VIBRANT

MEscope Application Note 19

Using a Stability Diagram to Estimate Modal Frequency & Damping

> The steps in this Application Note can be carried out using any MEscope package that includes the **VES-4600 Advanced Modal Analysis** option. Without this option, you can still carry out the steps in this App Note using the **AppNote19** project file. These steps might also require MEscope software with *a more recent release date*.

APP NOTE 19 PROJECT FILE

- To retrieve the Project file for this App Note, **click here** to download **AppNote19.zip**

This Project file contains *numbered Hotkeys & Scripts* for carrying out the steps of this App Note.

- *Hold down* **the Ctrl key** and *click* **on a Hotkey** to display its Script window

INTRODUCTION

In this Application Note a Stability diagram will be used to determine,

- The number of modes that are represented by resonance peaks in a set of **FRFs**
- A *stable estimate* of the modal frequency & damping of each mode

In MEscope, three different curve fitting methods can be used together with a Stability diagram to estimate frequency & damping. Two of the methods: the **AF Polynomial** and the **Z-Polynomial** method use the **Rational Fraction Polynomial** form of an FRF to curve fit the experimental data. The other method; the **Complex Exponential** method, curve fits a set of impulse response functions (**IRFs**) to estimate modal frequency & damping.

> The MEscope approach to curve fitting is to *use several different curve fitting methods* to estimate modal parameters (frequency, damping, & mode shape), and compare the results.
>
> If the *modal parameter estimates from different curve fitting methods are essentially the same*, the estimates did not depend on any one curve fitting method.

DETERMINING THE MODEL SIZE

> The first and most important step in the use of any curve fitting method is to determine *how many modes are represented by resonance peaks* in a set of measurements.

In MEscope, this can be done in two ways,

1. **Counting peaks** on a **Mode Indicator** curve
2. Using a **Stability diagram**

COUNTING PEAKS ON A MODE INDICATOR

As a first step in curve fitting, the resonance peaks on one or more Mode Indicator curves can be counted to determine how many modes of vibration are represented in a set of FRF measurements. When the **Count Peaks** command is executed on the **Mode Indicator** tab, the peaks above a **horizontal noise threshold line** are counted and displayed in the **Peaks** box on the tab.

> The same number in the **Peaks** box also appears in the **Modes** box on the **Frequency Damping** tab. This number is used as the **model size** by the **Polynomial** curve fitting method.

Each Mode Indicator is also used to locate resonances so that more accuracy can be gained by curve fitting in *smaller frequency cursor bands* surrounding each resonance peak.

Both the **Local & Global Polynomial** curve fitting methods use the number of peaks counted on the Mode Indicator as their curve fitting model size.

STABILITY DIAGRAM

When modes are ***heavily-coupled*** so that it is not clear how many resonances are represented in a particular frequency band, an ***incorrect number of peaks*** may be counted on a Mode Indicator. When resonance peaks cannot be accurately counted on a Mode Indicator curve, a **Stability diagram** should be used instead.

Using a **Stability diagram**, curve fitting is applied to the measurement data using model sizes ranging from **1 to a Maximum Model Size**.

If the **frequency & damping estimates** of a pole are ***essentially the same for different model sizes***, those pole estimates belong to a **Stable Pole Group.**

The **Stability diagram** is a plot of pole estimates for many different model sizes. In the diagram below, pole estimates are displayed for model sizes up to 50 modes. A **Modal Indicator** which shows three resonance peaks is plotted in the background.

In a Stability diagram, pole estimates are plotted on top of the **Mode Indicator**.
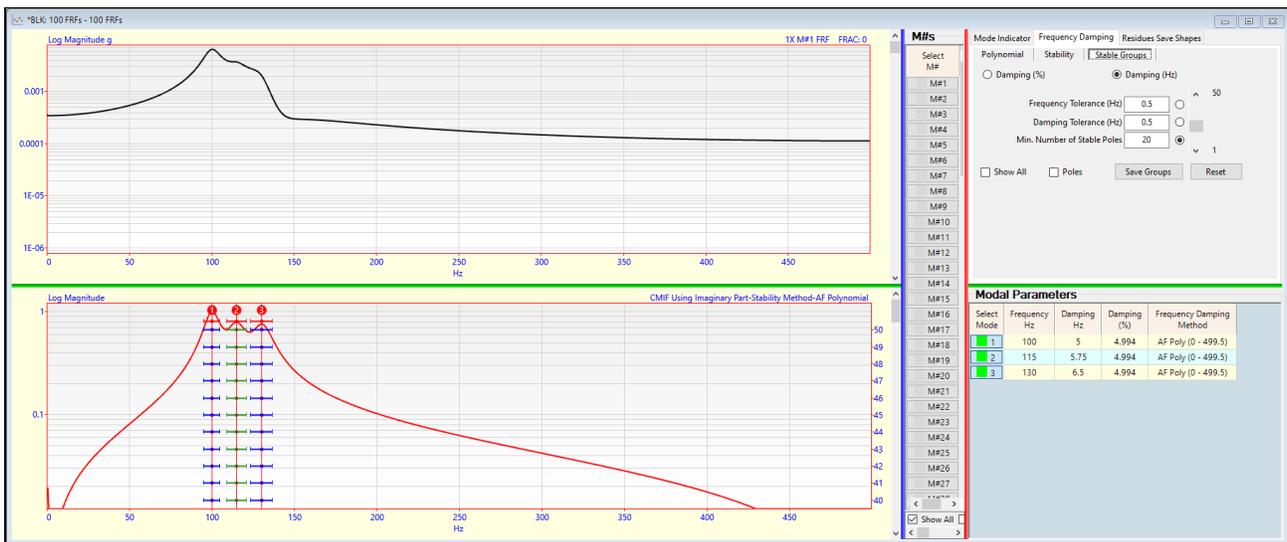
Stability Diagram Showing Three Stable Pole Groups.

Each pole is displayed with,

- A **vertical line & dot** at the **frequency** of the pole
- A **horizontal line** equal in length to the **half power point damping** of the pole
- Placing the mouse pointer ***near a pole*** will ***display its frequency & damping estimates*** as shown above

STABLE POLE GROUP

A group of poles is called **Stable Pole Group** if each pole estimate within the group has frequency & damping estimates within the user-specified tolerances on the **Stable Groups** curve fitting tab. Three different tolerances are used to define a **Stable Pole Group,**

- **Frequency Tolerance** (in Hz)
- **Damping Tolerance** (in Hz or %)
- **Minimum Number of Stable Poles** that satisfy the frequency & damping tolerances

In the figure above, *at least 20 poles* with *frequency & damping estimates within 0.5 Hz of each other* were required to define the three Stable Pole Groups.

All the poles in a Stable Pole Group are *displayed using the same color*.

The **second & third Contour Colors** in the Data Block (**BLK**) Options box are used to *alternately color* successive Stable Pole Groups.

THREE CLOSELY-COUPLED MODES

Three closely-coupled modes, with **known frequency & damping** and mode shapes with **100 random M#s**, were used to synthesize time waveforms using the **File | New | Data Block** command. The dialog box for creating the 100 time waveforms is shown below.

Dialog for Creating the New Data Block with 100 Time Waveforms.

These parameters in the dialog box created a new Data Block with **100 time waveforms** in it. Each **M#** is an impulse response function (**IRF**) containing three modes at **100, 115, & 130 Hz,** each with **5% damping** and a residue with *random magnitude & phase*. Each time waveform has **2000 samples**, with **0.001 seconds resolution** between samples spanning a **total time of 2 seconds**.

- A New Data Block **BLK: 100 FRFs** was already created using the parameters shown above and is saved in this Project

- Execute **Transform | Inverse FFT** in **BLK: 100 FRFs** to transform the **FRFs** to **IRFs**, as shown below
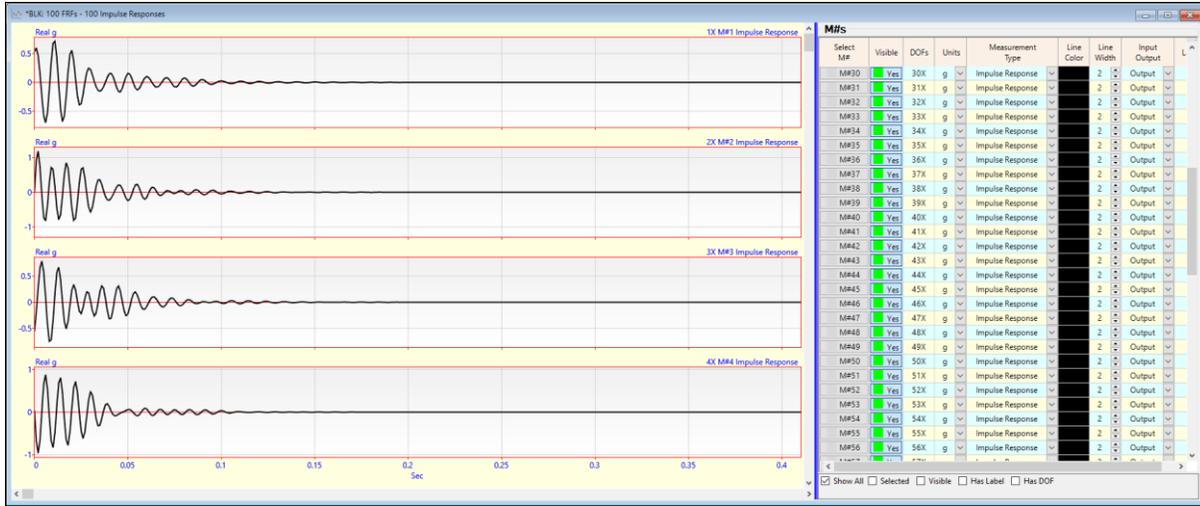
Data Block With 100 Impulse Response Functions.

- Execute **Transform | FFT** in the **BLK: 100 FRFs** window to transform the IRFs back to **FRFs**

USING A STABILITY DIAGRAM

During curve fitting, three **Curve Fitting** tabs are displayed *on the right*, and the **FRFs** and Mode Indicator are displayed *on the upper & lower left*, as shown below. The three tabs are for carrying out the three curve fitting steps,

1. **Count Peaks on a Mode Indicator**
2. **Estimate Frequency & Damping**
3. **Estimate Residues & Save Shapes**

A Stability diagram carries our Steps 1 & 2 *in a single step*.

STEP 1 - STABILITY DIAGRAM USING THE AF POLYNOMIAL METHOD

- *Press* **Hotkey 1 Stability using AF Poly**

When **Hotkey 1 is** *pressed*, the **AF Polynomial** method is used to curve fit the **FRFs** in **BLK: 100 FRFs**. When it is finished, three stable groups of pole estimates are displayed on top of the **Mode Indicator** as shown below. *This is a Stability diagram*.
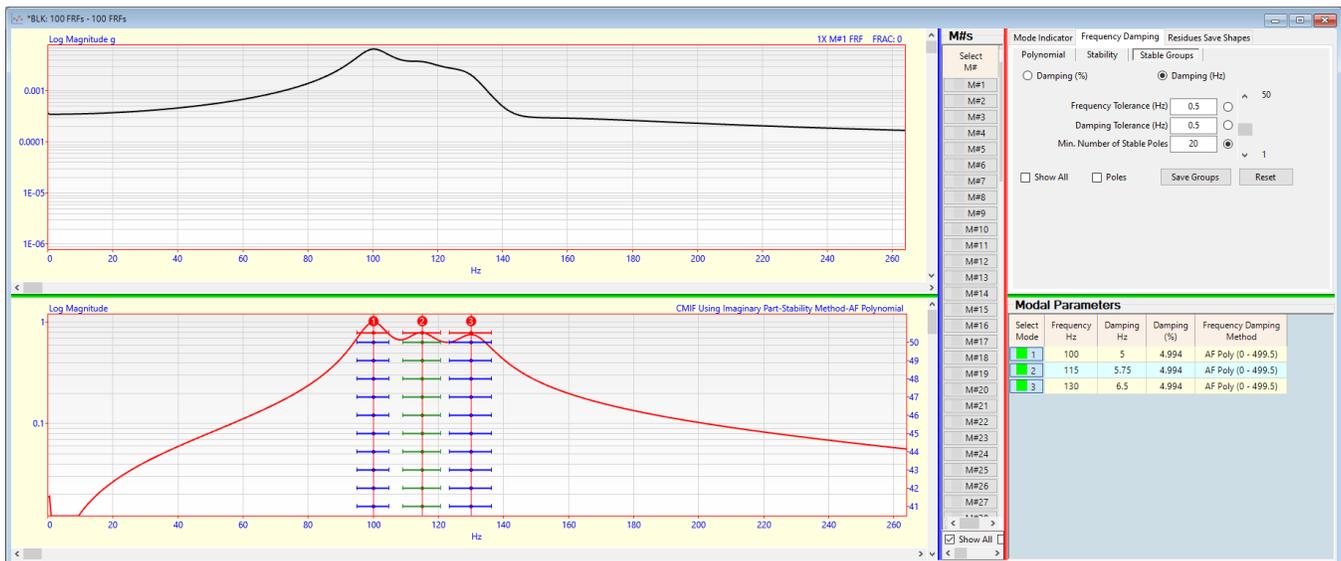
Stability Diagram Using the AF Poly Method.

COMPUTATION POLES

- *Check* the **Show All** box on the **Stable Groups** subtab on the **Frequency Damping** tab

In addition to the *three* **Stable Pole Groups**, many other pole estimates are displayed on the the Stability diagram. These are called *computational poles*. They could not be included in a **Stable Pole Group** because the three requirements for a **Stable Pole Group** were not met.

- *Un-Check* the **Show All** box on the **Stable Groups** subtab on the **Frequency Damping** tab
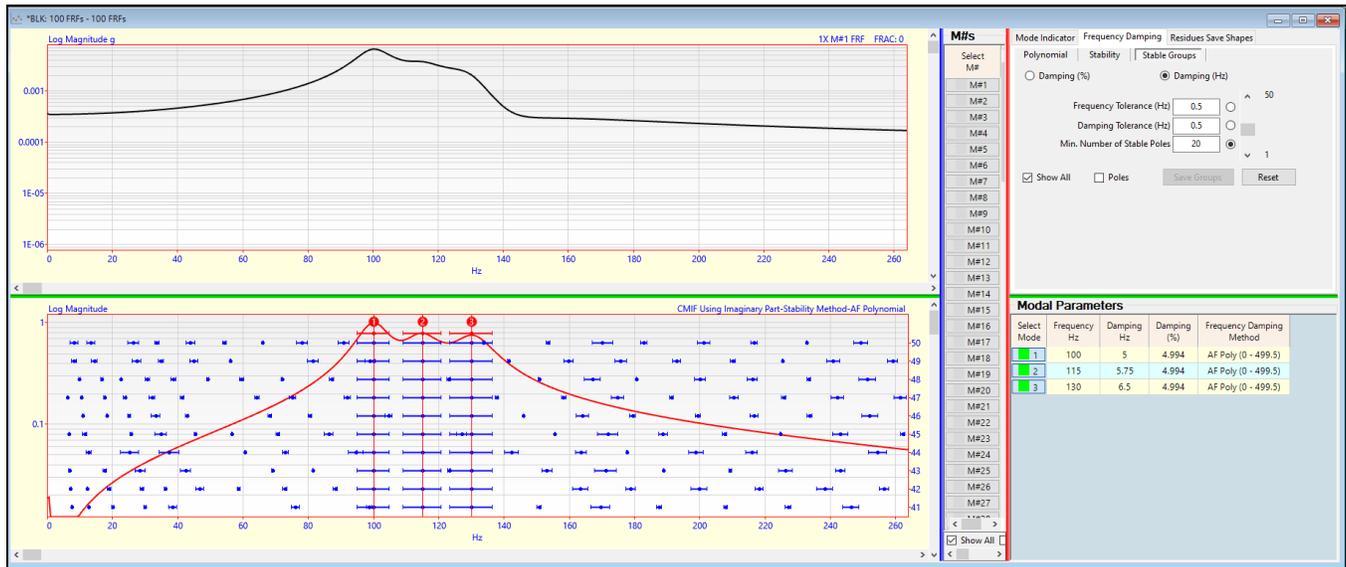
Stability Diagram Showing All Pole Estimates.

MIN. NUMBER OF STABLE POLES

In the figure above, the **Min. Number of Stable Poles = 20**. This was set by a parameter in the Script that was executed when **Hotkey 1 was** *pressed*. To be included in a **Stable Pole Group**, at least 20 pole estimates had to be within the **Frequency Tolerance** (**0.5 Hz**) and **Damping Tolerance** (**0.5 Hz**).

FREQUENCY & DAMPING TOLERANCES

You can decrease the frequency & damping tolerances on the **Stable Groups** subtab to further limit the number of poles in each **Stable Pole Group**. As soon as you change the tolerances, the Stability diagram will be redrawn to reflect your changes.

All the poles in a Stable Pole Group are *displayed using the same color*.

The **second & third Contour Colors** in the Data Block (**BLK**) **Options** box are used to *alternately color* successive Stable Pole Groups.

SAVING THE AVERAGE FREQUENCY & DAMPING OF A STABLE GROUP

There are two ways to save the average frequency & damping of the poles in a stable group.

1. *Press* the **Save Groups** button in the **Stable Groups** subtab
2. *Draw a selection box* around some poles in a stable group on the Stability diagram

SAVE GROUPS BUTTON

When this button is *pressed* the average frequency & damping of all the poles in each **Stable Pole Group** is saved in the **Modal Parameters** spreadsheet *on the right*, as shown below. The **Save Groups** button *was pressed* by the Script when **Hotkey 1** was *pressed*.
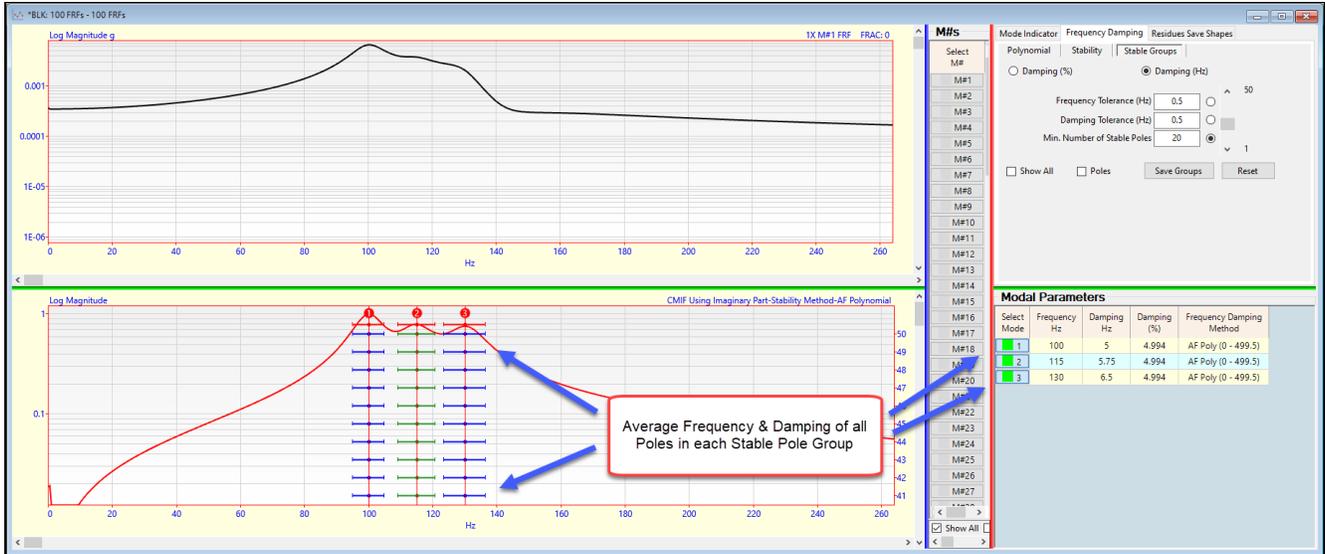
Stable Group Average Frequency & Damping Added to the Modal Parameters Spreadsheet.

POLES SELECTION BOX

To use the poles selection box to save the average frequency & damping of a **Stable Pole Group**,

- Execute **Curve Fit | Stability | Poles Selection Box**
- *Click & Drag* to draw a *selection box* around some of the poles on the Stability diagram
- Release the left mouse button to *save the* **average frequency & damping** in the **Modal Parameters** spreadsheet on the right

For this ideal case, curve fitting the **FRFs** in **BLK: 100 FRFs** *yielded the same frequency & damping values* that were used *to synthesize the* **100 IRFs** and hence the **FRFs** that were curve fit using the **AF Polynomial** method.

STEP 2 - STABILITY DIAGRAM USING THE COMPLEX EXPONENTIAL METHOD

- *Press* **Hotkey 2 Stability using Complex Exponential**

When **Hotkey 2 is** *pressed*, the **Stable Pole Group**s found by the **Complex Exponential** curve fitting method are displayed on the **Stability** diagram and the average frequency & damping of each **Stable Pole Group** is added to the **Modal Parameters** spreadsheet *on the right*, as shown below.

*Stability Diagram Showing **Stable Pole Group**s from Complex Exponential Curve Fitter*

- Scroll the bar *to the right* **of** the **Stability diagram** to display more of the poles in the stable groups, as shown above

STEP 3 - STABILITY DIAGRAM USING THE Z POLYNOMIAL METHOD

To use the Stability diagram together with the **Z Polynomial** curve fitting method

- *Press* **Hotkey 2 Stability using Z Poly**

When **Hotkey 3 is** *pressed*, the **Stable Pole Group**s found by the **Z Polynomial** curve fitting method are displayed on the **Stability** diagram, and the average frequency & damping of each **Stable Pole Group** is added to the **Modal Parameters** spreadsheet **on the right**, as shown below.
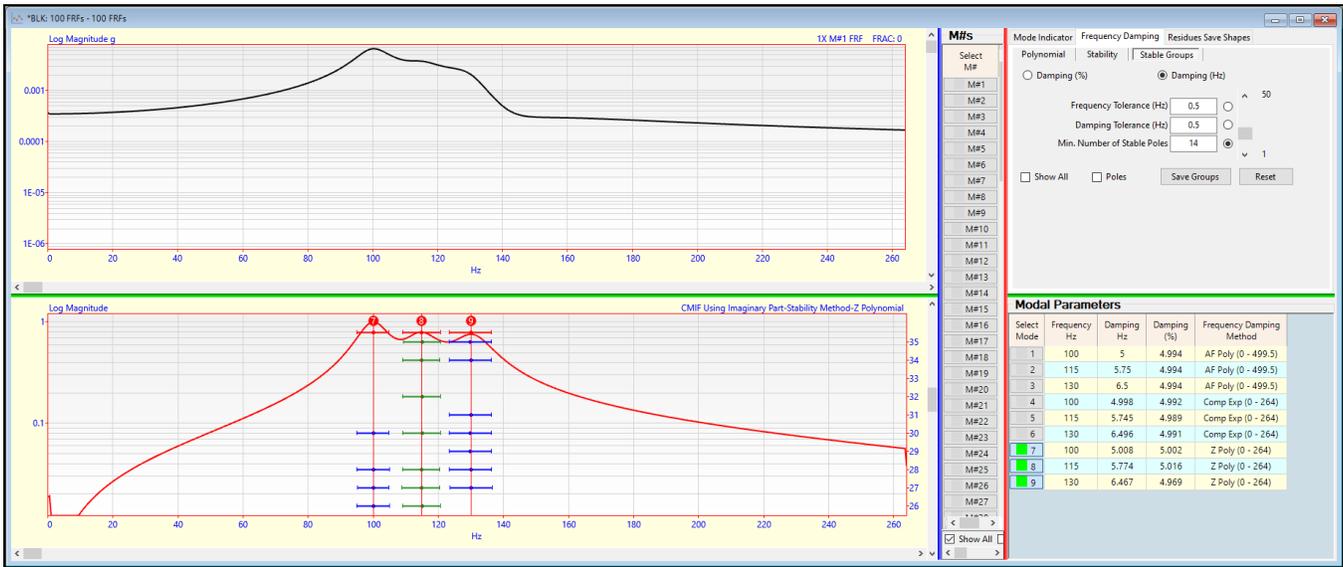
*Stability Diagram Showing **Stable Pole Group**s from the Z Polynomial Curve Fitter.*

To compare the frequency & damping estimates from all three curve fitting methods.

- Execute **Curve Fit | Modal Parameters | Sort by Frequency**

In the figure below, all three curve fitting methods provided *essentially the same frequency & damping estimates*.
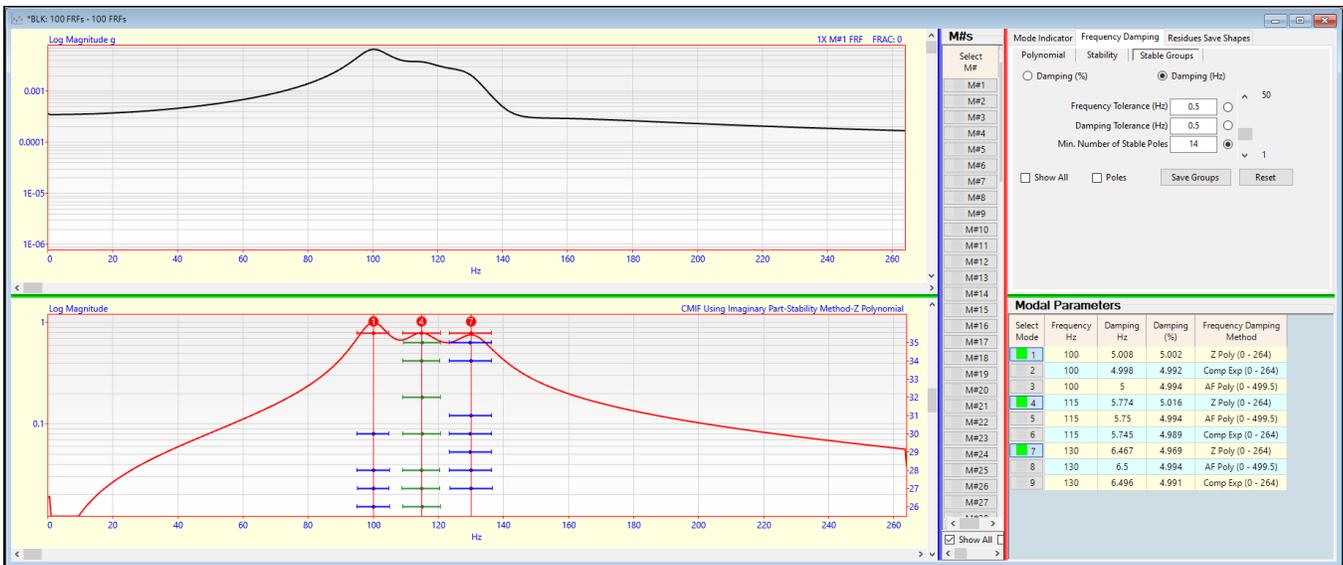
Modal Parameters Spreadsheet Listing Frequency & Damping Estimates Together.

ESTIMATING MODE SHAPES OF THE JIM BEAM

The Stability diagram will be used together with the three Stability curve fitting methods to estimate the frequency & damping of the modes of the Jim Beam structure shown below. This structure was tested by impacting it with an instrumented hammer in the vertical direction at Point 15, and roving a tri-axial accelerometer to 33 Points on the three plates for each acquisition of data. Three **FRFs** were calculated for each Point, giving a total of 99 **FRFs**. The **FRFs** are contained in the Data Block **BLK: Jim Beam FRFs.**
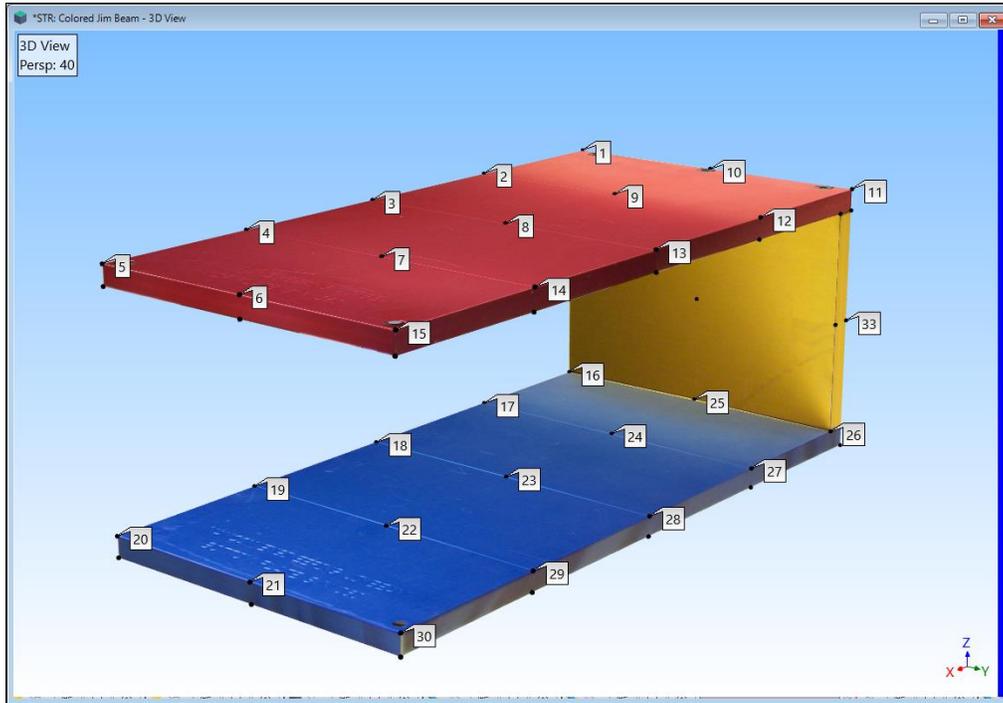
Jim Beam Showing 33 Roving Response Points

STEP 4 - MODE SHAPES USING THE AF POLYNOMIAL METHOD

- *Press* **Hotkey 4 Jim Beam using AF Poly**

When **Hotkey 4 is** *pressed*, the **AF Polynomial** method and a Stability diagram are used to estimate modal frequency & damping, and save the average values of each Stable Pole Group in **the Modal Parameters** spreadsheet. , **Residue** curve fitting was carried out by the Script to estimate the modal **residue** *for each mode and each* **FRF**. Then **Residue** mode shapes for 10 modes were saving in a Shape Table (**SHP: AF Poly Mode Shapes**).

After the modal frequency & damping have been estimated and saved in the **Modal Parameters** spreadsheet, **Residue** curve fitting was carried out by the Script to estimate the **modal residue** *for each mode and each* **FRF**. This curve fitting step is also down by *pressing* **Residues** on the **Residues Save Shapes** tab.

After a modal residue is estimated for each mode in each FRF, a **red Fit Function** is synthesized and overlaid on each FRF. A close match between the **red Fit Function** and the FRF indicates that the modal parameter estimates represent an accurate dynamic model of the of the test article.

- Use the scroll bar to the *right of the* **FRFs** to display each FRF overlaid with its **Fit Function**
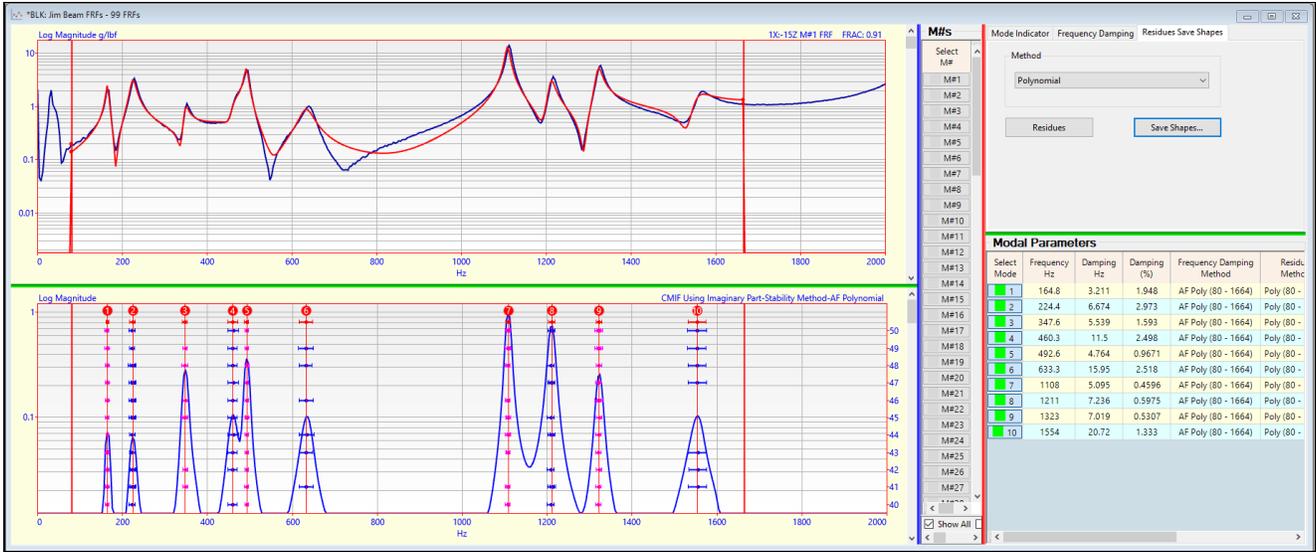
Stability Diagram Showing 10 Stable Pole Groups from AF Poly.

STEP 5 - MODE SHAPES USING THE COMPLEX EXPONENTIAL METHOD

- *Press* **Hotkey 5 Jim Beam using Complex Exponential**
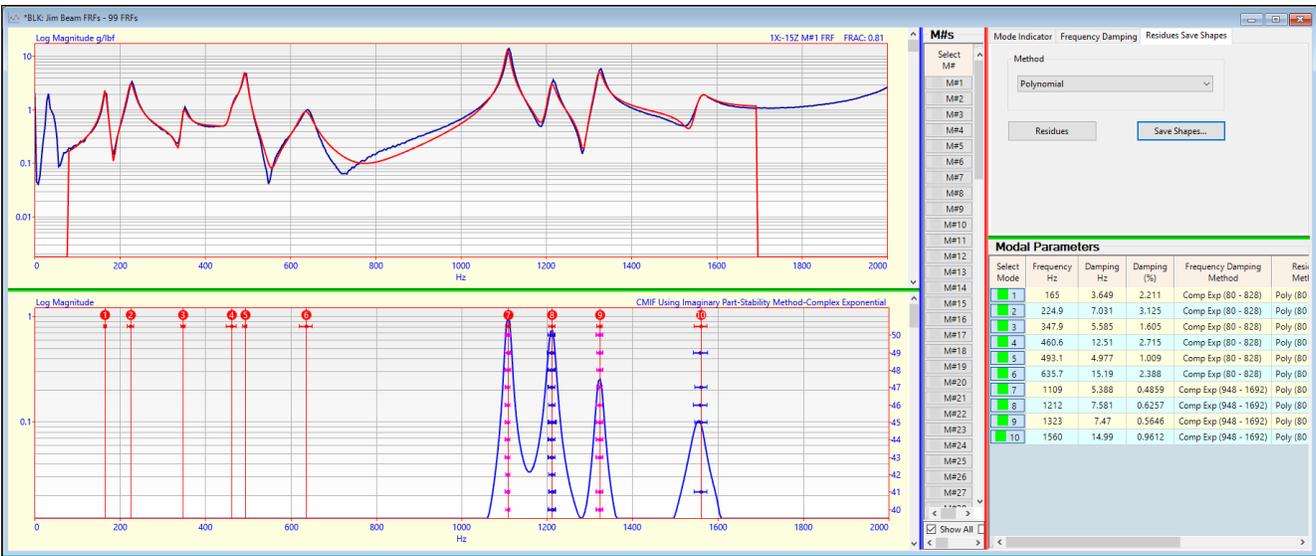
Stability Diagram Showing 10 Stable Pole Groups from Complex Exponential within two Cursor Bands.

The figure above shows the Stability diagram and an FRF *overlaid with a* **red Fit Function**. Following the Stability diagram curve fitting step using the **Complex Exponential** method, Residue curve fitting was carried out by the Script to estimate the modal residue for each mode and each FRF. Then **Residue mode shapes** for 10 modes were saving in a Shape Table (**SHP: Complex Exp Mode Shapes**).

STEP 6 - MODE SHAPES USING THE Z POLYNOMIAL METHOD

- *Press* **Hotkey 6 Jim Beam using Z Poly**
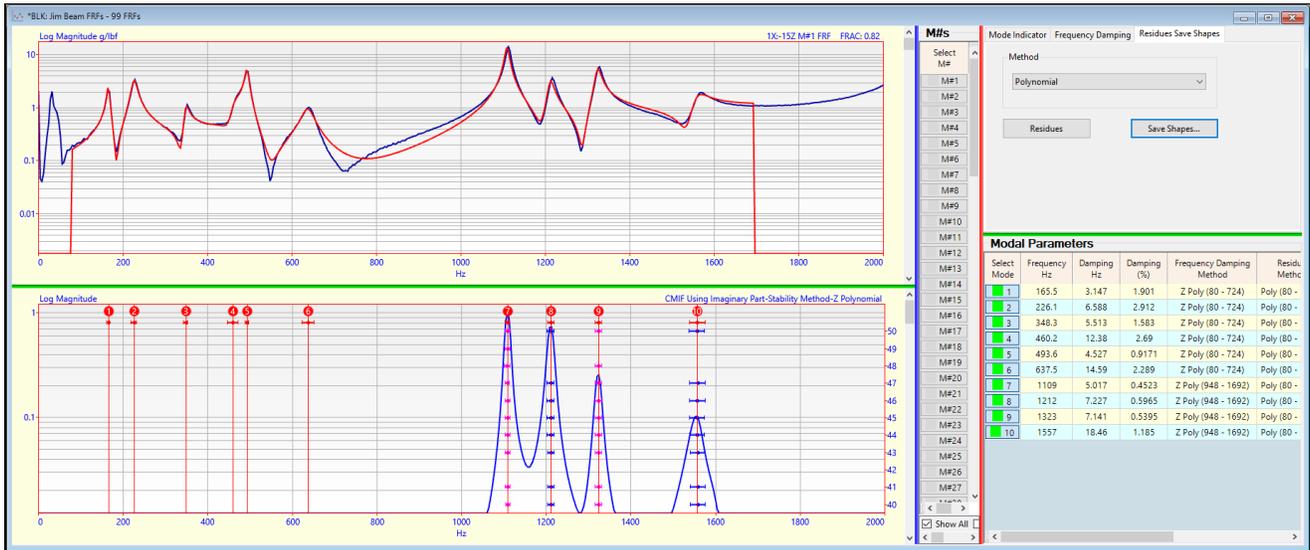
Stability Diagram Showing 10 Stable Pole Groups from Z Poly within two Cursor Bands.

The figure above shows the Stability diagram and an FRF *overlaid with a* **red Fit Function**. Following the Stability diagram curve fitting step using the **Z Polynomial** method, Residue curve fitting was carried out by the Script to estimate the modal residue for each mode and each FRF. Then **Residue mode shapes** for 10 modes were saving in a Shape Table (**SHP: Z Poly Mode Shapes**).

STEP 7 – COMPARING AF POLY & COMPLEX EXP MODE SHAPES

To compare the mode shapes from the **AF Polynomial** curve fitting method with the mode shapes from the **Complex Exponential** curve fitting method,

- *Press* **Hotkey 7 AF Poly vs Complex Exp Mode Shapes**

Sweep animation will begin displaying each mode shape in **SHP: AP Poly Mode Shapes** side-by-side with the *closest matching* mode shape in **SHP: Complex Exp Mode Shapes**, as shown below.

> The *closest matching* mode shape in **SHP: Complex Exp Mode Shapes** has a **Maximum MAC** with the mode shape displayed from **SHP: AP Poly Mode Shapes**.
>
> Two mode shapes that have a **MAC** ➔ *close to "1"* are *essentially the same mode shape*.

All 10 mode shape pairs have **MAC** ➔ *close to "1".* Since the mode shape estimates depend on the frequency & damping estimates, the *high* **MAC** *values indicate* that both curve fitters estimated modal parameters that were *essentially the same*.
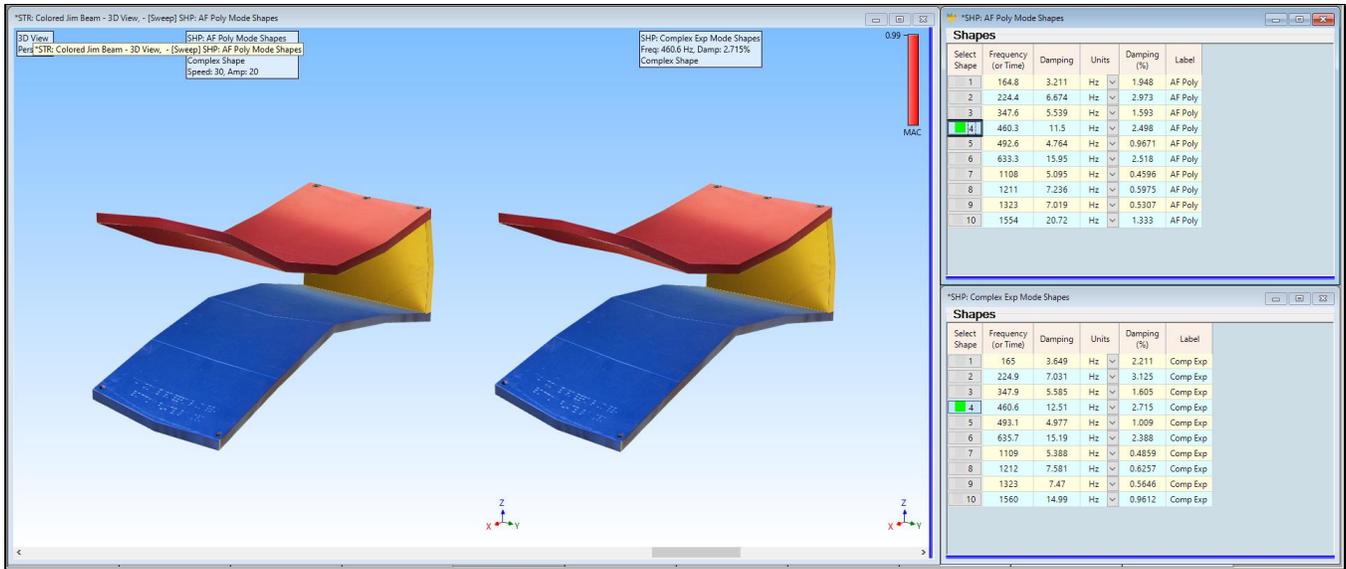
AF Polynomial verses Complex Exponential Mode Shapes.

STEP 8 – COMPARING COMPLEX EXP & Z POLYNOMIAL MODE SHAPES

To compare the mode shapes from the **Complex Exponential** curve fitting with the mode shapes from the **Z Polynomial** curve fitting method,

- *Press* **Hotkey 8 Complex Exp vs Z Poly Mode Shapes**

Sweep animation will begin displaying each mode shape in **SHP: Complex Exp Mode Shapes** side-by-side with the *closest matching* mode shape in **SHP: Z Poly Mode Shapes**, as shown below.

All 10 mode shape pairs have **MAC ➔ *close to "1"*.** Since the mode shape estimates depend on the frequency & damping estimates, the *high MAC values indicate* that both curve fitters estimated modal parameters that were *essentially the same*.
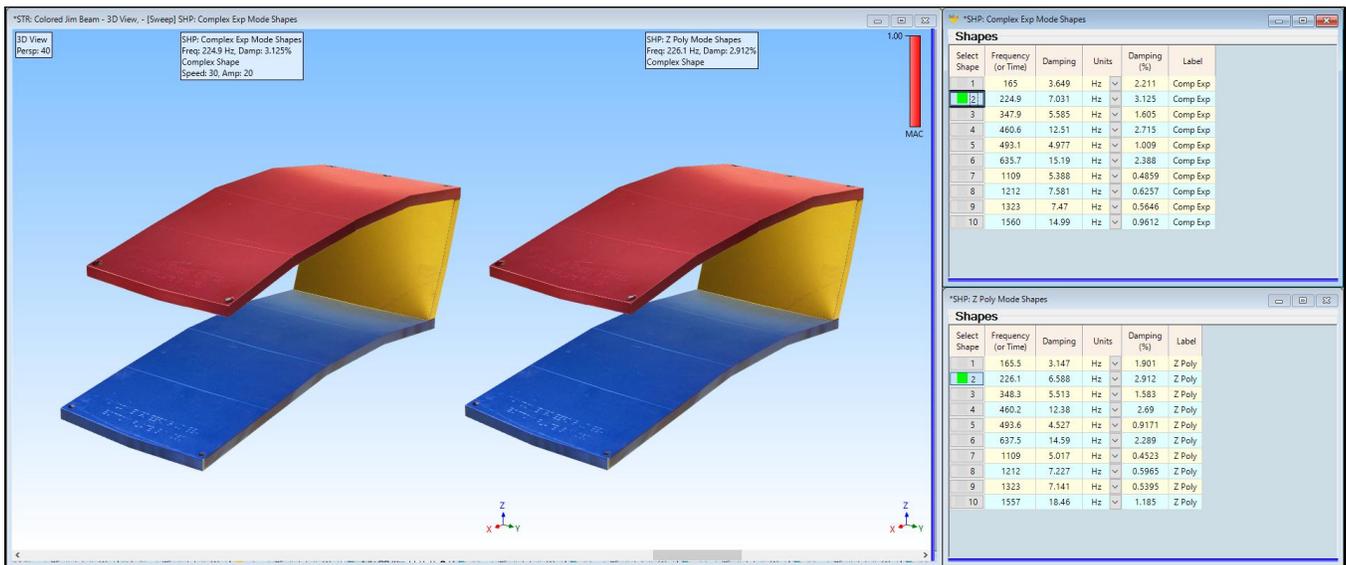
Complex Exponential verses Z Polynomial Mode Shapes

STEP 9 – REVIEW STEPS

To review all the steps of this App Note,

- *Press* **Hotkey 9 Review Steps**

CONCLUSIONS

Three different Stability diagram curve fitting methods were used to estimate the modal frequency & damping of three modes that were used to synthesize 100 Impulse Response Functions (**IRFs**). The synthesized IRFs were transformed (using the FFT) to their equivalent **FRFs**. which were then curve fit to display **Stable Pole Group**s on a Stability diagram. The three curve fitting methods estimated *essentially the same frequency & damping values.*

The Stability diagram was used again to estimate the frequency & damping of the modes of the Jim Beam structure by curve fitting 99 experimental **FRFs**. Using the frequency & damping estimates, a second **residue** curve fitting step was carried out to estimate 3-dimensional mode shapes for the modes of the Jim Beam.

Mode shape pairs with **Maximum MAC** values form two different curve fitting methods were displayed side-by-side in animation to validate the mode shape estimates.

> All 10 mode shape pairs from the **AF Polynomial** versus the **Complex Exponential** method and the **Complex Exponential** versus the **Z Polynomial** method had MAC ➔ *very close to "1*", indicating that all three curve fitting methods gave *essentially the same results*.
>
> Resonance peaks *could have be counted* on a **Mode Indicator** curve calculated from the data in this App Note.
>
> However, a Stability diagram can also be used to estimate modal frequency & damping from data where *it is not possible to resonance peaks*.