MEscope Application Note 16

Integration & Differentiation of FRFs and Mode Shapes

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** option. Without this option, you can still carry out the steps in this App Note using the **AppNote16** project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 16 PROJECT FILE

- To retrieve the Project file for this App Note, **click here** to download **AppNote16.zip**

This Project file contains numbered Hotkeys & Scripts for carrying out the steps of this App Note.

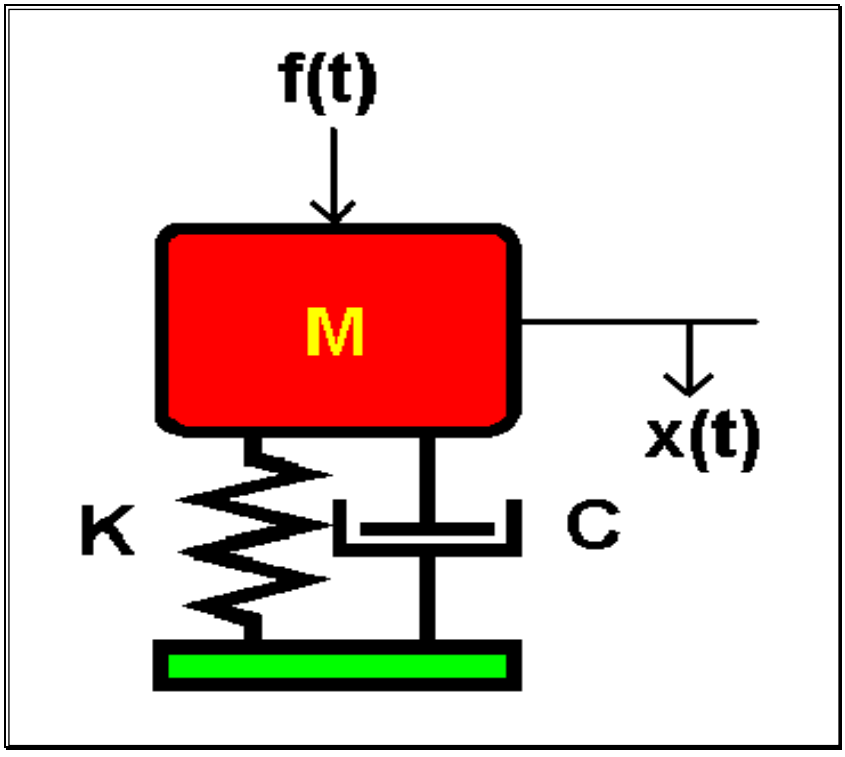
- ***Hold down* the Ctrl key** and ***click* on a Hotkey** to display its Script window

INTRODUCTION

Depending on the units of the sensors used to acquire real-world vibration data, both integration & differentiation can be used to convert the data to different units. Normally, integration & differentiation are only performed on time domain waveforms, but it will be shown in this App Note how integration & differentiation can also be performed on FRFs and mode shapes, specifically Residue mode shapes.

Integration & differentiation can be performed on time or frequency domain waveforms in any Data Block in MEscope. In addition, integration & differentiation can be performed on Residue mode shapes in any Shape Table in MEscope.

In this App Note, the formulas for FRFs in terms of modal parameters are first developed. These formulas are developed for the SDOF mass-spring-damper in the figure below. The formulas for an FRF in terms of modal parameters are also used for FRF-based modal parameter estimation (or curve fitting) in MEscope. The formulas for an IRF in terms of modal parameters give the clearest explanation of the role that each modal parameter plays in the vibration of a structure.



SDOF Mass-Spring-Damper.

The modal properties of real-world structures are analyzed using ***multi-degree-of-freedom*** (**MDOF**) dynamic models. In this App Note, the ***single degree-of-freedom*** (**SDOF**) mass-spring-damper model shown in the figure above will be used. The dynamics of real-world MDOF structures are better understood by analyzing the dynamics of this SDOF mass-spring-damper structure.

BACKGROUND MATH

The dynamic behavior of the SDOF mass-spring-damper in the figure above is represented by the single differential equation, equation (1) shown below. The dynamic behavior of an MDOF structure is represented by ***multiple equations of the same form*** as the equation below, but the mass, stiffness, and damping terms are replaced with a mass matrix multiplied by a vector of accelerations, a damping matrix multiplied by a vector of velocities, and a stiffness matrix multiplied by a vector of displacements.

The time domain equation of motion for the SDOF mass-spring-damper is represented by Newton's Second Law,

$$\mathbf{M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t)}$$

M= mass value

C = damping coefficient

K = spring stiffness

$\ddot{\mathbf{x}}(\mathbf{t})$ = acceleration

$\dot{\mathbf{x}}(\mathbf{t})$ = velocity

$\mathbf{x}(\mathbf{t})$ = displacement

$\mathbf{f}(\mathbf{t})$ = excitation force

LAPLACE TRANSFORM

By taking Laplace transforms of the terms in the equation above and setting initial conditions to zero, an equivalent frequency domain equation of motion results,

$$\mathbf{[M\,s^2 + C\,s + K]\ X(s) = F(s)}$$

X(s) = Laplace transform of the displacement

F(s) = Laplace transform of the force

$\mathbf{s = \sigma + j\omega}$ = Laplace variable (complex frequency

TRANSFER FUNCTION

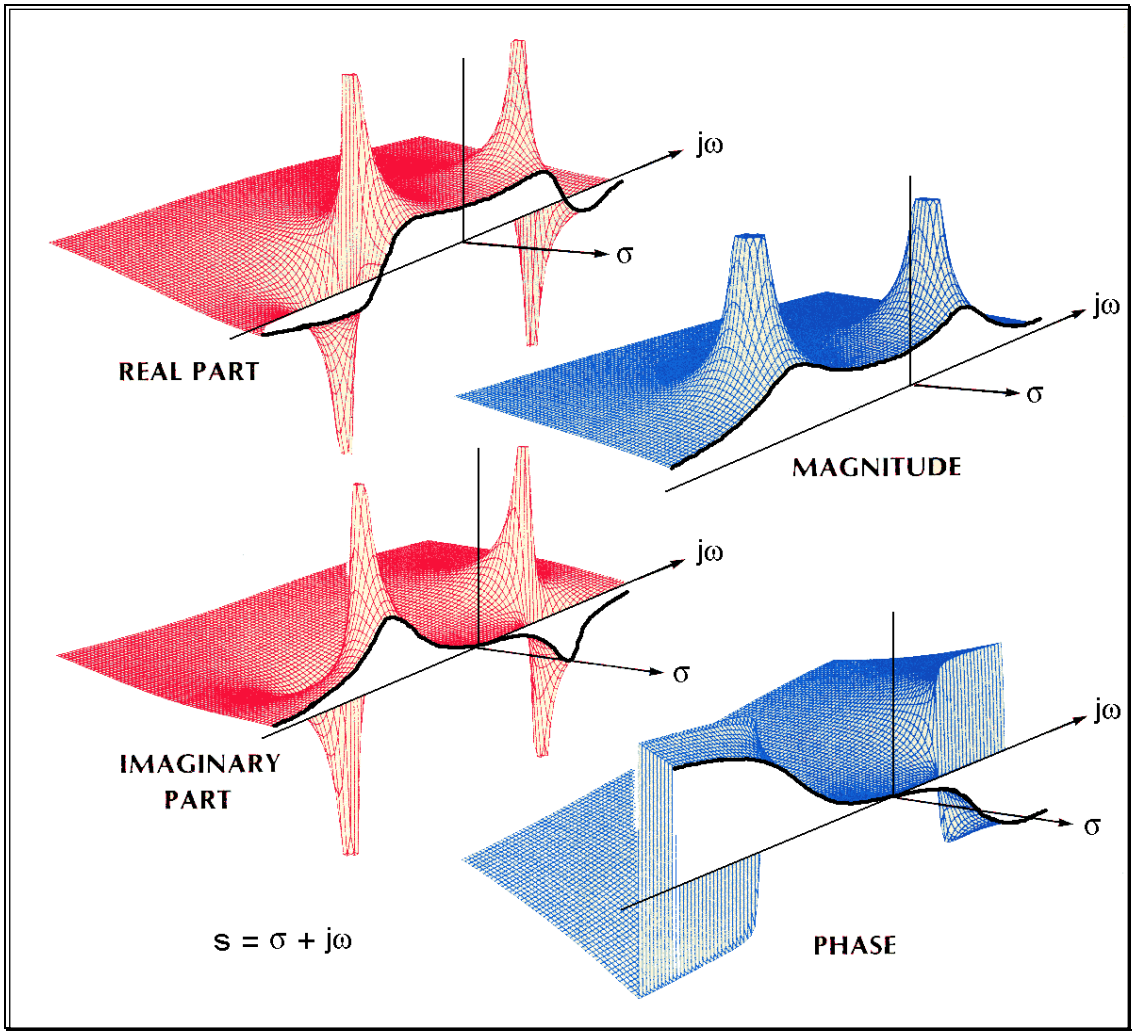
The equation above can be rewritten by dividing both sides by the coefficients of the left-hand side.

$$\mathbf{X(s) = \left(\frac{1}{M\,s^2 + C\,s + K} \right) F(s)}$$

The new coefficient on the right-hand side is called a **Transfer Function**.

$$\mathbf{H(s) = \frac{X(s)}{F(s)} = \left(\frac{1}{M\,s^2 + C\,s + K} \right)}$$

A Transfer Function is ***complex valued***, and therefore has ***real & imaginary*** parts, or equivalently ***magnitude & phase***. The two parts of the Transfer Function can be plotted on the complex Laplace plane (the **s**-plane), as shown in the figure below.



Transfer Function on the s-Plane.

POLES OF THE TRANSFER FUNCTION

The magnitude of the Transfer Function above has two peaks. These peaks are where the value of the Transfer Function ***goes to infinity***. The real and imaginary parts also show the same two peaks.

The Transfer Function above has a ***value of infinity*** for values on the **s**-Plane where its ***denominator is zero***. As the ***s variable approaches infinity***, the Transfer Function ***approaches zero***.

The denominator of the Transfer Function is a second order polynomial in the **s** variable, called the ***characteristic polynomial***. Since it is a second order polynomial, ***it has two roots*** (values of **s** where it will be zero).

The roots of the ***characteristic polynomial*** are called the ***poles*** of the Transfer Function

Poles always occur in pairs, one pole for a positive frequency and one for a negative frequency

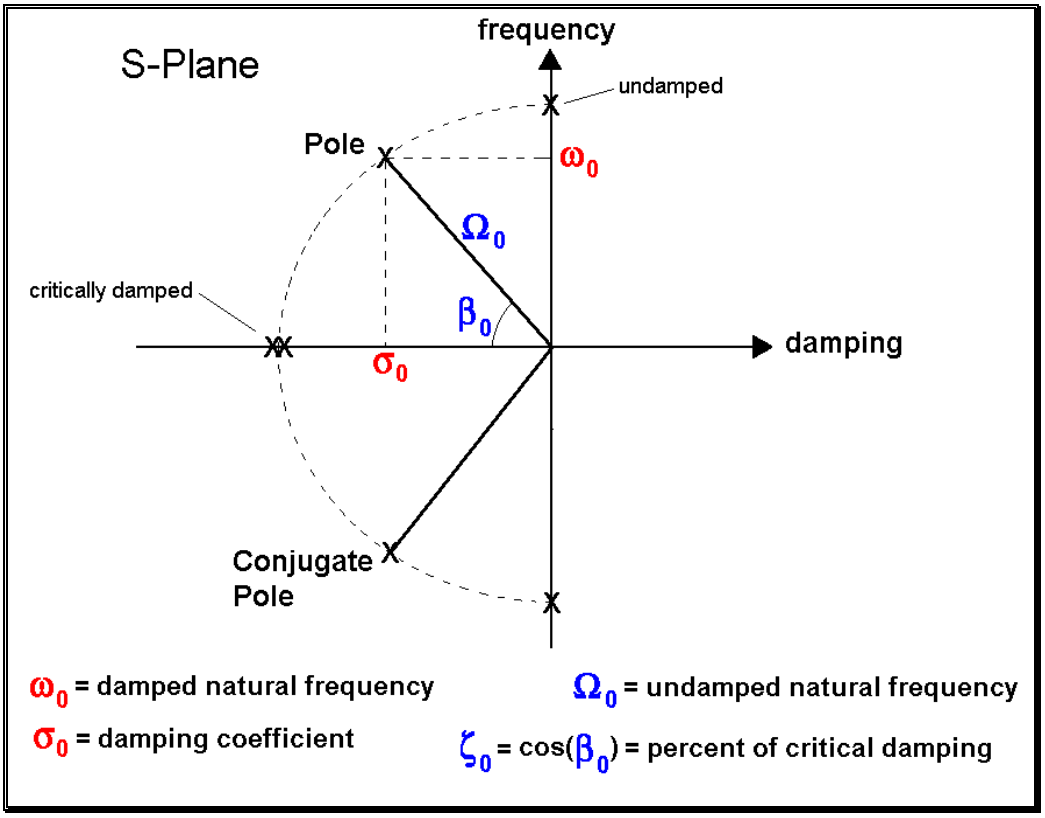
Poles are the locations in the **s**-plane where the Transfer Function ***has a value of infinity***

- The poles are also called **eigenvalues**.

$$\mathbf{p_0 = -\sigma_0 + j\omega_0}\ , \quad \mathbf{p_0{*} = -\sigma_0 - j\omega_0}$$

S-PLANE NOMENCLATURE

The ***real axis*** in the **s**-Plane is called the ***damping axis***, and the ***imaginary axis*** is called the ***frequency axis***. The locations of the poles in the **s**-Plane have also been given some other commonly used names, as shown in the figure below.



s-Plane Nomenclature.

MODAL PARAMETERS

The coordinates of the poles in the **s**-Plane are also modal parameters. Rewriting the equation for a Transfer Function in terms of its pole locations,

$$H(s) = \left(\frac{1/M}{s^2 + 2\sigma_0 s + \Omega_0^{\,2}} \right)$$

$$\sigma_0 = \frac{C}{2M}, \quad \Omega_0^{\,2} = \frac{K}{M}$$

σ_0 = **modal damping coefficient**

Ω_0 = **undamped modal frequency**

$\Omega_0^{\,2} = \sigma_0^{\,2} + \omega_0^{\,2}$

ω_0 = **damped modal frequency**

The ***percent of critical damping*** (ζ_0) is,

$$\zeta_0 = \frac{\sigma_0}{\Omega_0} = \frac{C}{2\sqrt{MK}}$$

FREQUENCY RESPONSE FUNCTION (FRF)

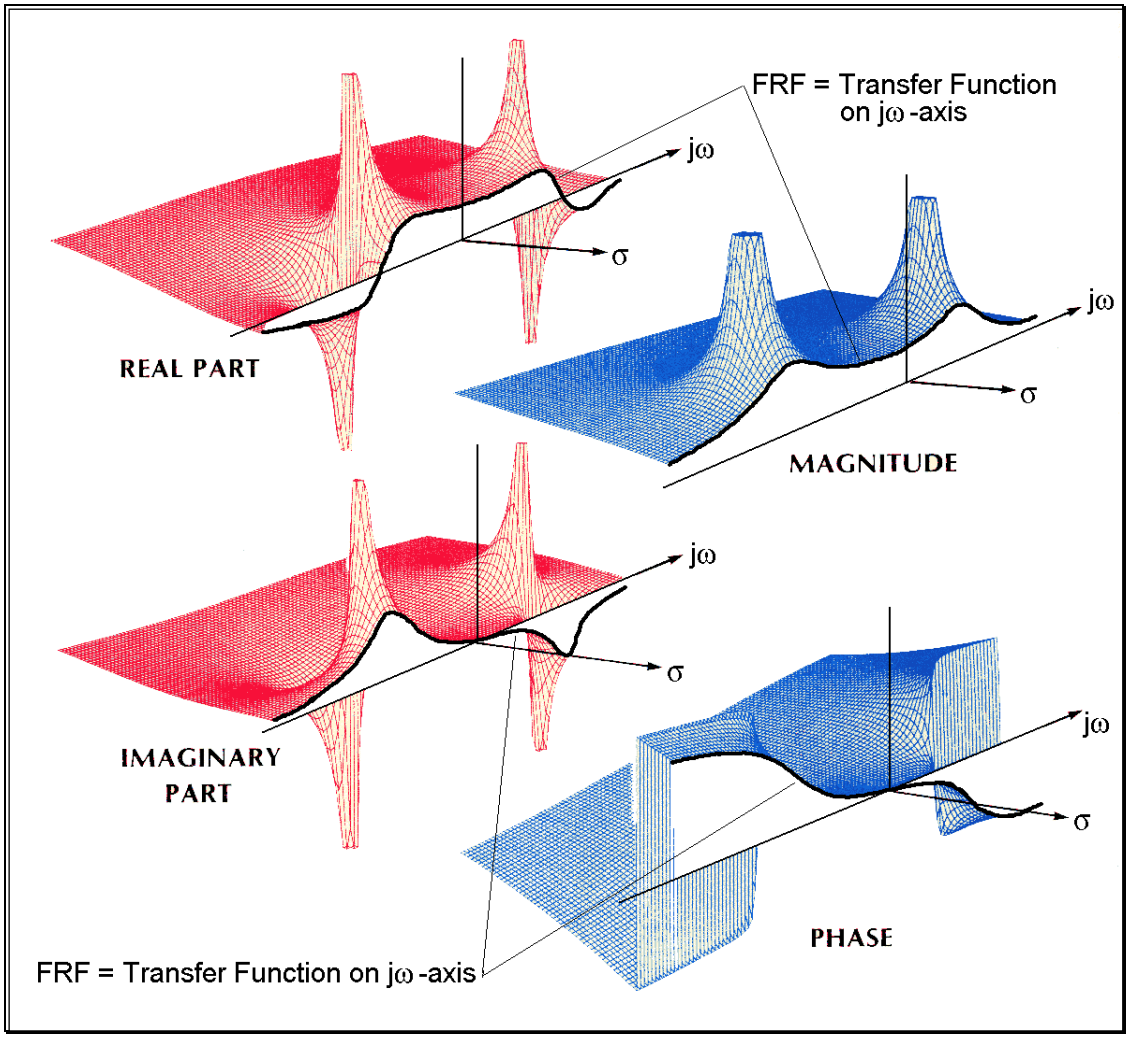
Notice in the figure below that the ***Transfer Function*** is only plotted for ***half of the s-Plane.*** That is, it has been only been plotted for ***negative values of damping axis*** (the real part of **s=σ+jω**). This was done so that the values of the Transfer Function along the ***frequency axis*** (the imaginary part of **s=σ+jω**) are clearly seen.

- The **Frequency Response Function** (**FRF**) is defined as the values of the Transfer Function ***only along the jω-axis*** (or the ***frequency axis***) in the s-plane

The FRF is merely the Transfer function evaluated along the line (**s=jω**) in the s-plane.

$$\mathbf{FRF} = \mathbf{H(j\omega)} = \mathbf{H(s)}\Big|_{s=j\omega}$$

$$= \frac{\mathbf{X(s)}}{\mathbf{F(s)}}\Big|_{s=j\omega} = \frac{\mathbf{X(j\omega)}}{\mathbf{F(j\omega)}}$$



FRF Plotted on the jω-axis.

The FRF for the SDOF mass-spring-damper is written by replacing the **s**-variable with $\mathbf{j\omega}$ in the equation above.

$$\mathbf{H(j\omega)} = \frac{(1/M)}{(j\omega)^2 + 2\sigma_0(j\omega) + \Omega_0^{\,2}}$$

PARTIAL FRACTION EXPANSION OF AN FRF

Using the poles of the characteristic polynomial, a ***partial fraction expansion*** can be performed on the equation for an FRF

$$H(j\omega) = \frac{1}{2j}\left[\frac{R_0}{j\omega - p_0} - \frac{R_0}{j\omega - p_0*}\right]$$

$$R_0 = 1/\omega_0 M$$

R_0 is called the ***modal residue***. It is the ***amplitude*** of the numerator of each resonance term when an FRF is written as a partial fraction expansion.

Comparing the equation above with the plot of the FRF in the figure above, it is clear that the FRF of the SDOF mass-spring-damper is a ***summation of two resonance curves***, each curve ***forming a peak*** at one of the two pole locations.

$$p_0 = -\sigma_0 + j\omega_0 \quad \& \quad p_0* = -\sigma_0 - j\omega_0$$

In ***partial fraction expansion form***, the FRF for the SDOF mass-spring-damper is ***fully represented*** by a ***modal frequency*** $(j\omega_0)$, ***modal damping*** (σ_0), and ***modal residue*** (R_0)

RESIDUE UNITS

From the equation above, it is clear that,

Residue units = (**FRF units**) x (**radians/second**)

This is because the units of the FRF denominator are (radians/second), or 2π **Hz**.

- Residues have engineering units and they also have unique values

For the SDOF mass-spring-damper, the residue is ***part of a vector with two components***. The second component corresponds to the ***ground***, to which the spring & damper are attached. The ground has ***no motion***, so the second component of the residue vector has a value of ***zero***.

Residue mode shape: In MEscope, when residues are assembled into a vector, they are called a ***Residue mode shape***

MODE SHAPE

One final step is to represent the FRF in terms of a ***mode shape instead of residues***.

$$H(j\omega) = \frac{1}{2j}\left[\frac{\{u_0\}^2}{j\omega - p_0} - \frac{\{u_0\}^2}{j\omega - p_0*}\right]$$

$$\{u_0\} = \begin{Bmatrix} \frac{1}{\sqrt{A\omega_0 M}} \\ 0 \end{Bmatrix}$$

A = a scaling constant

- The mode shape $\{u_0\}$ of the SDOF mass-spring-damper is a vector with ***two components***
- The ***second component*** of $\{u_0\}$ corresponds to the ***ground***, where there is ***no motion***
- The ***scaling constant*** (**A**) is necessary because the components of a mode shape ***do not have unique values***
- Only the ***"shape" (one component relative to another***) of a mode shape ***is unique***. For this reason, a mode shape is also called an ***eigenvector***

In the above equation, the FRF of the SDOF mass-spring-damper is fully represented by a pair of eigenvalues (poles) and a pair of eigenvectors (mode shapes).

In ***partial fraction expansion form***, the FRF for the SDOF mass-spring-damper is fully represented by a ***modal frequency*** $(j\omega_0)$, ***modal damping*** (σ_0), and a ***mode shape*** $\{u_0\}$

MODE SHAPE VERSUS RESIDUE MODE SHAPE

A Mode Shape vector is different than a Residue mode shape vector.

The **Mode Shape** vector $\{u_0\}$ has arbitrary values and no engineering units.

The **Residue** mode shape vector $\{R_0\}$ has specific values and engineering units.

For the **scaling constant A=1**, the **Residue** mode shape and the **Mode Shape** vector are related by the equation,

$$\{u_0\} = \begin{Bmatrix} \frac{1}{\sqrt{A\omega_0 M}} \\ 0 \end{Bmatrix} = \begin{Bmatrix} \frac{1}{\sqrt{\omega_0 M}} \\ 0 \end{Bmatrix} = \begin{Bmatrix} \sqrt{R_0} \\ 0 \end{Bmatrix}$$

IMPULSE RESPONSE FUNCTION (IRF)

An Impulse Response Function (**IRF**) is the ***Inverse FFT*** of an **FRF.**

An IRF can also be written in terms of modal parameters, and it ***provides the best physical meaning of the modal parameters***. Applying the Inverse FFT to the equations above gives,

$$h(t) = FFT^{-1}\left(\frac{1}{2j}\left[\frac{R_0}{j\omega - p_0} - \frac{R_0}{j\omega - p_0 *}\right]\right)$$

$$h(t) = \frac{T}{2j}\left[R_0 e^{p_0 t} - R_0 e^{p_0 * t}\right]$$

$$h(t) = T\,|R_0|\,e^{-\sigma_0 t}\left(\sin(\omega_0 t + \alpha_0)\right)$$

$$\alpha_0 = \text{phase angle of } R_0$$

These equations show how the IRF is expressed in terms of each modal parameter.

The IRF is a sinusoidal function $\left(\sin(\omega_0 t + \alpha_0)\right)$ which describes the sinusoidal oscillation of the SDOF, hence ω_0 is called ***modal frequency***

σ_0 is the coefficient in the exponential term $(e^{-\sigma_0 t})$ which defines the envelope of decay of the SDOF, hence σ_0 is called the ***modal damping coefficient*** or ***modal damping decay constant***

In a real-world structure, the decay of each mode is caused by ***a combination of damping mechanisms*** within or about the structure. For the SDOF mass-spring-damper, the modal damping is caused by the viscous damper attached between the mass and ground.

In a real-world structure, the ***viscous damping of the surrounding air*** is a ***significant damping force***, hence a viscous damping model is used in Experimental Modal Analysis (**EMA**) to model real-world damping.

IRF UNITS

In the equation above, the IRF has two terms, an exponential function $(T|R_0|e^{-\sigma_0 t})$ and a sinusoidal function $(\sin(\omega_0 t + \alpha))$. The exponential decay $(e^{-\sigma_0 t})$ and sinusoidal function are dimensionless.

Only T and R_0 have units,

Impulse Response units = (**Seconds**) x (**Residue units**)

or,

Impulse Response units = **FRF units**

DIFFERENTIATING AN IRF

Differentiation changes vibration data for ***displacement units*** to ***velocity units*** and from ***velocity units*** to ***acceleration units***. Differentiating the **IRF** with **(displacement/force) units** with respect to time yields,

$$\frac{dh(t)}{dt} = \frac{d}{dt}\left(\frac{T}{2j}\left[R_0 e^{p_0 t} - R_0 e^{p_0^* t}\right]\right)$$

$$= \frac{T}{2j}\left[R_0 \frac{d\left(e^{p_0 t}\right)}{dt} - R_0 \frac{d\left(e^{p_0^* t}\right)}{dt}\right]$$

$$= \frac{T}{2j}\left[R_0 p_0 e^{p_0 t} - R_0 p_0^* e^{p_0^* t}\right]$$

Differentiating an IRF is equivalent to multiplying its modal residues by their respective poles.

$$R_{Velocity} = R_{Displacement}\, p$$

$$R^*_{Velocity} = R^*_{Displacement}\, p^*$$

For the SDOF mass-spring-damper, both poles have real residues (R_0).

For real-world structures, each ***complex conjugate pair*** of poles has a ***complex conjugate pair*** of residues.

An IRF with (**velocity/force**) units ***can be differentiated*** to yield an IRF with (**acceleration/force**) units. Each residue with (**velocity/force-sec**) units is ***multiplied by its pole*** to obtained a residue with (**acceleration/force**) units. In summary,

$$R_{Acceleration} = R_{Velocity}\, p = R_{Displacement}\, p^2$$

The same formula applies for the ***conjugate residues***. A ***conjugate residue*** is ***multiplied by its conjugate pole***. Three equations relating residues with (**displacement/force-sec**), (**velocity/force-sec**), and (**acceleration/force-sec**) units can be written,

$$R_{Acceleration} = R_{Velocity}\, p = R_{Displacement}\, p^2$$

$$R_{Velocity} = R_{Displacement}\, p = \frac{R_{Acceleration}}{p}$$

$$R_{Displacement} = \frac{R_{Velocity}}{p} = \frac{R_{Acceleration}}{p^2}$$

Integration or differentiation of an IRF is equivalent to ***dividing or multiplying*** each of its modal residues by its associated pole.

DIFFERENTIATING AN FRF

Differentiation of an IRF can be written in terms of its corresponding Transfer Function using the following derivative formula for Laplace Transforms,

$$\begin{aligned}
L\{h'(t)\} &= sH(s) - h(0^+) \\
&= \frac{R_0}{2j}\left[\frac{s(p_0 - p_0^*)}{(s-p_0)(s-p_0^*)}\right] - \frac{R_0}{2j}\left[e^0 - e^0\right] \\
&= \frac{R_0}{2j}\left[\frac{s(p_0 - p_0^*) + p_0 p_0^* - p_0 p_0^*}{(s-p_0)(s-p_0^*)}\right] \\
&\quad - \frac{R_0}{2j}[1-1] \\
&= \frac{1}{2j}\left[\frac{R_0 p_0}{s-p_0} - \frac{R_0 p_0^*}{s-p_0^*}\right]
\end{aligned}$$

The derivative of the IRF is obtained by evaluating the right-hand side of the equation above along the frequency axis ($s = j\omega$).

Assuming $h(0^+) = 0$, ***differentiating*** an IRF is the same as multiplying its corresponding FRF by $j\omega$.

Any time waveform can be ***differentiated*** by ***multiplying*** its corresponding frequency domain function by $j\omega$.

Multiplying the FRF for the SDOF mass-spring-damper by $j\omega$

$$H_{Velocity}(j\omega) = \frac{(j\omega/M)}{(j\omega)^2 + 2\sigma_0\, j\omega + \Omega_0^{\,2}}$$

Performing a partial fraction expansion of this FRF gives,

$$R_{Velocity} = R_{Displacement}\; p_0$$

Differentiation or integration of an IRF ***is the same as multiplying or dividing*** its corresponding FRF by $j\omega$.

Differentiation or integration of an IRF ***is the same as multiplying or dividing*** its residues by their respective poles.

In MEscope all time waveforms are ***differentiated or integrated*** by ***multiplying or dividing*** their corresponding frequency domain function by $j\omega$.

DOUBLE DIFFERENTIATING AN IRF

Now that the relationship between ***differentiation & integration*** of an IRF and ***multiplying or dividing*** its FRF by $j\omega$ has been established, what about ***double integration & differentiation*** of an IRF?

Writing the formula for ***double differentiation*** of the IRF in terms of its Laplace Transform gives,

$$\begin{aligned}
L\{h''(t)\} &= s^2H(s) - sh(0^+) - h'(0^+) \\
&= \frac{R_0}{2j}\left[\frac{s^2(p_0 - p_0^*)}{(s-p_0)(s-p_0^*)}\right] - 0 - \frac{R_0}{2j}\left[p_0 - p_0^*\right] \\
&= \frac{R_0}{2j}\left[\frac{p_0p_0(s-p_0^*)}{(s-p_0)(s-p_0^*)} - \frac{p_0^*p_0^*(s-p_0)}{(s-p_0)(s-p_0^*)}\right] \\
&= \frac{1}{2j}\left[\frac{R_0p_0^{\,2}}{s-p_0} - \frac{R_0p_0^{*2}}{s-p_0^*}\right]
\end{aligned}$$

The second derivative of the IRF can be obtained by evaluating the equation along the frequency axis (**s=jω**). The following conclusion can be drawn,

Double differentiation or double integration of an IRF is the same as multiplying or dividing its residues by the square of their poles.

But is ***double differentiation or double integration*** of an IRF the same as multiplying or dividing its corresponding FRF by $(j\omega)^2$?

Let's check by multiplying the FRF for the SDOF mass-spring-damper by $(j\omega)^2$.

$$H_{Acceleration}(j\omega) = \frac{\left((j\omega)^2/M\right)}{(j\omega)^2 + 2\sigma_0\ j\omega + \Omega_0^{\,2}}$$

Performing a partial fraction expansion of this FRF gives a ***different result***,

$$H(j\omega) = \frac{1}{M} + \frac{1}{2j}\left[\frac{R_{Accel}}{j\omega - p_0} - \frac{R_{Accel}}{j\omega - p_0*}\right]$$

$$R_{Acceleration} = R_{Displacement}\ p_0^{\,2}$$

The partial fraction expansion above has an extra term in it, (**1/M**). Evaluating $h'(0^+)$ gives,

$$\begin{aligned}
h'(0^+) &= \frac{R_0}{2j}\left[p_0e^0 - p_0^*e^0\right] \\
&= \frac{R_0}{2j}\left[-\sigma_0 + j\omega_0 - (-\sigma_0 - j\omega_0)\right] \\
&= \frac{1}{2jM\omega_0}\left[2j\omega_0\right] \\
&= \frac{1}{M}
\end{aligned}$$

The velocity term $h'(0^+)$ at a time which is a ***little greater than zero*** (0^+) ***is not zero***

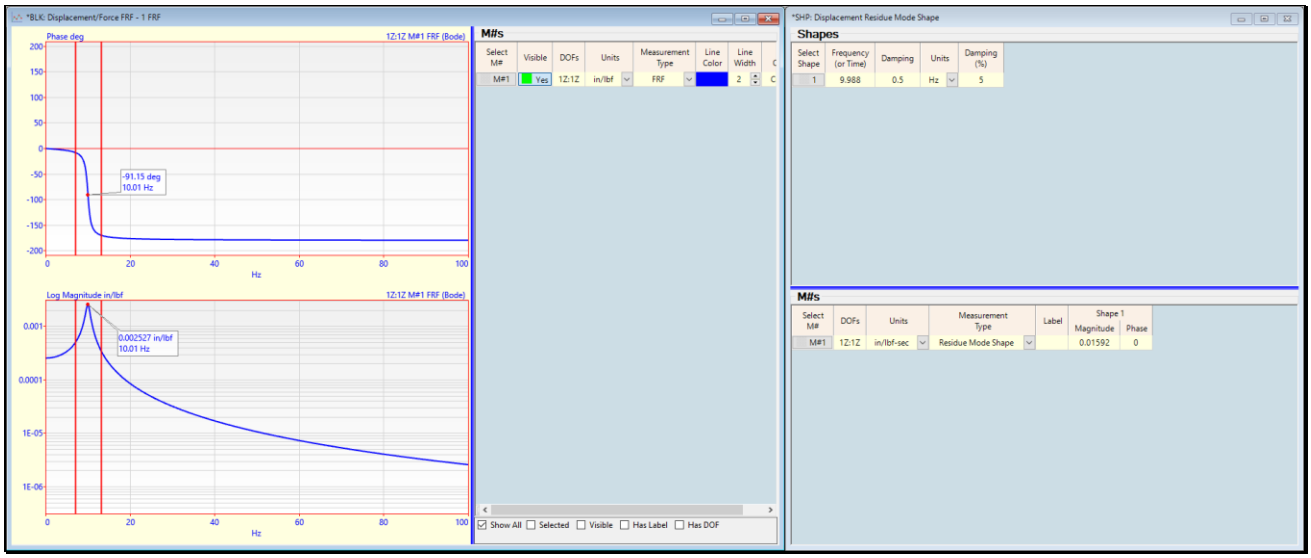
Double differentiating or integrating an IRF is not the same as multiplying or dividing its corresponding FRF by $(j\omega)^2$

STEP 1 - SYNTHESIZING A (DISPLACEMENT/FORCE) FRF

To synthesize the driving point **FRF 1Z:1Z** for the SDOF mass-spring-damper,

- ***Press* Hotkey 1 Synthesize FRF**

The FRF is synthesized and displayed next to the Shape Table with the Residue mode shape in it, **SHP: Residue mode shape**.



Driving Point (Displacement/Force) FRF and Residue mode shape Table.

An FRF, like a Transfer Function, defines the dynamic characteristics between two DOFs of a structure. MDOF systems have many DOF pairs for which FRFs can be calculated, either from measured data or from modal parameters. The mode shape of the SDOF mass-spring-damper has only one meaningful DOF. The other DOF is for the ***ground point*** where there is ***no motion***.

Any FRF with its **Roving DOF *equal to its* Reference DOF** is called a **Driving Point FRF**.

The driving point **FRF 1Z:1Z** was synthesized using the Residue mode shape with **DOFs 1Z:1Z** in the **SHP: Residue mode shape** window.

STEP 2 - DIFFERENTIATING TO OBTAIN A (VELOCITY/FORCE) FRF

There are two ways to obtain a (**Velocity/Force**) FRF,

1) Multiply the Residue mode shape with (**displacement/force-sec**) units by its pole to obtain a (**velocity/force-sec**) residue, and synthesize a new FRF using the new Residue mode shape.

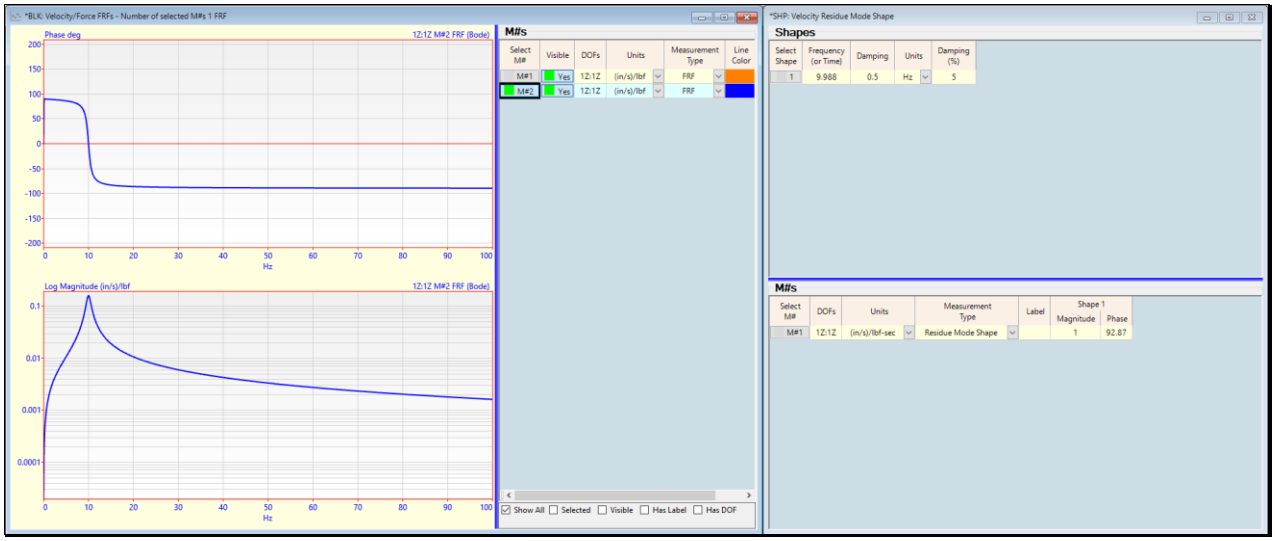
2) Multiply the (**Displacement/Force**) FRF by **jω**.

It has already been shown by the math above that ***both methods give the same result***.

In MEscope, **method #1** is carried out by executing **Tools | Differentiate** followed by the **Tools | Synthesize FRFs** command in a Shape Table window.

- ***Press* Hotkey 2 Velocity/Force FRF**

Notice that the Residue mode shape in **SHP: Residue mode shape** now contains ***velocity units*** in its numerator.



(Velocity/Force) FRFs Overlaid

Method #2 is carried out by executing **Tools | Differentiate** on the FRF in the **BLK: Displacement/Force FRF** window. When **Hotkey 2 was *pressed***, the (**displacement/force**) FRF is added to the **BLK: Velocity FRF** window, and is ***differentiated*** by ***selecting*** **M#2** and executing **Tools | Differentiate**. Then, the two **M#s *are overlaid***.

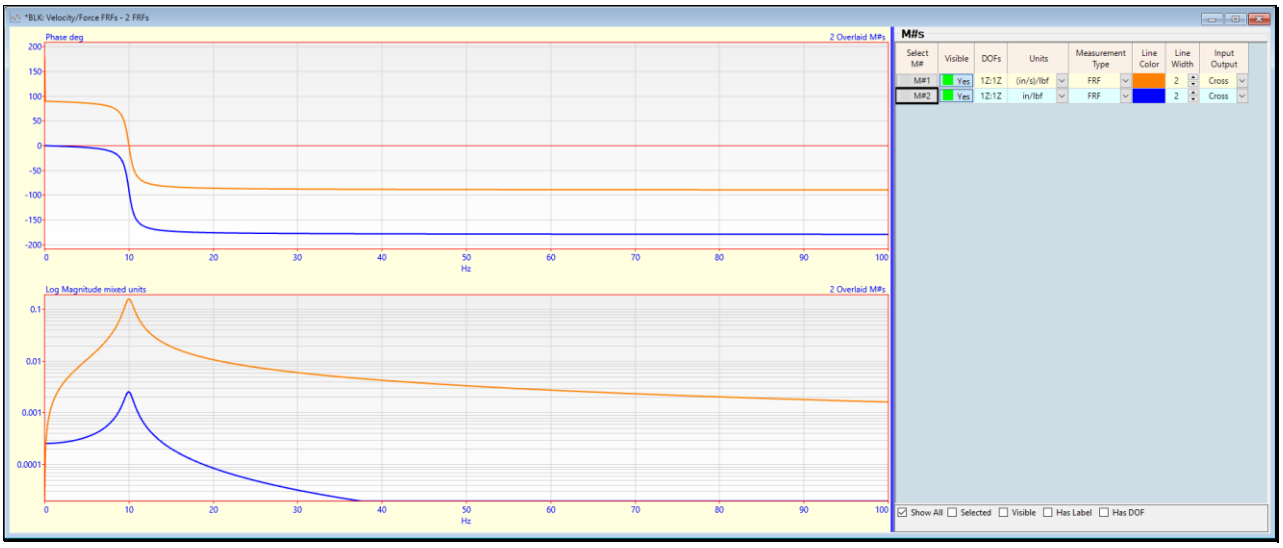
- ***Alternately click*** on a **Select M#2** in **BLK: Velocity/Force FRFs** to display **M#2** as shown above

The two FRFs ***are exactly the same but have different colors,*** so the color will change from orange to blue as **Select M#2 is *pressed***.

STEP 3 - (DISPLACEMENT/FORCE) VS. (VELOCITY/FORCE) FRF

- ***Press* Hotkey 3 Displacement vs. Velocity**

When **Hotkey 3 is *pressed***, the (**Displacement/Force**) FRF is overlaid with the (**Velocity/Force**) FRF.



(Displacement/Force) vs. (Velocity/Force) FRF

Multiplying the (**Displacement/Force**) FRF by $\mathbf{j\omega}$ caused the phase to ***change by 90 degrees*** and the magnitude to be ***zero at DC (zero frequency)***.

STEP 4 - DIFFERENTIATING TO OBTAIN AN (ACCELERATION/FORCE) FRF

There are three ways to obtain an (**Acceleration/Force**) FRF,

1) Multiply the Residue mode shape with **(velocity/force-sec)** units by its pole and synthesize a new FRF using the Residue mode shape with **(acceleration/force-sec)** units.

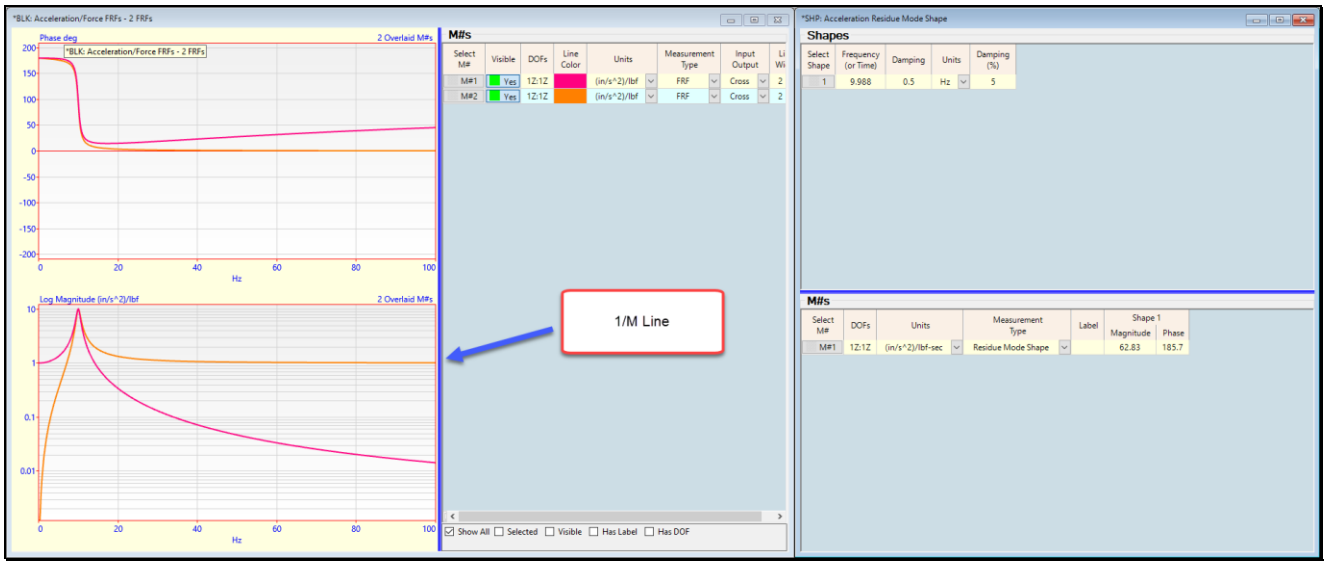
2) Multiply a (**Displacement/Force**) FRF by $(j\omega)^2$

3) Multiply a (**Velocity/Force**) FRF by $j\omega$

We have already concluded from the math that ***these three methods will not yield the same results*** because ***there is an extra term involved***.

- ***Press* Hotkey 4 Acceleration/Force FRF**

Notice that the Residue mode shape shown below contains ***acceleration units*** in its numerator.



(Acceleration/Force) FRFs Overlaid.

When **Hotkey 4 was *pressed***, the (**Acceleration/Force**) FRF **in red** was synthesized from the Residue mode shape with ***acceleration units*** in the numerator. The (**Acceleration/Force**) FRF **in orange** was created by ***multiplying a* (Velocity/Force)** FRF by $j\omega$.

The phases of the two (**Acceleration/Force**) FRFs are similar, but their magnitudes ***are different.***

The **1/M line** in **M#2**, the extra term in its partial fraction expansion, is ***clearly visible***.

STEP 5 – REVIEW STEPS

To review all the steps of this App Note,

- ***Press* Hotkey 5 Review Steps**

CONCLUSIONS

The following conclusions can be drawn from the steps of this App Note,

Multiplying or dividing an FRF by $\mathbf{j\omega}$ ***is the same*** as differentiating or integrating its IRF.

Multiplying or dividing an FRF by $(\mathbf{j\omega})^2$ ***is not the same*** as double differentiating or integrating its IRF.

Multiplying or dividing Residue mode shapes ***by their Pole is the same*** as differentiating or integrating the IRF corresponding to an FRF that is synthesized from the residues.

Multiplying or dividing Residue mode shapes ***by their Pole squared is the same*** as double differentiating or integrating the IRF corresponding to an FRF that is synthesized from the residues.

Any time waveform can be ***differentiated or integrated*** by ***multiplying or dividing*** its corresponding frequency domain function by $\mathbf{j\omega}$.