MEscope Application Note 17

Modal Analysis of a Mass-Spring-Damper

The steps in this Application Note can be carried out using any Package that includes the **VES-3600 Advanced Signal Processing**, **VES-4000 Modal Analysis** & **VES-8000 FEA** options. They can also be carried out using the **AppNote17** project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 17 PROJECT FILE

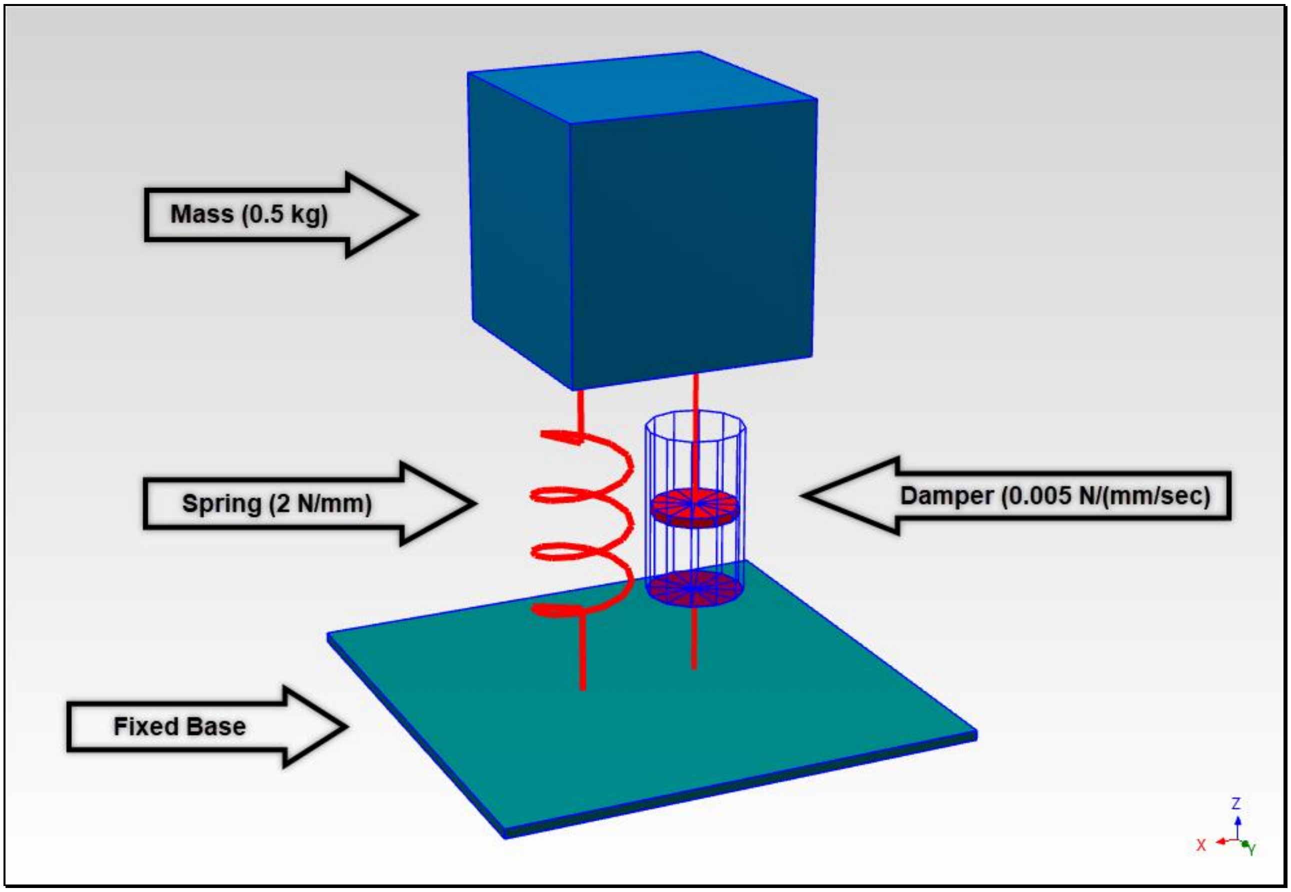
- To retrieve the Project for this App Note, **click here** to download **AppNote17.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down*** **the Ctrl key** and ***click*** **on a Hotkey** to display its Script window

INTRODUCTION

In this Application Note, **FEA** mass, **FEA** spring & **FEA** damper elements are added to the 3D model shown below. Its single **FEA** mode shape is then calculated and displayed in animation on the model. Finally, the **FEA** mode shape is used to synthesize an FRF and its physical significance is examined. The 3D model was built using the steps in **App Note 07**.



3D Model of a Mass-Spring-Damper.

3D GRAPHICAL MODEL

In MEscope, a 3D graphical model is required to display Operating Deflection Shapes (ODS's) and mode shapes in animation. The 3D model of the Mass-Spring-Damper model above was created in **App Note 07** using **five substructures**.

Building a model using substructures makes setting up the model for **FEA** modeling and shape animation much easier.

ADDING FEA OBJECTS TO THE MODEL

A 3D model like the one above would normally be used to display experimental ODS or mode shape data, but in this App Note, **FEA** elements (called **FEA Objects**) are added to the 3D model to turn it into an **FEA** model. The following three **FEA** Objects are required to model the dynamics of this single degree-of-freedom (**S-DOF**) structure,

- **Point mass**
- **Linear spring** between the MASS and the fixed BASE
- **Linear damper** between the MASS and the fixed BASE

STEP 1 - MASS POINT & BASE POINT

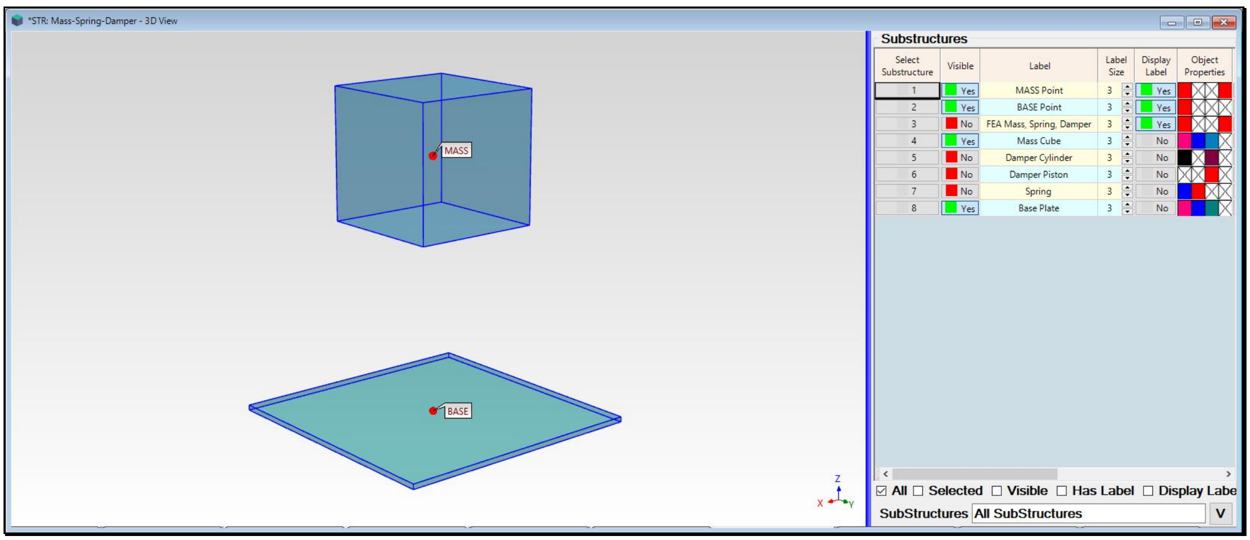
To add the Point mass to the 3D model, a new Point is added to the **center of the Mass Cube**, as shown below. To attach the spring & damper between the mass and the (fixed) ground, a new Point is also added to the **center of the Base Plate**.

- ***Press* Hotkey 1 Mass Point & Base Point**

Only the **Mass Cube** and the **Base Plate** are visible, and their surfaces are **transparent**.

- A Point is added to the **center of the Mass Cube**, and a second Point is added to the **top center of the Base Plate**

The two new Points were added to the end of the **Points** spreadsheet. Their Coordinates place them in the center of the Mass Cube & Base Plate Substructures.

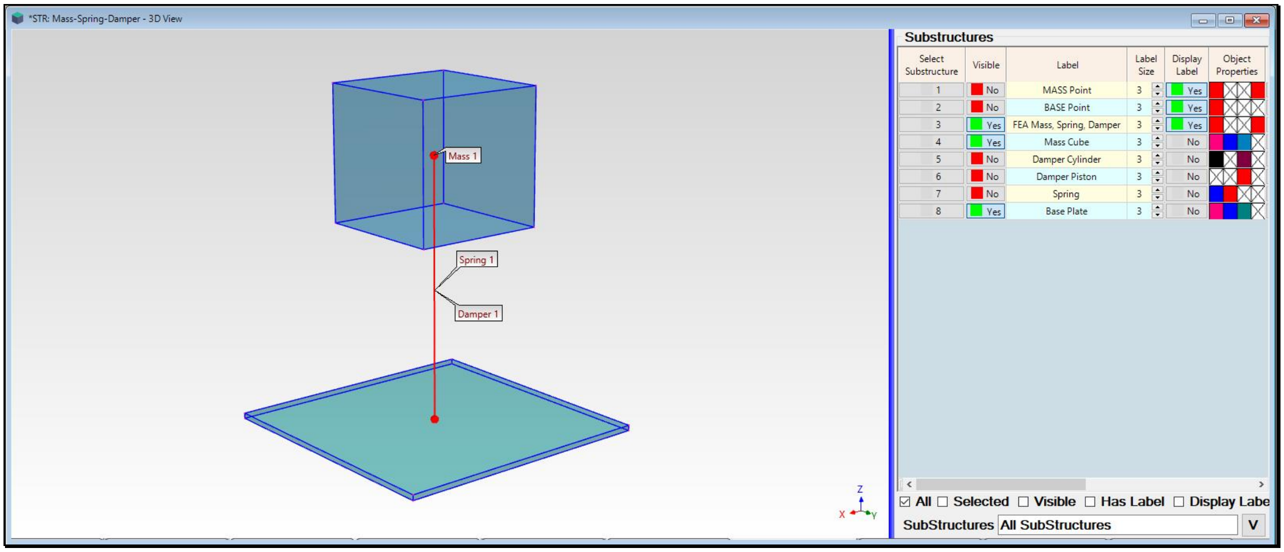


3D Model Showing Mass & Base Points.

STEP 2 - FEA MASS, SPRING, & DAMPER

- ***Press* Hotkey FEA Mass, Spring, & Damper**

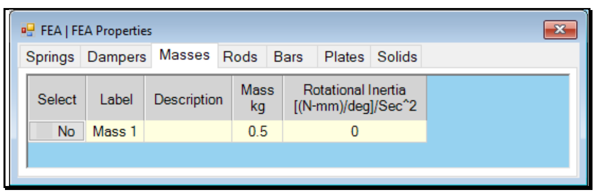
An **FEA Mass** is added to the **MASS Point**, and an **FEA Spring** & **FEA Damper** are each added between the **MASS Point** and the **BASE Point**.



*3D Model Showing **FEA** Mass, **FEA** Spring, **FEA** Damper'*

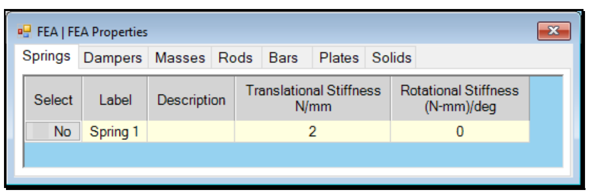
To complete the **FEA** model, the physical properties of the **FEA** mass, **FEA** spring & **FEA** damper must be entered into the **FEA Properties** box and referenced by their label in the **FEA Properties** column . To confirm these properties,

- Execute **FEA | FEA Properties** to open the **FEA** Properties box
- ***Click*** on the **Masses** tab
- Confirm a mass of **0.5 kg** as shown below



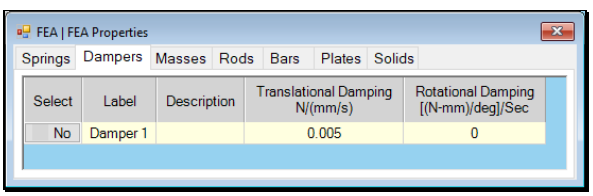
Select	Label	Description	Mass kg	Rotational Inertia [(N-mm)/deg]/Sec^2
No	Mass 1		0.5	0

- ***Click*** on the **Springs** tab
- Confirm a spring of **2 N/mm** as shown below



Select	Label	Description	Translational Stiffness N/mm	Rotational Stiffness (N-mm)/deg
No	Spring 1		2	0

- ***Click*** on the **Dampers** tab
- Confirm a damper of **0.005 N/(mm/sec)** as shown below

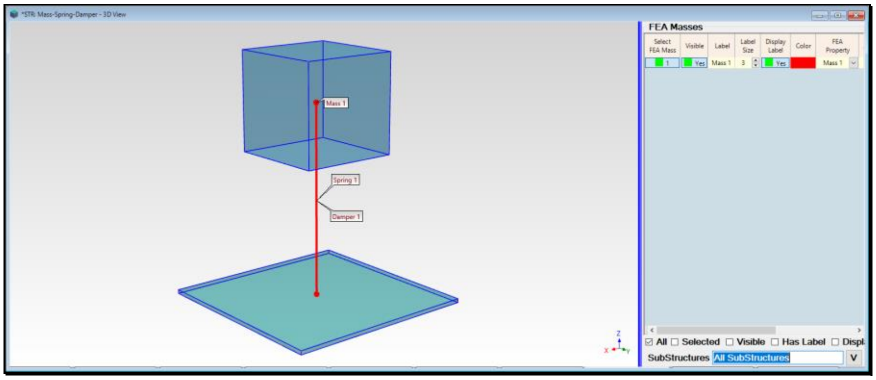


Select	Label	Description	Translational Damping N/(mm/s)	Rotational Damping [(N-mm)/deg]/Sec
No	Damper 1		0.005	0

- After each **FEA** Object is added to a 3D model, its physical properties must be defined **on the appropriate Tab** in the **FEA | FEA Properties** box and **referenced by label** in the **FEA Property** column of its Objects spreadsheet

To display the **FEA Masses** spreadsheet and verify the **Mass 1** property,

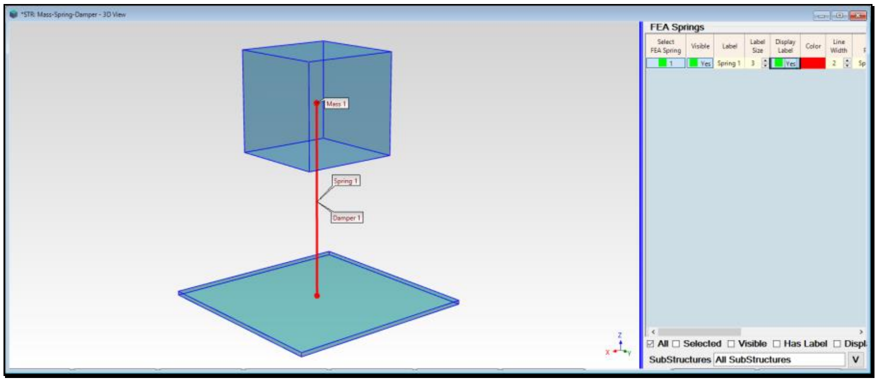
- Execute **FEA | FEA Masses**
- ***Toggle*** the **Display Label** to show & hide the **Mass 1** property label



FEA *Masses Spreadsheet Showing the* ***Mass 1*** *Property.*

To display the **FEA Springs** spreadsheet and verify the **Spring 1** property,

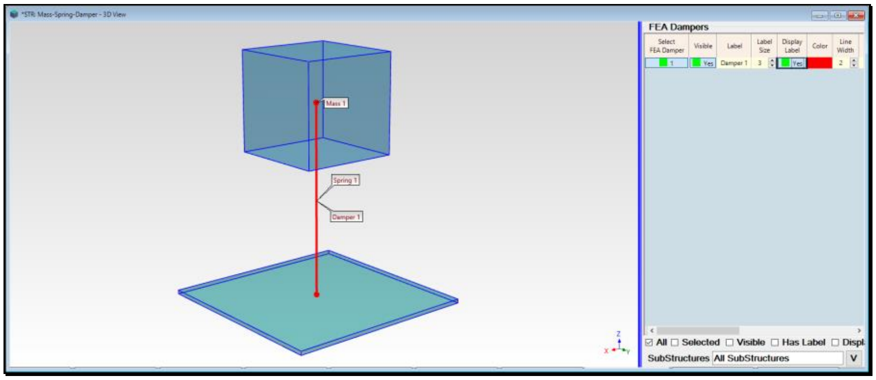
- Execute **FEA | FEA Springs**
- ***Toggle*** the **Display Label** to show & hide the **Spring 1** property label



FEA *Springs Spreadsheet Showing the* ***Spring 1*** *Property.*

To display the **FEA Dampers** spreadsheet and verify the **Damper 1** property,

- Execute **FEA | FEA Dampers**
- ***Toggle*** the **Display Label** to show & hide the **Damper 1** property label



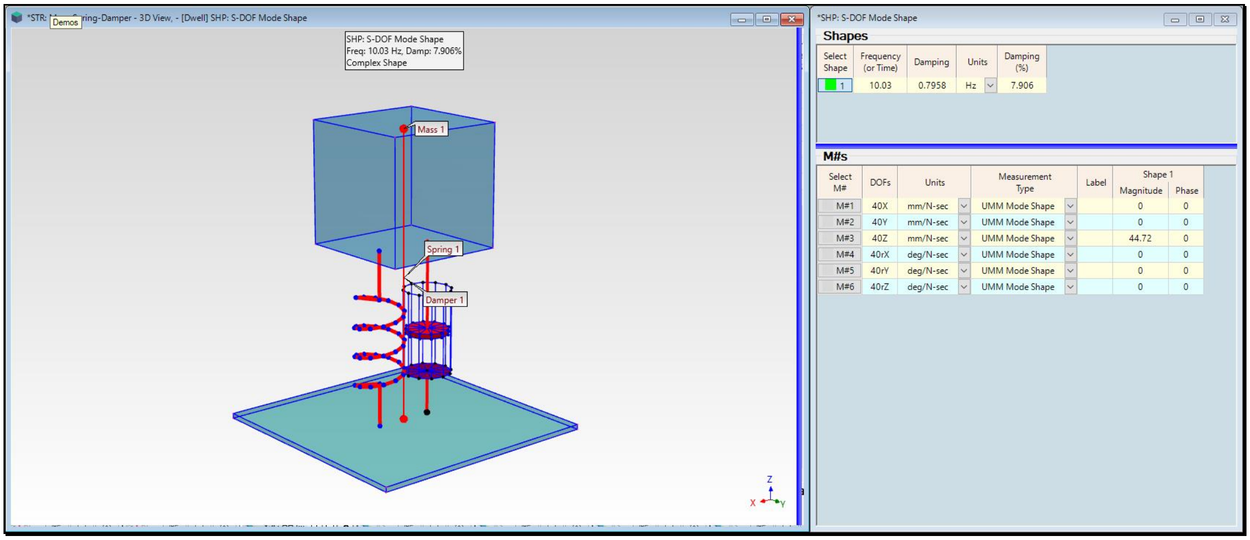
FEA *Dampers Spreadsheet Showing the* ***Damper 1*** *Property.*

STEP 3 - CALCULATING THE FEA MODE SHAPE

The **FEA** model is now ready to be solved for the single **FEA** mode shape of the Mass-Spring-Damper.

- ***Press* Hotkey 3 FEA Mode Shape**

The **FEA** model of the Mass-Spring-Damper is solved for its mode shape, which is displayed in **SHP: S-DOF Mode Shape** ***on the right*** and animated on the 3D model ***on the left***.



*Animation of **FEA** Mode Shape*

- The **FEA** solver in MEscope can only calculate a mode shape for **an un-damped FEA model**

Using the **mode shape of the undamped FEA model**, **SDM** is used to calculate a the new **damped FEA mode shape** of the **FEA** model with the damper added to it.

SDM uses the **M# Links** created by the **FEA** solver to link the **undamped FEA mode shape** to the **FEA** model and solve for the **mode shape of the damped model**.

MODAL FREQUENCY & DAMPING

The modal parameters in **SHP S-DOF Mode Shapes** ***agree precisely*** with the analytical definitions of these parameters.

Millimeter (**mm**) units were internally converted to Meters (**m**) to make them consistent with Newtons (**N**) and kilograms (**kg**).

The **un-damped natural frequency (f_n)** of the Mass-Spring-Damper is calculated with the formula,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{2000}{0.5}} = 10.07 \text{ Hz}$$

The **percent of critical damping (ζ)** is calculated with the formula,

$$\zeta = \frac{C}{\sqrt{4KM}} = \frac{5}{\sqrt{4 \times 2000 \times 0.5}} = 7.906\ \%$$

The **damped natural frequency (f_d)** is calculated with the formula,

$$f_d = f_n\sqrt{1-\zeta^2} = 10.03 \text{ Hz}$$

STEP 4 - INTERPOLATING THE DEFLECTION OF THE MASS-SPRING-DAMPER

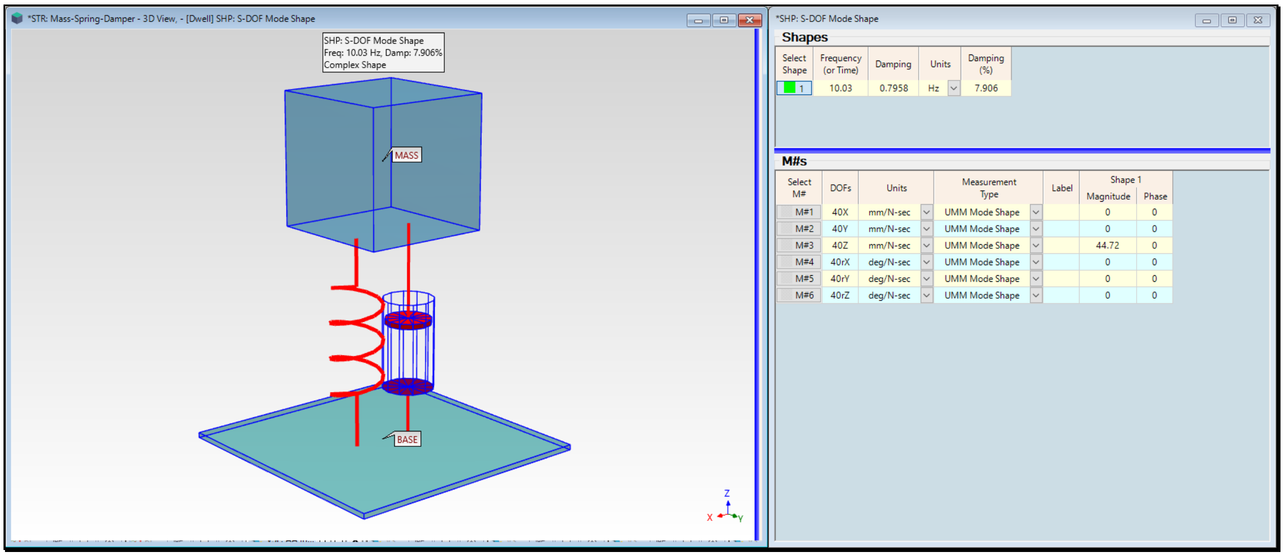
- **Press Hotkey 4 Interpolate Other Points**

The damped mode shape is displayed in animation on the 3D model of the Mass-Spring-Damper.

In the previous step, the MASS Point had motion and the BASE Point was fixed. Those were the only two Points required in the calculation of the **FEA** mode shape.

When **Hotkey 4 is *pressed***, **Interpolated M# Links** are created for all the other Points on the the Mass-Spring-Damper model.

- The **Mass Cube** & **Damper Piston** Points are interpolated from the deflection of the **MASS** Point
- The **Base Plate** & **Damper Cylinder** Points are interpolated from the fixed **BASE** Point
- The **Spring** Points are interpolated between the deflection of the **MASS** Point and the fixed **BASE** Point



Select Shape	Frequency (or Time)	Damping	Units	Damping (%)
1	10.03	0.7958	Hz	7.906

Select M#	DOFs	Units	Measurement Type	Label	Shape 1 Magnitude	Phase
M#1	40X	mm/N-sec	UMM Mode Shape		0	0
M#2	40Y	mm/N-sec	UMM Mode Shape		0	0
M#3	40Z	mm/N-sec	UMM Mode Shape		44.72	0
M#4	40rX	deg/N-sec	UMM Mode Shape		0	0
M#5	40rY	deg/N-sec	UMM Mode Shape		0	0
M#6	40rZ	deg/N-sec	UMM Mode Shape		0	0

Mass-Spring-Damper With All Other Points Interpolated From MASS & BASE Points.

STEP 5 - DRIVING POINT FRF

- ***Press* Hotkey 5 Driving Point FRF**

When **Hotkey 5 is *pressed***, a Data Block window with the **driving point FRF** at the **MASS** Point is opened as shown below.

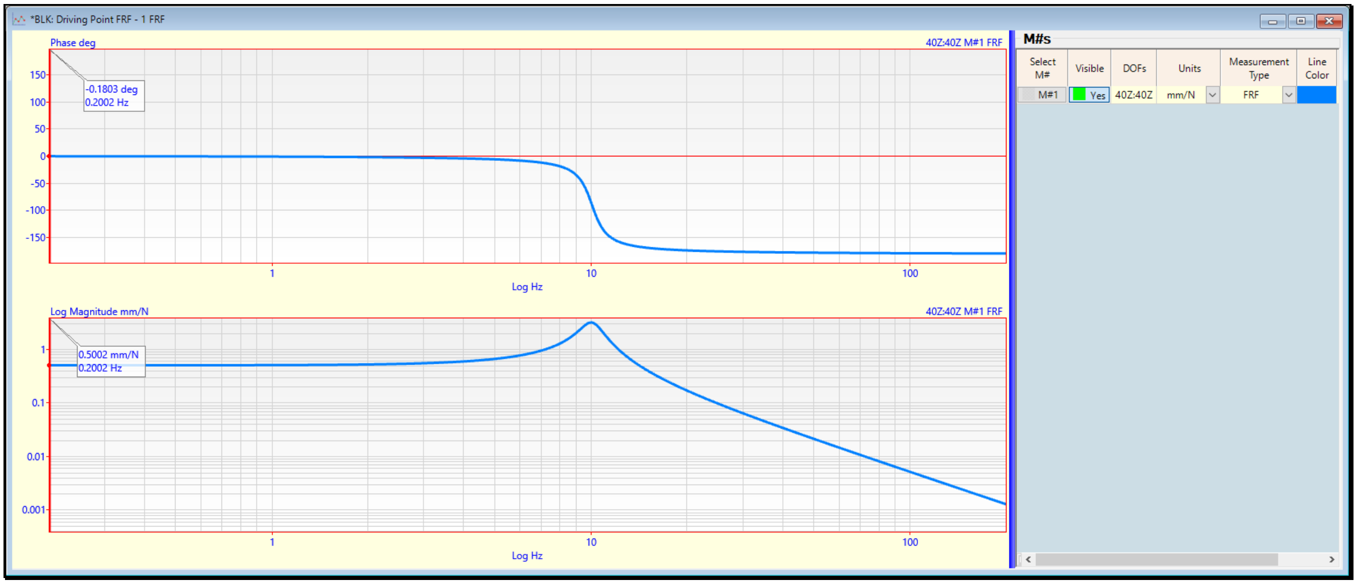
The **M#**s of the **FEA** mode shape in **SHP: S-DOF Mode Shape** have **Measurement Type ➔ UMM Mode Shape**.

When the **FEA** mass matrix is pre- & post-multiplied by a **Unit Modal Mass (UMM)** mode shape the result is **unity "1"**.

MODAL MODEL

Mode shapes that are scaled to **Unit Modal Mass (UMM)** preserve the dynamic properties of the structure from which they were derived, and are therefore called a **Modal Model**.

Because the mode shape in **SHP: S-DOF Mode Shape** is a Modal Model of the Mass-Spring-Damper, it can be used to synthesize a **driving point FRF** at the **MASS** Point.



Log Magnidude & Phase of the Driving Point FRF at the MASS Point.

STIFFNESS LINE

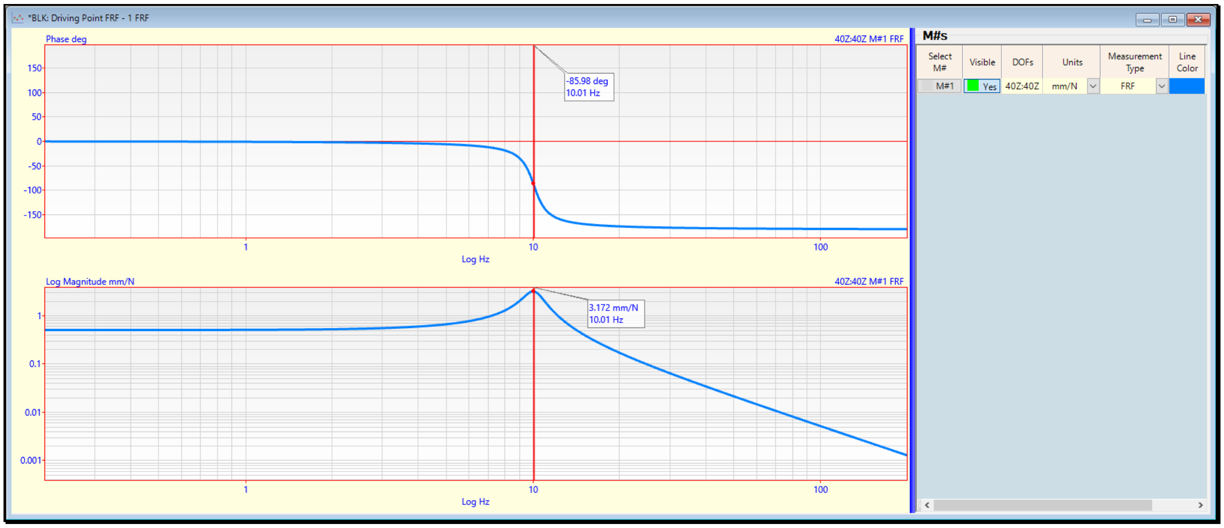
The magnitude of the FRF at the Line cursor **near 0 Hz is 0.5 mm/N**. This is called the flexibility of the structure.

- **Stiffness = 1/Flexibility ➔ 2 N/mm**

The horizontal asymptote at frequency approaches **DC (zero Hz)** is also called the **stiffness line**.

QUALITY FACTOR

- ***Drag*** the **Line** cursor to the resonance peak on the FRF, as shown below



Q Factor at the Resoance Peak.

The FRF amplitude at the resonance is **3.172 mm/N**. The **Quality Factor, Q** is a measure of the **amplification** of an S-DOF at resonance. It is the ***inverse*** of the **Loss factor**, or ***twice*** the damping ratio (ζ).

$$\mathbf{Q = \frac{1}{2\zeta} = 6.325\%}$$

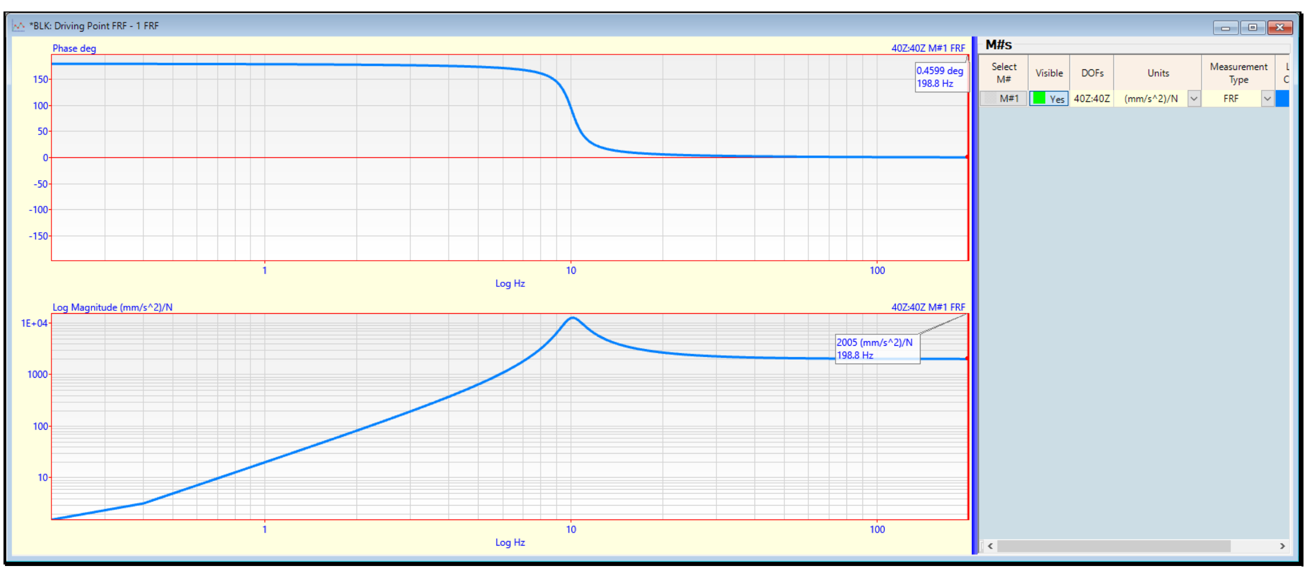
If the **Quality Factor, Q** is multiplied by the stiffness line value (**0.5 mm/N**) the peak amplitude of the FRF is the result

$$\mathbf{FRF_{peak} = Q \; x \; 0.5 = 3.163 \; mm/N}$$

This verifies that the **Spring & Damper** values are correctly represented in the **Modal Model** of the Mass-Spring-Damper.

MASS LINE

- Execute **Tools | Differentiate *twice*** in the **BLK: Driving Point FRF** window
- Display the **Line** cursor and position it **at a high frequency**, as shown below



Twice Differentiated Driving Point FRF with Line Cursor on the Mass Line Asymptote.

- The magnitude of FRF at the **Line** cursor on the **Mass Line** asymptote is **2005 (mm/sec^2)/N ➔ 2 (m/sec^2)/N**
- The ***inverse*** of the **Mass Line** value is the mass: **0.5 N/(m/sec^2) ➔ 0.5 kg**

This verifies that the mass of the S-DOF is also correctly represented in the **Modal Model** of the Mass-Spring-Damper.

IMPULSE RESPONSE FUNCTION

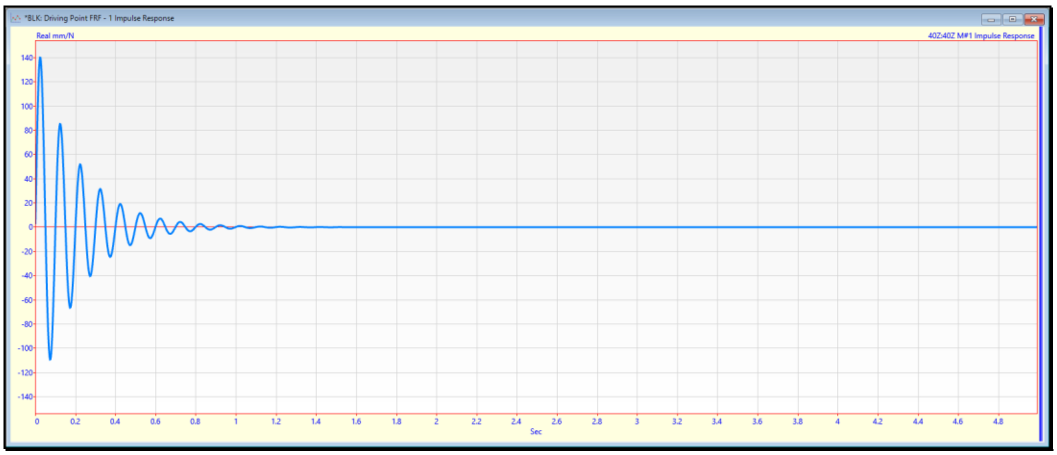
- ***Execute*** **Transform | Inverse FFT in BLK: Driving Point FRF**

The **Inverse FFT** of an **FRF** is the Impulse Response Function (IRF)

Just as the **Driving Point FRF** is a complete representation of the dynamics of the Mass-Spring-Damper, the **IRF** is also a complete time domain representation of the dynamics of the Mass-Spring-Damper.

The **IRF** shows the response of the structure to an impulsive force applied to the Mass at **DOF 39Z**.

The **IRF** decays exponentially due to the damping in the structure. This is characteristic of all damped structures, and is a another way to check the validity of a set of **FRFs** that have been calculated from real world vibration data



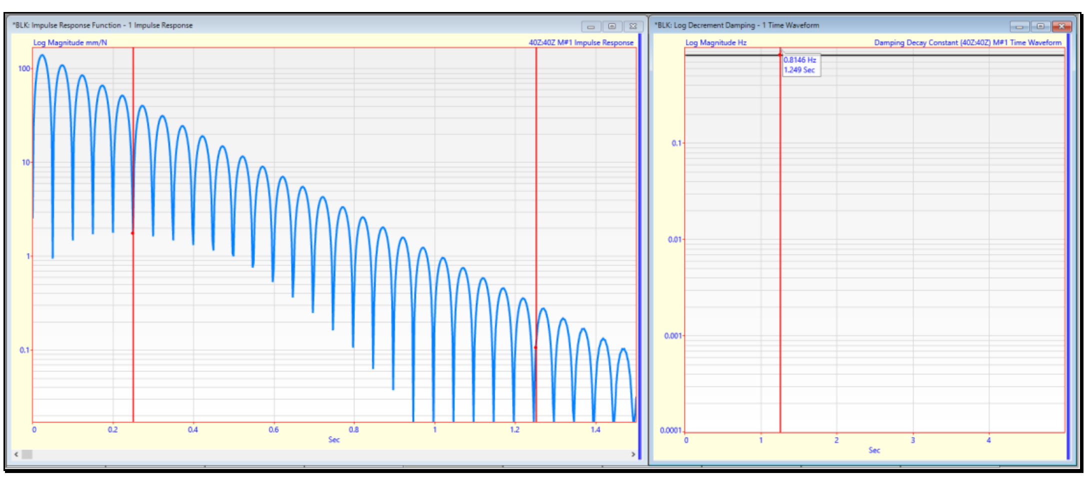
IRF of the Mass-Spring-Damper.

STEP 6 - LOGARITHMIC DECREMENT

- ***Press*** **Hotkey 6 Logarithmic Decrement**

On the ***left side*** is the **Logarithmic Magnitude** display of the **Impulse Response Function (IRF).** The peak of each cycle of the sine wave response lies on a **straight line with a negative slope**.

The **slope of the line** is a measure of the **modal damping in Hz** called the **Logarithmic Decrement**.



Logarithmic Decrement of the Mass-Spring-Damper.

The Data Block on the **right side** contains the **Logarithmic Decrement** in Hz.

- The Line cursor value (**0.8146 Hz*)*** is ***close to*** **the modal damping** listed in **SHP: S-DOF Mode Shape**

STEP 7 - REVIEW STEPS

To review all the steps of this App Note,

- ***Press*** **Hotkey 7 Review Steps**