M# PAIRS CORRELATION

The **M# Pairs Correlation** command calculated the **MAC** & **SDI** between ***M#s with matching*** **DOFs** in the two Data Blocks. These **MAC** & **SDI** values are saved as two shapes in a Shape Table. Each shape component is the **MAC** or **SDI** value between ***all samples of the two*** **M#s** ***with a matching*** **DOF**.

The Magnitude Ranking plot **on the lower-right** ranks the **MAC** values of each pair of **M#s** with matching DOFs in the Calculated & Measured response Data Blocks.

Several **MAC** values ➔ ***greater than*** **0.9**. The rest of the **MAC** values ➔ ***below*** **0.9**.

Low **MAC** values mean that the **modal model** of the bridge was ***not as accurate*** a representation for the dynamics of the bridge ***over the entire frequency span*** **3 to 30 Hz**, when matching pairs of response Fourier spectra are numerically compared with each other.

STEP 7 - REVIEW STEPS

To review all the steps of this App Note,

- ***Press*** **Hotkey 7 Review Steps**

CONCLUSIONS

For Measurement Set [**1**], the **FRF-based** Calculated responses ***closely matched all the Measured responses***, whereas the Calculated **Mode Shape-based** responses only matched ***near the frequencies of the mode shapes*** and only for ***a few matching*** **DOFs**. It can be concluded from these results that the **FRF** matrix model more accurately modeled the structural dynamics of the bridge, which it should have because the FRFs were ***originally calculated from the*** **Input & Output time waveforms**.

Although the modal model included what appeared to be all the significant modes of the bridge, over the 3 to 30 Hz frequency span, nevertheless they still represented a truncated form of the bridge dynamics.

FRF INTERPOLATION

To calculate responses using the MIMO equation, the **FRFs** ***were interpolated between samples*** to match the frequency-axis parameters of the Fourier spectra of the force **Input** waveforms. The Block Size of **BLK: Bridge FRFs** is 1024 samples. The Fourier spectra of the force **Input** waveforms had a spectrum Block Size of 32,768 samples. Therefore, to multiple the **FRFs** by the **Input** Fourier spectra, linear interpolation was performed to ***expand each FRF*** from 1024 samples to 32,768 samples.

The close correlation of the Calculated & Measured response waveforms verifies that the linear dynamics of the bridge were preserved even in the heavily-interpolated **FRFs**.

TRUNCATED MODAL MODEL

When the modal model (consisting of eight multi-reference mode shapes) was used to calculate the bridge response waveforms, the ***required*** **FRFs** ***were synthesized*** using the mode shapes to match the 32,768 samples of the frequency-axis of the Fourier spectra of the force **Inputs**.

The synthesized **FRFs** yielded a noise-free **FRF** matrix model, but the ***truncated dynamics represented by the*** **modal model** only gave a close matching of the Calculated & Measured response Fourier spectra ***near the frequencies of the mode shapes*** and for ***only a few matching*** **DOFs**.