MEscope Application Note 20

Calculating FRFs from a Multi-Shaker Bridge Test

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** option. Without this option, you can still carry out the steps in this App Note using the **AppNote20** project file. These steps might also require MEscope software with ***a more recent release date***.

APP NOTE 20 PROJECT FILE

- To retrieve the Project file for this App Note, **click here** to download **AppNote20.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down*** **the Ctrl key and *click on a* Hotkey** to open its Script window

INTRODUCTION

In the frequency domain, driving forces and response motions of a vibrating structure are related to one another by the following MIMO equation. Multiple response **Outputs** ($\{X(f)\}_{Nx1}$)at **N-DOFs** (degrees-of-freedom or points & directions) are equal to multiple force **Inputs** ($\{F(f)\}_{Mx1}$) applied at **M-DOFs** multiplied by an (**N by M)** matrix ($[H(f)]_{NxM}$) of Frequency Response Functions (**FRFs**).

$$\{X(f)\}_{Nx1} = [H(f)]_{NxM} \cdot \{F(f)\}_{Mx1}$$

{F(f)} is an ***M-vector*** containing the Fourier spectra of excitation forces (**Inputs**) at **M-DOFs**

{X(f)} is an ***N-vector*** containing the Fourier spectra of responses (**Outputs**) at **N-DOFs**

[H(f)] is an **(N by M)** ***rectangular matrix*** of **Multiple Reference FRFs**

Each **DOF** of the Input & Output vectors contains a ***point number & direction***. Each **FRF** defines the dynamic properties of a structure between an Input **DOF** and an Output **DOF**.

If any two elements of the above MIMO equation are provided, the third element can be calculated using one of the following Data Block window commands in MEscope.

- **Transform | H1 FRFs ➔** calculates Multiple Reference H1 **FRFs** in the (**N by M) FRF matrix** given an ***M-vector*** of force **Input TWFs** and an ***N-vector*** of response **Output** waveforms
- **Transform | H2 FRFs ➔** calculates Multiple Reference H2 **FRFs** in the **(N by M) FRF matrix** given an ***M-vector*** of force **Input** waveforms and an ***N-vector*** of response **Output** waveforms
- **Transform | Outputs ➔** calculates an ***N-vector*** of response Output **TWFs** given an **(N by M) FRF matrix** and an ***M-vector*** of force **Input** waveforms (***see* App Note 21**)
- **Transform | Inputs ➔** calculates an ***M-vector*** of force **Input** waveforms given an **(N by M) FRF matrix** and an ***N-vector*** of response **Output** waveforms (***see* App Note 23**)
- **Transform | Sinusoidal ODS ➔** calculates an **ODS *N-vector*** given an **(N by M) FRF matrix** and an ***M-vector*** of sinusoidal force **Inputs** (*see* App Note 22)

In this App Note, the **Transform | H1 FRFs** command is used to calculate **Multiple Reference H1 FRFs** from multiple excitation force **Input TWFs** (**TWFs**) and multiple response **Output TWFs**. When **Multiple Reference H1 FRFs** are calculated, other functions such as **Multiple Coherence** and **Partial Coherence** and **Auto & Cross Spectra** can also be calculated.

Z24 Bridge Viewed from the Bern-to-Zurich Highway A1

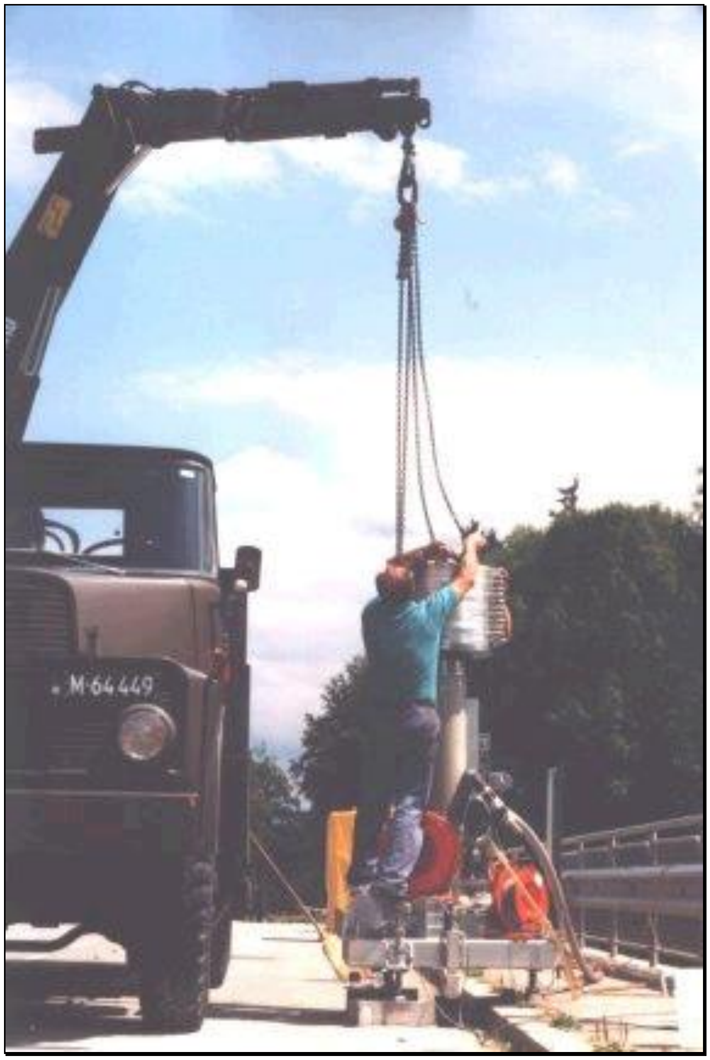
MULTIPLE MEASUREMENT SETS

The data used in this App Note was acquired in ***multiple*** **Measurement Sets** from the bridge shown in the figure above.

The bridge was tested using two hydraulic shakers with random forcing-functions applied to the shakers. Because of acquisition hardware limitations, **nine Measurement Sets** of force & response data were acquired. Each Measurement Set contains force & response data that was ***simultaneously acquired***.

The nine Measurement Sets of force & acceleration response **TWFs** are contained in an MEscope Data Block **BLK: Z24 Bridge 2 Shaker Test**. Multiple Measurement Sets of data are automatically post-processed by the commands in the **Transform** menu in MEscope.

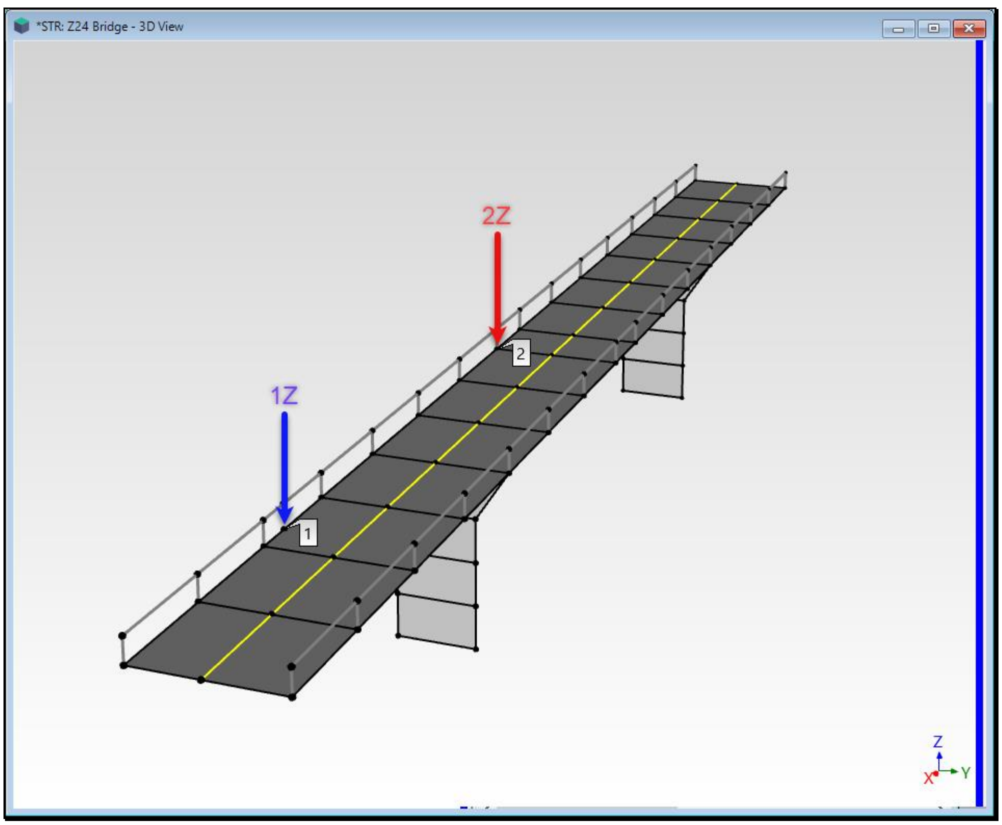
Deck of Z24 Bridge During Tests (The Replacement Bridge is Adjacent.)



Installing one of the two shakers used to Excite the Bridge.

Installing the Accelerometers for one Measurement Set.

Close-up of a Seismic Accelerometer.



3D Model Showing Fixed Shaker Locations.

The ***larger shaker*** excited the bridge at **DOF 1Z** while the ***smaller shaker*** excited it at **DOF 2Z** as shown above.

The shakers ***operated simultaneously*** and remained fixed throughout the test.

We will see from the data that the ***larger shaker*** had a ***much stronger influence*** on the response of the bridge than the ***smaller shaker***.

NINE MEASUREMENT SETS

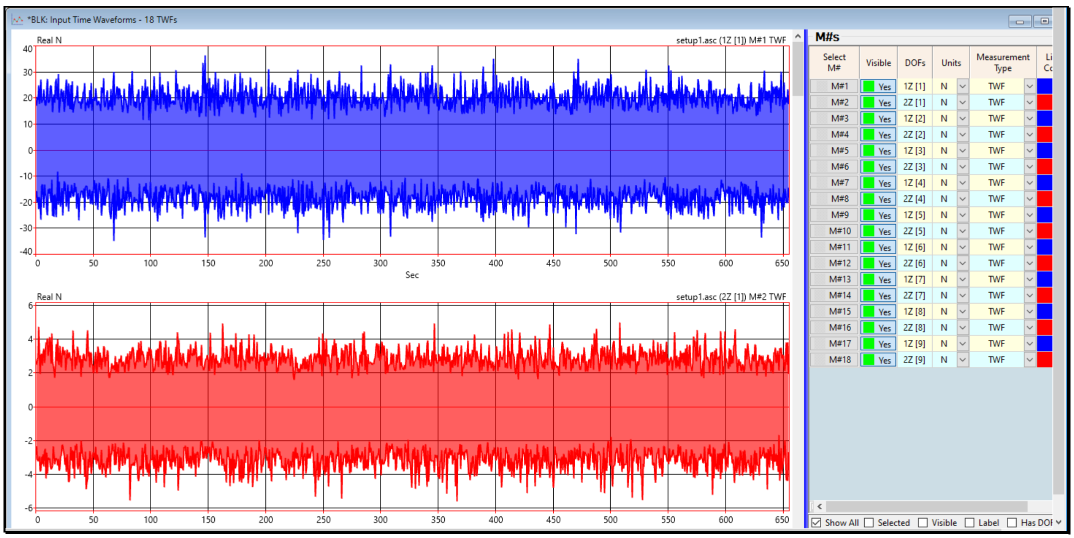
The data was collected in ***9* Measurement Sets**. Bridge response motions were measured using seismic accelerometers at **75 *different* DOFs**. The accelerometers were moved to ***different unique* DOFs** before each Measurement Set was acquired. During the acquisition of each Measurement Set, the two force signals applied by the shakers at **DOFs 1Z** & **2Z** were ***simultaneously acquired*** along with unique acceleration responses. Each Measurement Set also contained the ***same acceleration responses at* DOFs (1Z, -2Y, & 2Z)**.

- Forces were ***simultaneously acquired*** at **DOFs 1Z & 2Z** along with accelerations at **72 unique DOFs** in 9 Measurement Sets.
- Forces were ***simultaneously acquired*** at **DOFs 1Z** & **2Z** along with accelerations at **DOFs (1Z, -2Y, & 2Z) *in all*** Measurement Sets.

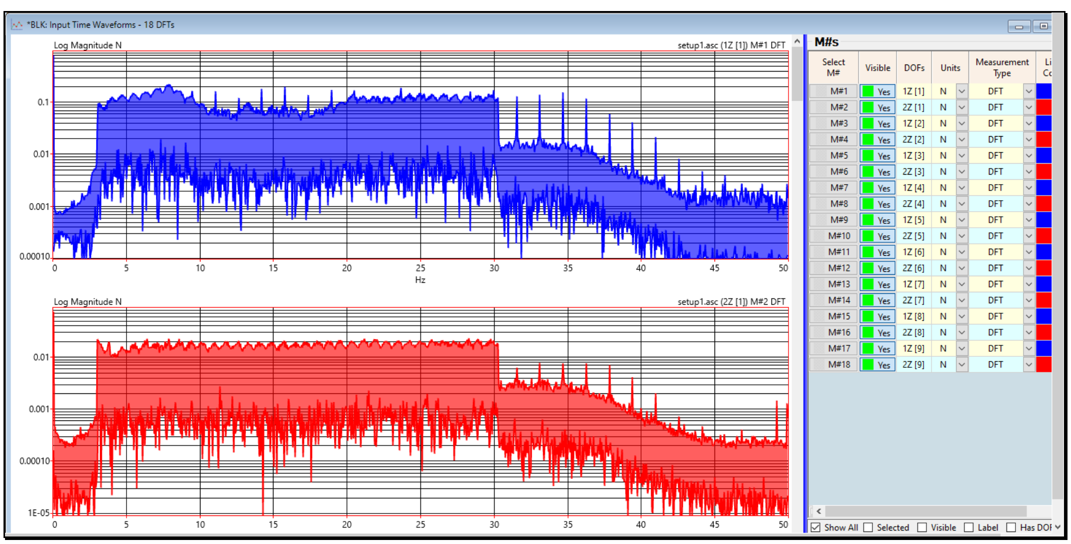
The forces & response accelerations at the same **DOFs** (**1Z**, **-2Y**, & **2Z)** in each Measurement Set will be used to examine the ***consistency*** among the 9 Measurement Sets as well as to confirm ***structural reciprocity***.

SHAKER FORCE SIGNALS

The shakers were driven with computer-generated ***white random noise*** spanning a **3 Hz** to **30 Hz** frequency range. The shaker **TWFs** and their spectra for Measurement Set [1] are shown in the figure below.



*TWFs of Shaker Force Signals Applied to **DOFs** 1Z & 2Z During Acquisition of Measurement Set [1].*



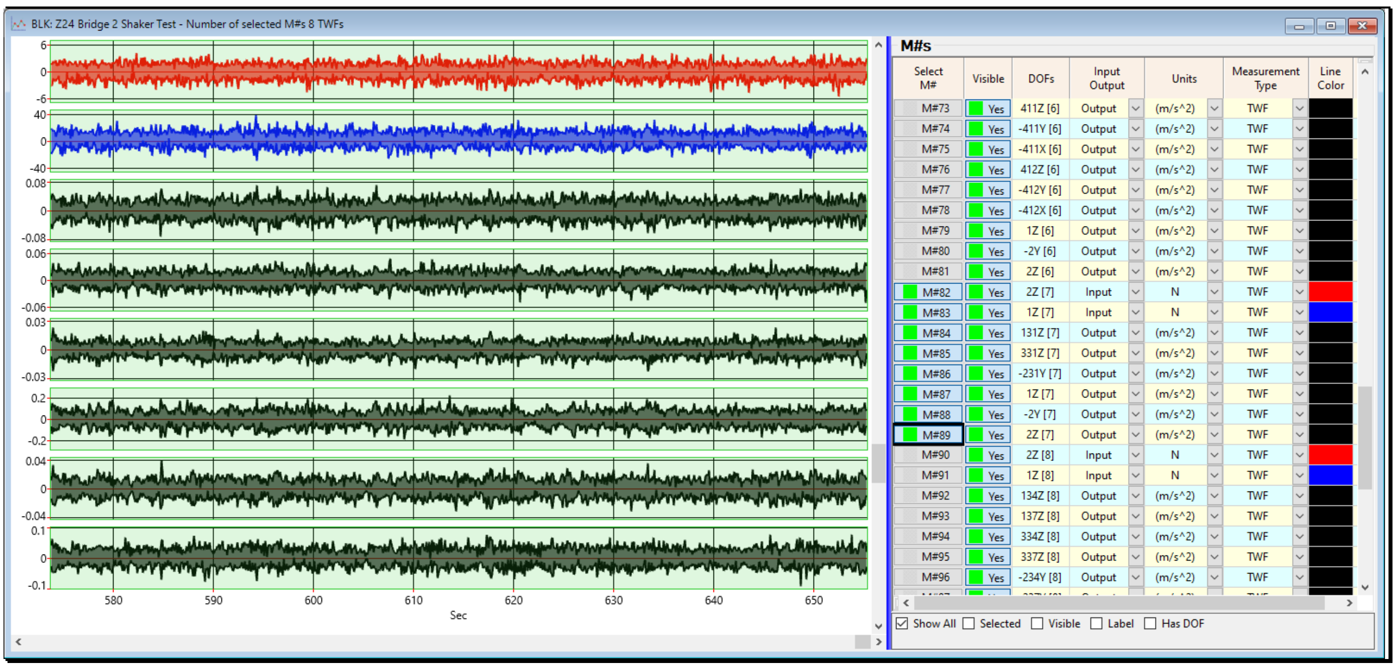
DFTs of Shaker Force Signals Applied to DOFs 1Z & 2Z During Acquisition of Measurement Set [1].

Measurement Set	Force DOFs 1Z, 2Z	Common Response DOFs 1X, -2Y, 2Z	Roving DOF M#s	TOTAL M#s
1	**2**	**3**	**12**	**17**
2	**2**	**3**	**6**	**11**
3	**2**	**3**	**9**	**14**
4	**2**	**3**	**6**	**11**
5	**2**	**3**	**6**	**11**
6	**2**	**3**	**12**	**17**
7	**2**	**3**	**3**	**8**
8	**2**	**3**	**12**	**17**
9	**2**	**3**	**6**	**11**
TOTALS	**18**	**27**	**72**	**117**

Each Measurement Set contains a different number of **M#s**. The table above lists the number of **TWF M#s** acquired in each Measurement Set.

All the **TWFs** in each Measurement Set were used to calculate **FRFs** & **Coherences**.

The **M#s** for **Measurement Set [7]** in **BLK: Z24 Bridge 2 Shaker Test** are shown below.



Measurement Set [7] (8 Selected M#s in BLK: Z24 Bridge 2 Shaker Test).

STEP 1 - CALCULATING FRFs & COHERENCES

- ***Press* Hotkey 1 Multi-Ref FRFs & Coherence**

In this step, **FRFs** & **Coherences** are calculated from the **9 Measurements Sets** of **TWF** data. Each Measurement Set contains **TWFs** acquired at the force **DOFs** (**1Z & 2Z**) together with some of the **72 *unique response* DOFs** plus **27 *common response* DOFs** that are contained in all Measurement Sets.

BLK: Z24 Bridge 2 Shaker Test contains **117 M#s**. Each **TWF** in **BLK: Z24 Bridge 2 Shaker Test** contains **65,536 samples** of data. This Block Size could be used to calculate ***one* Digital Fourier Transform (DFT)** with Block Size of **32,768**.

When a ***smaller* Spectrum Block Size** is chosen, the amount of **TWF** data required permits the averaging together of ***many Auto and Cross spectra***.

When ***Hotkey 1 is pressed*** **FRFs & Coherences** are calculated using the following Script parameters.

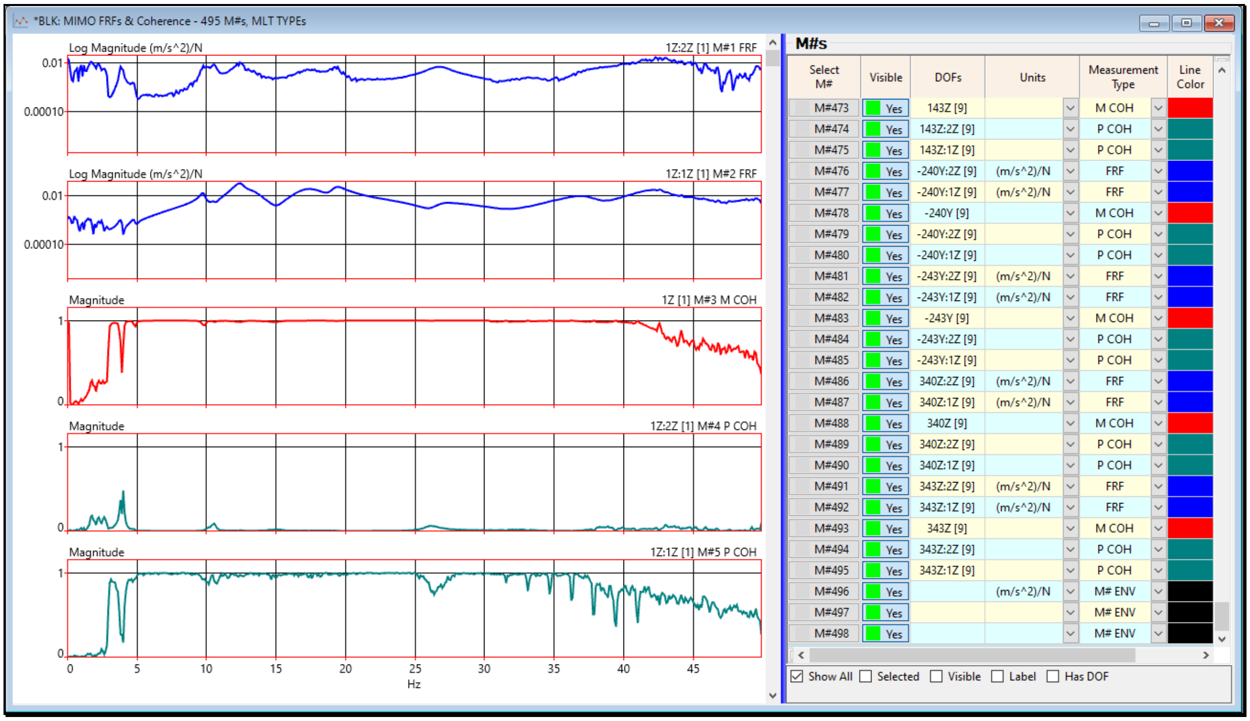
- Spectrum Block Size ➔ 512
- Number of Averages ➔ 128
- Spectrum Averaging ➔ Stable
- Percent Overlap Processing ➔ **50%**

These parameters create spectrum averaging using **50 Percent Overlap Processing**. **1024 samples of TWF data** are required to calculate each **DFT with 512 samples** in it. Therefore 128 spectra are calculated using **50% "new" data** and **50% "old" data** from the previous sampling window of **TWF** data.

The M#s in **BLK: Z24 Bridge 2 Shaker Test** contain random waveforms which are ***not periodic, (not completely contained), within their TWF sampling window***. As a result, the spectra calculated from these **TWFs** will contain ***significant leakage***.

Leakage in a **DFT** means that data at the resonance peaks ***will leak into adjacent frequencies***, thus causing errors at and around resonance peaks. Applying a **Hanning** window to each **TWF** prior to transforming it to its **DFT** ***reduces leakage at its resonance peaks*** and in the resulting **FRF**.

When the calculations are completed, the **BLK: MIMO FRFs & Coherence** window will open, as shown below.



*2 **FRFs**, 1 Multiple Coherence, & 2 Partial **Coherences** for Response* **DOF** 1Z.

- Use the **scroll bars *on the right side*** of each Data Block to scroll through a display of the **M#s**

The **BLK: MIMO FRFs & Coherence** window contains **495 M#s** consisting of the following,

- **198 FRFs**
- **144 FRFs** with **unique DOFs** (***72 unique*** **DOFs** paired with 2 force **DOFs**)
- **54 FRFs with the same DOFs** (**6 response DOFs** (**1X, -2Y, 2Z**) paired with **2 force DOFs** for each Measurement Set)
- **99 Multiple Coherences** (one for each unique response **DOF** plus one for **DOFs** (**1X, -2Y, 2Z**) from each Measurement Set
- **198 Partial Coherences** (one for each response **DOF** and 2 force **DOFs**)

STEP 2- FRFs WITH THE SAME DOFs

- ***Press*** **Hotkey 2 Overlay Redundant FRFs**

Redundant data was acquired in every Measurement Set. In Step 1, ***6 FRFS with the same DOFs*** were calculated for each Measurement Set. These 54 **FRFs** will be used to check for ***consistency*** of the test data and ***structural reciprocity***.

FRFs were calculated between the force **DOFs** (**1Z & 2Z**) and the same response **DOFs** (**1X, -2Y, 2Z**)

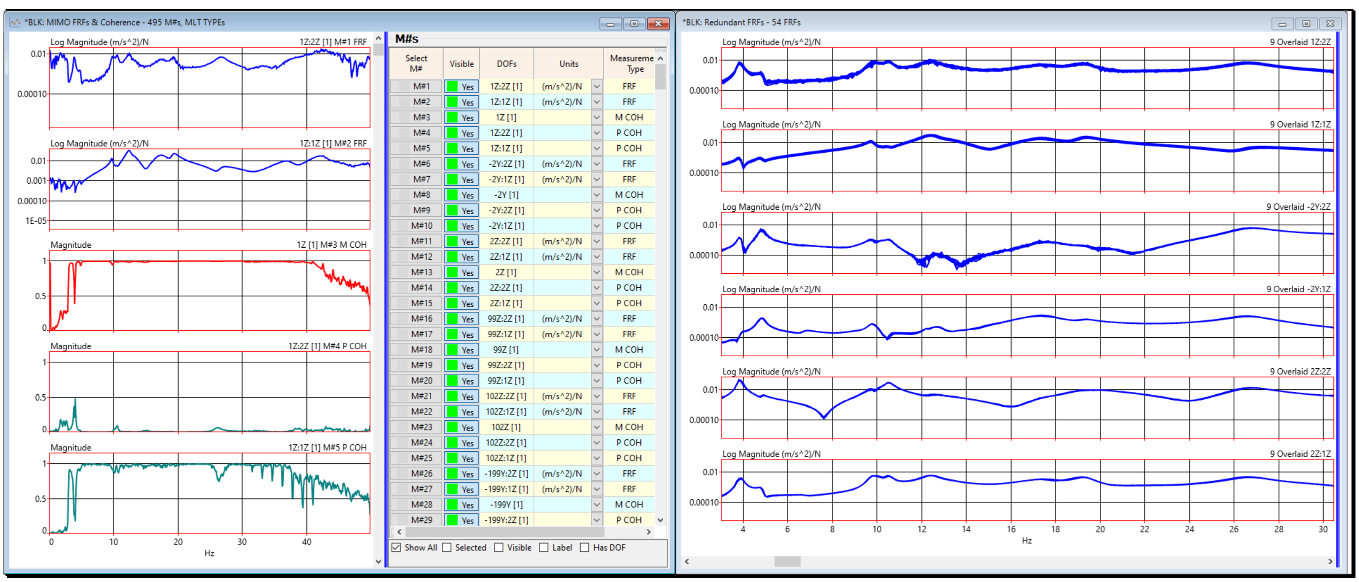
When **FRFs** with the same **DOFs** are overlaid, they should be the same.

The overlaid **FRFs** are shown below. The **FRFs** & **Coherences** calculated in **Step 1** are displayed on the left and **6 FRFs** with the same **DOFs** in all 9 Measurement Sets are overlaid on the right.

- Since the two shakers ***only excited the bridge from 3 to 30 Hz***, the overlaid **FRFs** are only displayed over that frequency span

The overlaid **FRF** log magnitudes on the right verify that the data acquisition in all 9 Measurement Sets ***was very consistent***.

The overlaid FRFs also show that the FRFs with 1Z as a reference are more consistent than the FRFs with 2Z as a reference.

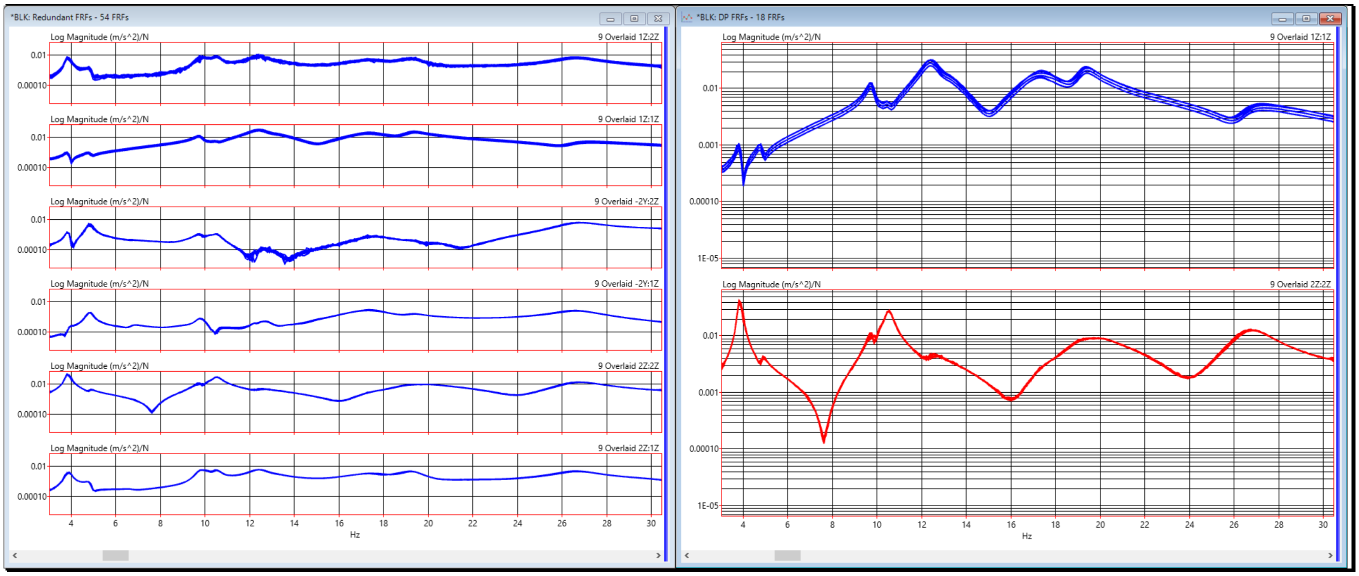


*Six Groups of Overlaid **FRFs** with Redundant **DOFs** (3 to 30 Hz)*

STEP 3 – DRIVING POINT FRFS

In this step only the Driving Point **FRFs** (**1Z:1Z** & **2Z:2Z**) are overlaid.

- ***Press* Hotkey 3 Driving Point FRFs**



*Driving Point **FRFs** Overlaid (3 to 30 Hz)*

The 6 groups of **FRFs** with the same **DOFs** from all 9 Measurement Sets are overlaid on the left, and the **two Driving Point FRFs from all 9 Measurement Sets** are overlaid on the right.

The nine **Z2:Z2 FRFs** (for the **smaller shaker**) are nearly identical.

The differences between the nine **Z1:Z1 FRFs** (for the **larger shaker**) are basically small gain changes.

These driving point **FRFs** verify that the structure was undergoing ***linear stationary vibration*** while data was being acquired for the 9 Measurement Sets.

STEP 4 – STRUCTURAL RECIPROCITY

- ***Press* Hotkey 4 Reciprocity**

Structural reciprocity ➔ an **FRF** calculated from a response at **DOF A *due to excitation at* DOF B** is the same as the **FRF** calculated from a response at **DOF B *due to excitation at* DOF A**.

Structural reciprocity *is usually assumed* when a modal test is performed on a structure.

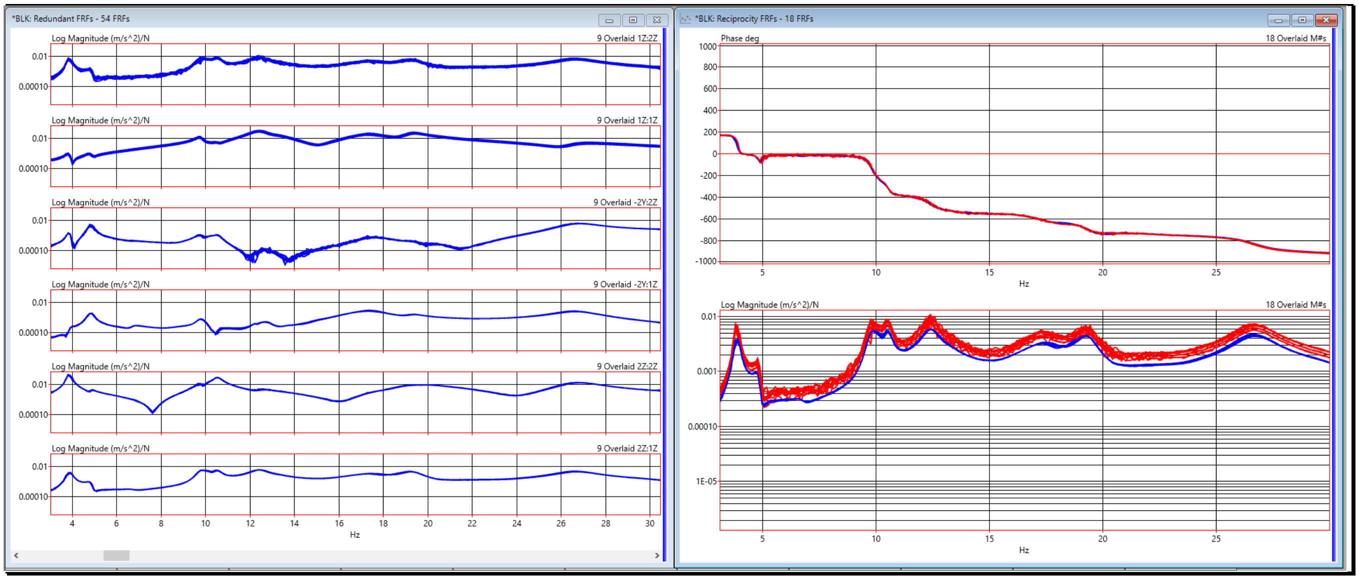
Reciprocity can be checked by overlaying the nine **FRFs** with **DOFs 2Z:1Z** with the nine **FRFs** with **DOFs 1Z:2Z**. ***When Hotkey 4 was pressed***, pairs of these two **FRFs** were calculated from all 9 Measurement Sets of **TWF** data.

Structural reciprocity is a ***basic property of a linear dynamic system***.

Structural reciprocity ***is usually assumed*** during a modal test.

When **structural reciprocity *is valid***, mode shapes can be extracted from ***any row or column of* FRFs** in the **FRF matrix** of the MIMO equation.

If structural reciprocity is not valid, the ***entire* FRF matrix *must be measured*** to capture all the dynamic properties of a structure.



Nine ***2Z:1Z FRFs*** *and Nine* ***1Z:2Z FRFs*** *Overlaid on the Right.*

The **18 overlaid FRFs** on the right above confirm **structural reciprocity** between the two shaker locations.

- The 18 unwrapped phases ***are the same***
- The 18 overlaid log magnitudes ***are essentially the same***

These overlaid plots confirm that the bridge was behaving dynamically in a ***linear symmetric manner***.

STEP 5 – MULTIPLE COHERENCE

- ***Press* Hotkey 5 Multiple Coherence**

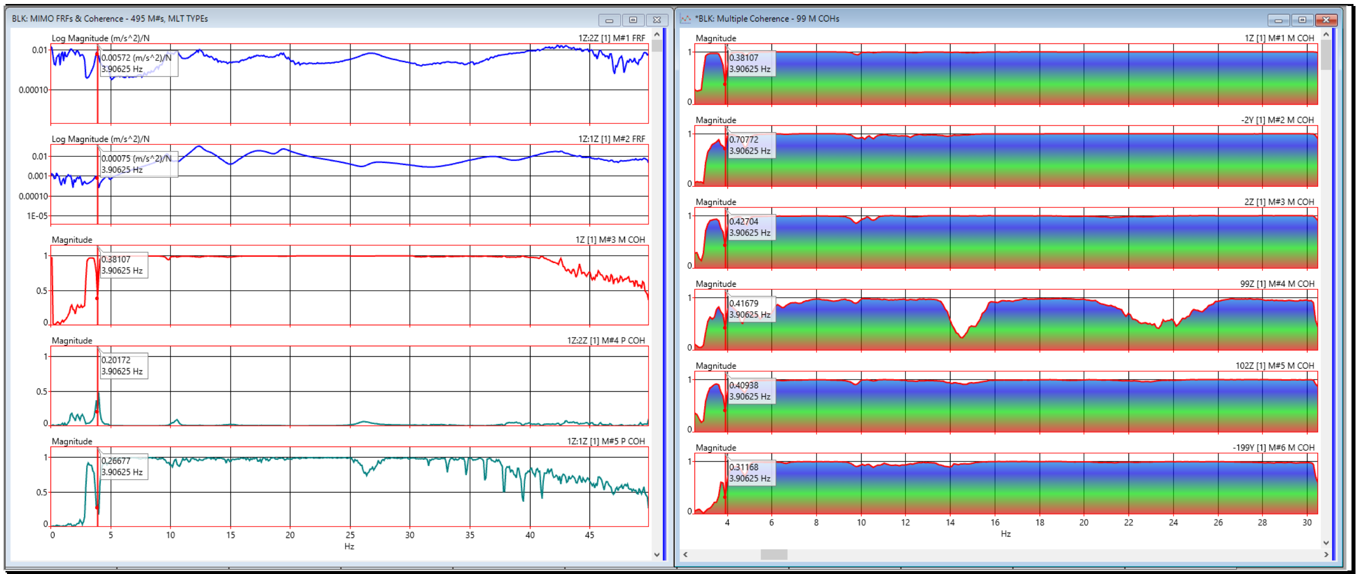
Multiple and **Partial Coherence** can also be used to verify the validity of multi-shaker measurements.

Like the Ordinary Coherence, Multiple Coherence and Partial Coherence also ***have values between* 0 & 1**.

Coherence ➔ **"1"** means that all the measured response is ***linearly related to (or caused by)*** the measured force.

Coherence ➔ ***less than* "1"** means the response and the force ***are unrelated (or uncorrelated)***.

In this case, **Multiple Coherence** answers the question, ***"At each frequency, how much of the response at each DOF is linearly related to the two forces measured at DOF 1Z & DOF 2Z?"***



*Multiple **Coherences** for 99 Response **DOFs**.*

- Use the **scroll bar** on the right side to scroll through the Multiple **Coherences**

In the figure above, the **Multiple Coherences** (**M#1 & M#3**) on the right verify that the **FRFs *accurately model*** the response at both **DOFs (1Z & 2Z)** due to excitations at **DOFs 1Z** & **2Z**, except near the first resonance peak at **3.9 Hz**.

- The cursor values in the Data Block on the left show that at **3.9 Hz**, the Multiple Coherence (**M#3**) dips to ***a value much less than 1***

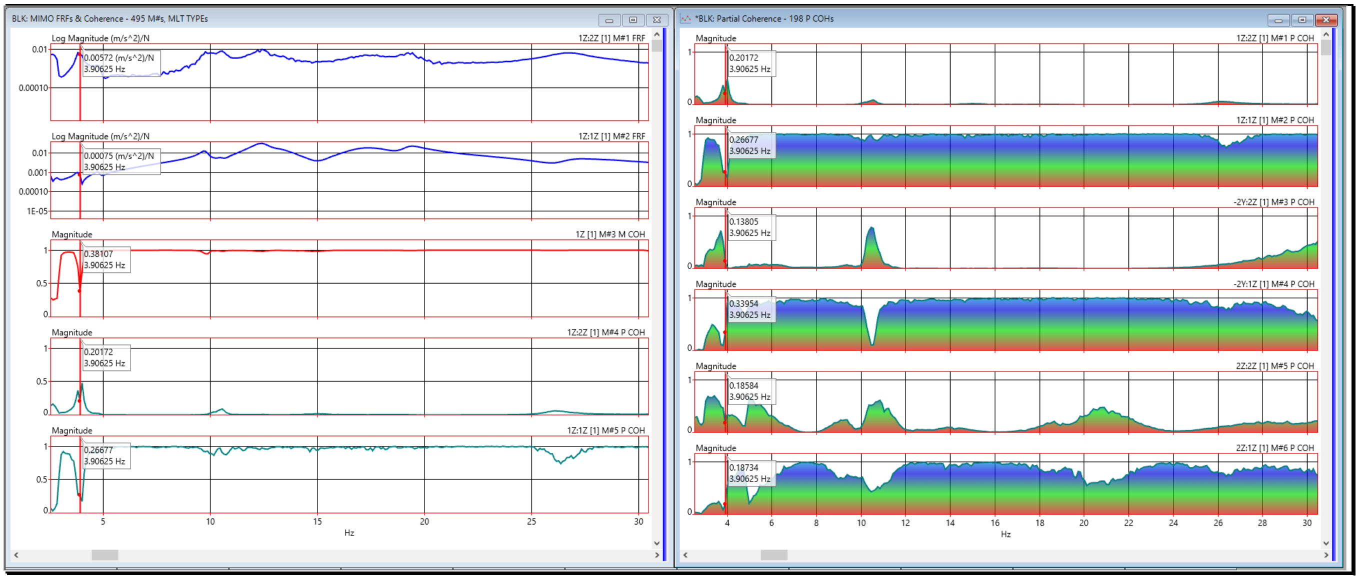
STEP 6 – PARTIAL COHERENCE

- ***Press* Hotkey 6 Partial Coherences**

Since there are two excitation shakers, there are **two Partial Coherences** for each response.

Partial Coherences answer a different question: "At each frequency, how much of a response is linearly related to the force at DOF 1Z and how much response is linearly related to the force at **DOF 2Z**?"

For any given response **DOF**, the ***sum of all* Partial Coherences** at each frequency should be approximately equal to the **Multiple Coherence**.



Partial Coherences *for Response* ***DOFs*** *1Z, -2Y, 2Z.*

On the right side of the figure above, the **two Partial Coherences** for the **DOF 1Z** (**M#1 & M#2**) show that its response is ***almost completely caused*** by the force applied at **DOF 1Z**.

The **Partial Coherence 1Z:2Z (M#2)** ***is nearly zero*** for all frequencies.

The **Partial Coherence** for **DOFs -2Y & 2Z (M#3 through M#6 on the right)** indicates that those two responses are ***caused primarily by*** the shaker at **DOF 1Z**.

The shaker at **1Z** is located ***20 meters away from*** **Point 2**!

STEP 7 - ODS's OF THE BRIDGE

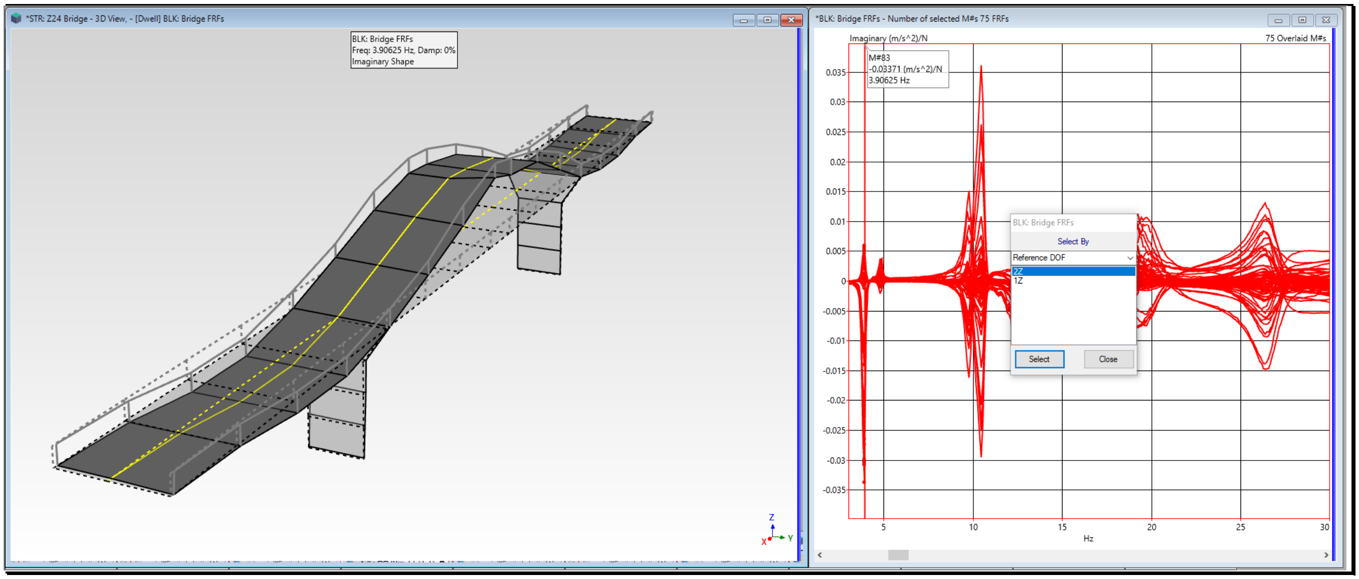
- ***Press* Hotkey 7 Bridge ODS's**

Animation of the **ODS** at **3.9 Hz** will begin, as shown below.

When distinct resonance peaks are clearly visible in a set of multi-reference **FRFs**, the **ODS** at the frequency of a resonance peak can only be displayed by selecting **FRFs** from ***one reference DOF at a time***.

Displaying **ODS**'s from different reference **DOFs** of multi-reference **FRF** data is a good way to determine whether an ***ODS is dominated by a single mode shape***.

If the **ODS** displayed from one reference **DOF** ***is the same as*** the **ODS** displayed from another reference **DOF**, a ***single mode shape is dominating the ODS*** at that frequency.



Since BLK: Bridge **FRFs** contains multi-reference **FRFs**, **ODS**'s can only be displayed ***from one reference DOF at a time***.

- Select a Reference **DOF** (**1Z** or **2Z)** in the M# **Select By** dialog box displayed in front of **BLK: Bridge FRFs**
- ***Drag*** the Line cursor to display the **ODS** at the overlaid resonance peaks in **BLK: Bridge FRFs**

Display of the animated **ODS**'s at the resonance peaks in **BLK: Bridge FRFs** show that each **ODS** is ***dominated by one of the mode shapes*** of the bridge.

STEP 8 – REVIEW STEPS

To review all the steps of this App Note,

- ***Press* Hotkey 8 Review Steps**