MEscope Application Note 15

Multi-Reference Curve Fitting to Find Closely-Coupled Modes

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** & **VES-4600 Advanced Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the **AppNote15** project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 15 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote15.zip**

This Project file contains numbered Hotkeys & Scripts for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click* on a Hotkey** to display its Script window

WHEN IS MULTI-REFERENCE MODAL TESTING NECESSARY?

Multi-Reference Modal Analysis is necessary when a structure has resonances that occur under one of the following conditions,

- **Closely-coupled Modes:** One resonance peak represents ***two or more modes***
- **Repeated Roots:** Two or more modes having the ***same natural frequency*** but ***different mode shapes***
- **Local Modes:** ***Different resonance peaks*** appear in FRFs ***from different references***

In each of the above cases, ***Multi-reference curve fitting is required*** to properly extract all modal parameters from a set of FRFs.

SINGLE-REFERENCE VERSUS MULTI-REFERENCE FRFS

The term ***Multi-Reference Modal Analysis*** means that FRFs are acquired using ***two or more references*** (fixed excitation sensors or fixed response sensors). Multi-Reference FRFs must be curve fit using ***Multi-Reference curve fitting***. Multi-reference FRFs correspond to ***multiple rows or columns*** of the FRF matrix in the MIMO dynamic model of a structure. (***See* App Note 42** for details.)

A single-reference set of FRFs,

- Is the ***minimum requirement*** for extracting experimental modal parameters from FRFs
- Is obtained by exciting the structure with a ***fixed single exciter*** or using ***a fixed single response sensor***
- Is not enough data for extracting ***closely-coupled modes, repeated roots***, or ***local modes*** of a structure

A multi-reference set of FRFs,

- Is obtained by exciting the structure with ***multiple (fixed) exciters*** or using ***multiple (fixed) response sensors***
- Is necessary for extracting ***closely-coupled modes, repeated roots***, or ***local modes*** of a structure
- Is useful for extracting modes when a structure has ***high modal density*** (many resonance peaks in small frequency bands)

MULTI-REFERENCE MODAL TESTING

A ***Multi-reference modal test*** is done using either ***multiple (fixed) exciters*** with sensors to measure the excitation forces or using ***multiple (fixed) response sensors***.

MULTI-SHAKER TEST

Large structures with ***non-linear dynamic behavior*** are typically tested using multiple shakers, driven by ***broad-band*** excitation signals.

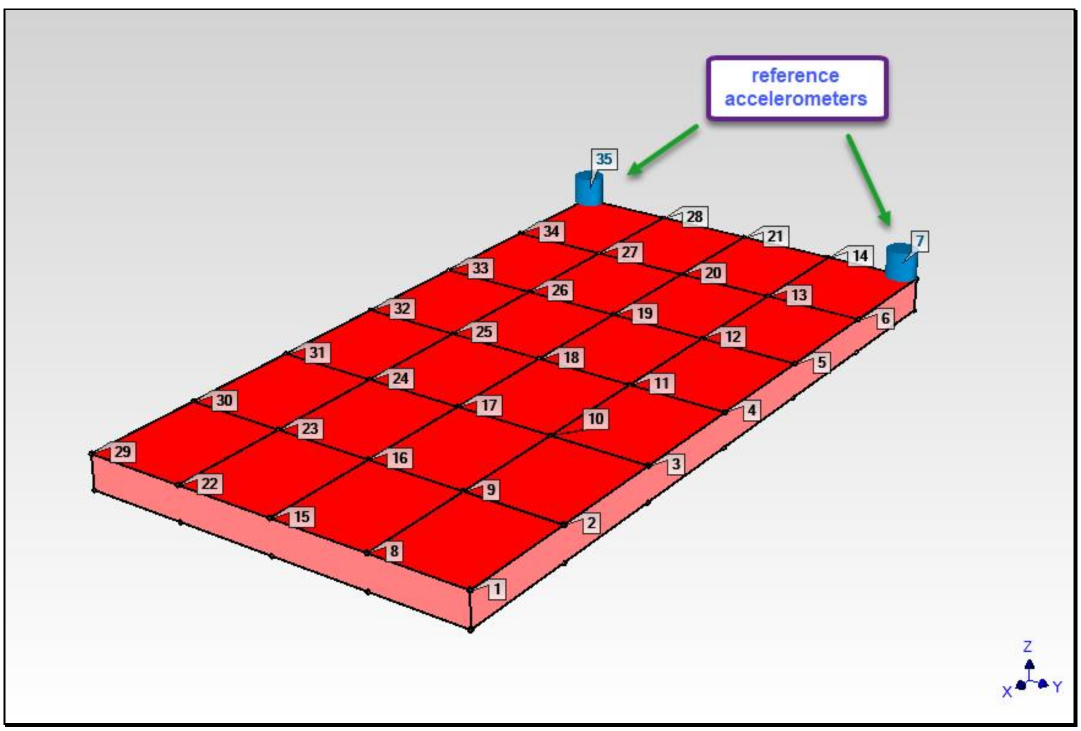
- **Linear Dynamics:** Any structure that exhibits ***linear stationary behavior*** is said to exhibit linear dynamics
- **Non-Linear Dynamics**: Any structure that ***does not exhibit linear stationary behavior*** is said to exhibit non-linear dynamics
- **Linear Stationary Behavior**: The resonances (modes of vibration) of the structure ***do not change with excitation force levels***, and its physical properties (mass, stiffness, damping & boundary conditions) ***do not change during a modal test***

In a Multi-Shaker test, ***two or more (fixed) shakers*** are used to ***simultaneously*** excite the structure.

- The shakers must be driven with ***uncorrelated broad band signals***
- Random excitation together with ***spectrum averaging*** is used to ***"average out" the non-linear dynamic behavior*** of the structure from the spectra and hence the FRFs
- An FRF is calculated between each response & each excitation force
- **Multiple Coherence** is calculated to indicate the ***participation of all shakers*** in each response
- **Partial Coherence** is calculated to indicate the ***participation of each shaker*** in each response
- The FRFs are elements of ***two or more columns of the FRF matrix*** in a **MIMO model** of the structure

MULTI-REFERENCE ROVING IMPACT TEST

In a Multi-reference roving impact test, ***two or more (fixed) response sensors*** are used, and the structure is excited one DOF at a time with a roving impactor. An example with two reference accelerometers is shown below.



Multiple-Reference Roving Impact Test Using Two Reference Acclerometers.

- A Multi-reference roving impact test is the ***same as performing two or more single-reference roving impact tests***. It takes no more time to complete than a single-reference roving impact test
- FRFs are calculated for ***two or more rows of the FRF matrix*** in a MIMO model of the structure

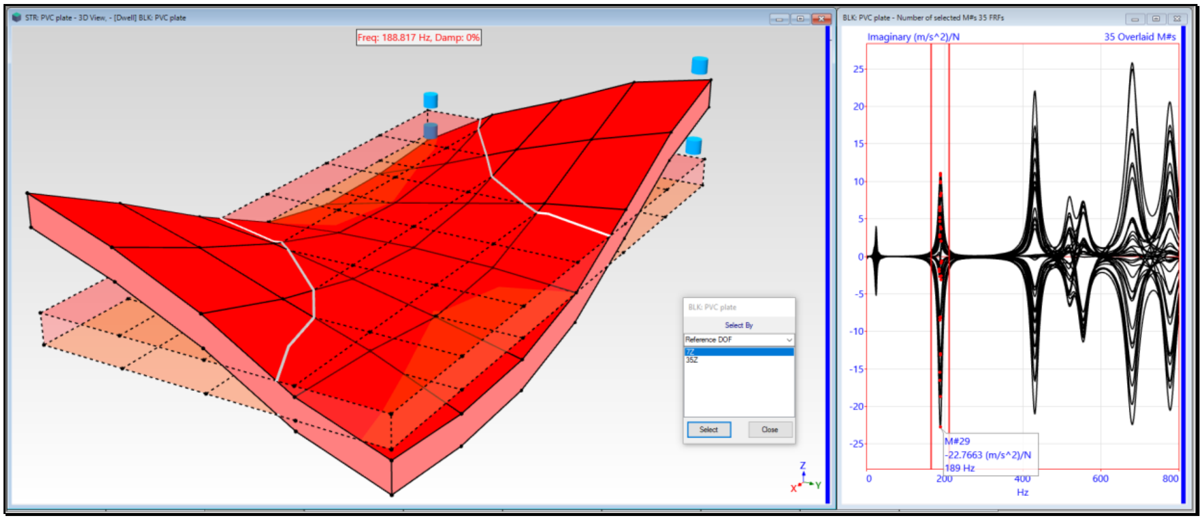
STEP 1 - ODS ANIMATION REVEALS CLOSELY-COUPLED MODES

The PVC plate structure shown below has two ***closely-coupled*** modes at about **189 Hz**. Animated deflection of the ODS at **189 Hz** will reveal the presence of those two modes.

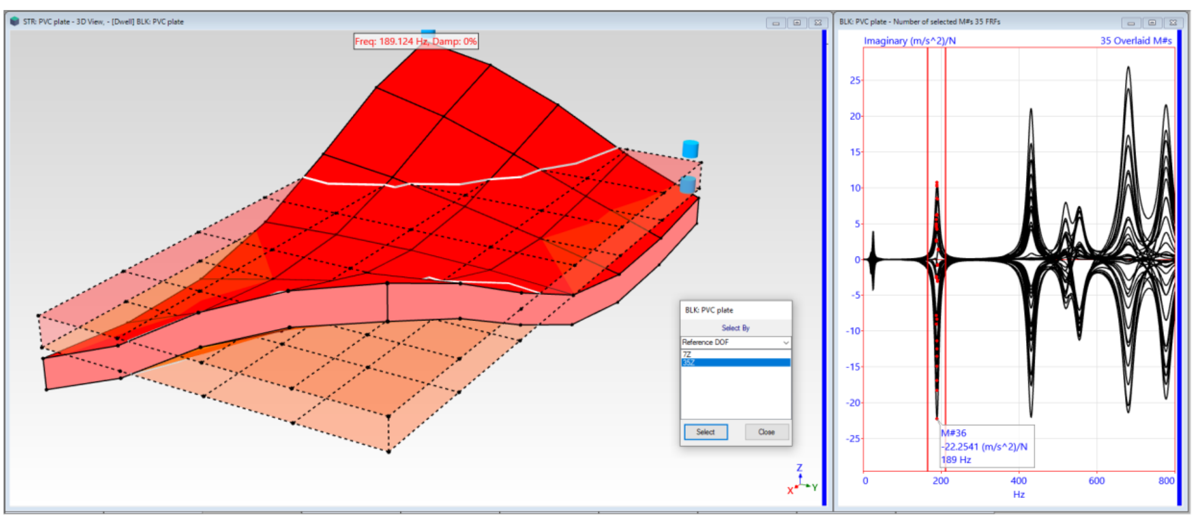
- ***Press* Hotkey 1 Animate ODS's**

The **Imaginary** part of the **M#s** in **BLK: PVC plate** are displayed in **Overlaid** format, and the **Peak** cursor is surrounding a resonance peak at about **189 Hz**, as shown below

- Select reference **DOF 7Z** and then reference **DOF 35Z** in the selection box



ODS animation from Reference 7Z.



ODS animation from Reference 35Z.

- If it is ***dominated*** by a ***single mode shape***, an ODS ***will not change*** if displayed from one reference and then from another
- If an ODS changes when displayed from one reference and then from another, this is a ***clear indication*** that the ***ODS is dominated*** by ***more than mode shape***
- ***Drag*** the **Peak** cursor band to surround the peaks at **431 Hz**
- Select reference **DOF 7Z** and then reference **DOF 35Z**

The ODS ***does not change*** when displayed from one reference and then the other. This ***verifies*** that the ODS at the **431 Hz** peak is ***dominated by the mode shape of a single resonance.***

Displaying the ODS from different references of Multi-reference FRF data is the best way to verify whether the ODS at a resonance peak **is *dominated by a single mode shape***.

If the ODS changes when displayed from one reference and them from another, this reveals that ***two or more closely-coupled modes are dominating the* ODS**

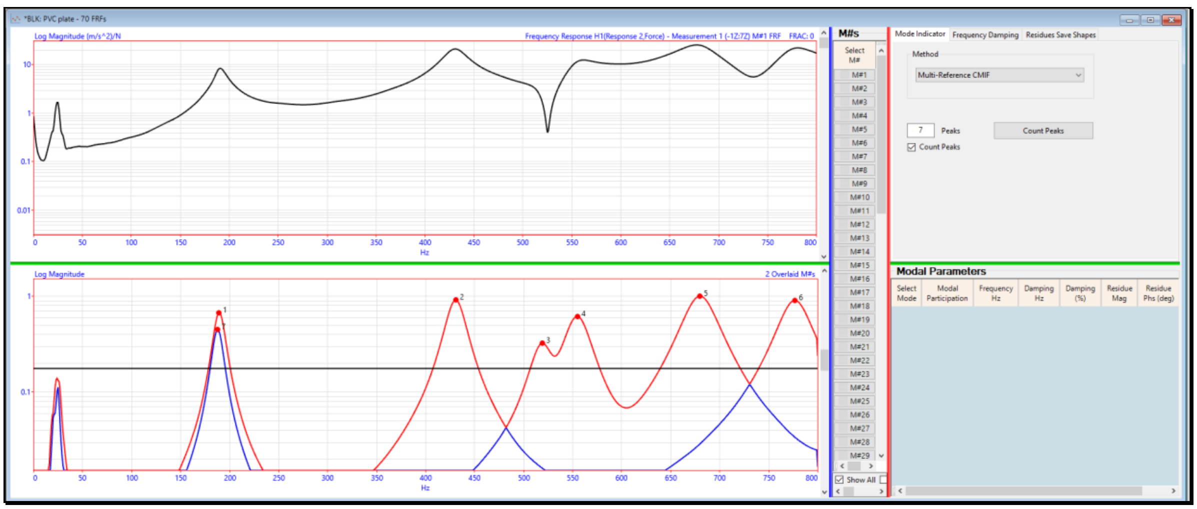
STEP 2 - MULTI-REFERENCE MODE INDICATOR

The **first step of all curve fitting** is to determine **how many modes** are represented by resonance peaks in a set of FRF data.

- ***Press* Hotkey 2 Multi-Ref Mode Indicator**

Curve fitting has begun in the **BLK: PVC plate** window as shown below.

- **Two Multi-Reference CMIF** curves are shown on the ***lower left***, one for each reference of FRF data
- ***Seven peaks*** have been counted on the ***two Indicator curves*** and the number of peaks in displayed in the **Peaks** box on the **Mode Indicator** tab
- The ***seven resonance peaks*** are numbered and indicated with **red dots** on the Mode Indicator curves
- Both Indicator curves have **red dots *near 189 Hz***, indicating ***two closely-coupled modes***



Multi-Reference CMIFs Showing 7 Resonance Peaks.

In this App Note, Multi-reference curve fitting starts by ***counting the resonance peaks*** on two Multi-reference Mode Indicator curves. MEscope contains two types of Multi-reference Mode Indicators, **Multi-Reference CMIF** and **Multi-Reference MMIF**.

Both Mode Indicators are calculated from Multi-Reference FRF data, resulting in an Indicator curve ***for each reference of* FRF *data***. **Modal Participation** curves are also calculated for each reference of FRF data.

- The **Mode Indicator** curves are used for ***resonance peak counting***
- The **Modal Participation** curves are used for ***weighting the FRF data*** during curve fitting

STEP 3 – MULTIPLE REFERENCE QUICK FIT

- ***Press* Hotkey 3 Multi-Ref Quick Fit**

When a Multi-Reference Mode Indicator is chosen on the **Mode Indicator** tab, the Multi-Reference Polynomial curve fitting method is automatically chosen on the **Frequency Damping** and on the **Residues Save Shapes** tab.

The Quick Fit command completes the curve fitting process in ***one step*** by ***successively executing*** commands on the **Mode Indicator**, **Frequency Damping**, and **Residues Save Shapes** tabs using the methods selected on each of those tabs.

Modal parameters for the seven peaks counting on the Mode Indicator curves are estimated and listed In the **Modal Parameters** spreadsheet as shown below. The curve fitting **red Fit Functions** have also been calculated.

Each Fit Function is **overlaid in red** on its corresponding FRF.

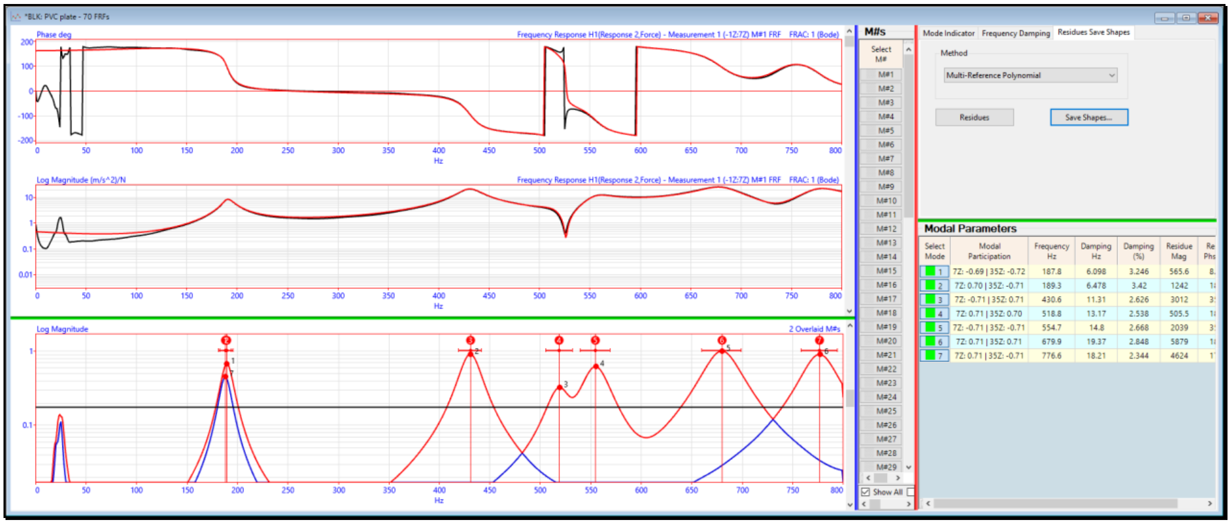
- ***Scroll*** the bar ***to the right*** of the FRFs to display each **red Fit Function** overlaid on its FRF

The modal parameters of ***two closely-coupled modes*** at **188 Hz** & **189 Hz** have been estimated.

The **Modal Participation** of each mode in each reference of FRF data is also listed in the **Modal Parameters** spreadsheet.

Modal Participation was used to ***weight the FRF data*** during curve fitting.

The Participations show that ***both references*** of FRF data ***were equally weighted*** during curve fitting.



Multi-Reference Quick Fit Results.

STEP 4 - FIRST BENDING & FIRST TORSIONAL MODE

In the previous step, a **Multi-Reference Quick Fit** was performed on the FRF data in the **BLK: PVC plate** Data Block and modal parameters for two ***closely-coupled modes*** **(188 Hz & 189 Hz)** were estimated. In this step their mode shapes are displayed in animation of the PVC plate model.

- ***Press*** **Hotkey 4 Quick Fit Mode Shapes**

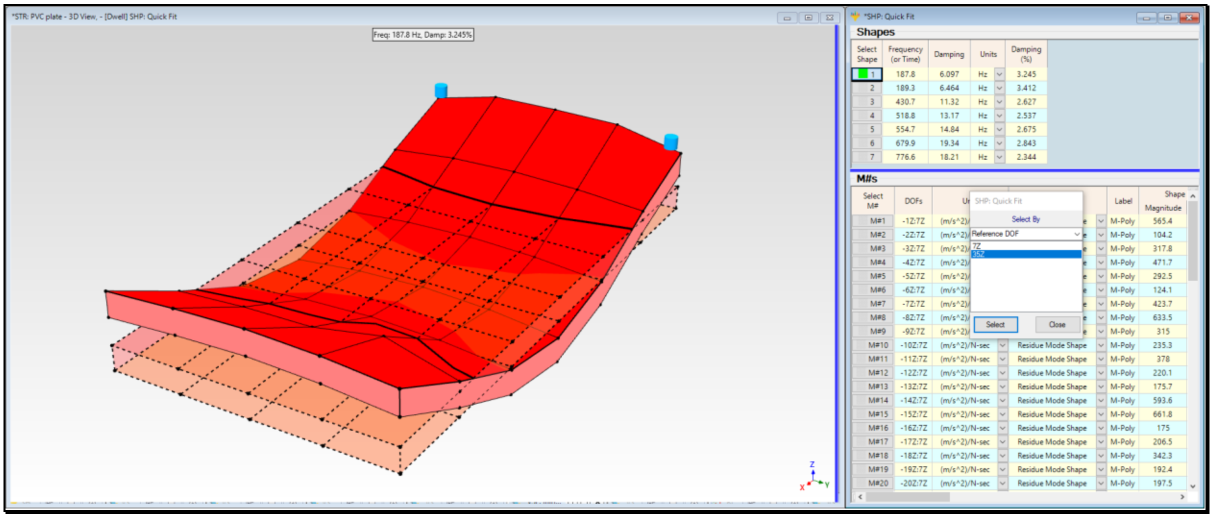
Shape animation of Shape **#1** in **SHP: Quick Fit** will begin.

- In the **Reference DOF selection** box that opens, select reference **DOF 7Z** and then reference **DOF 35Z**

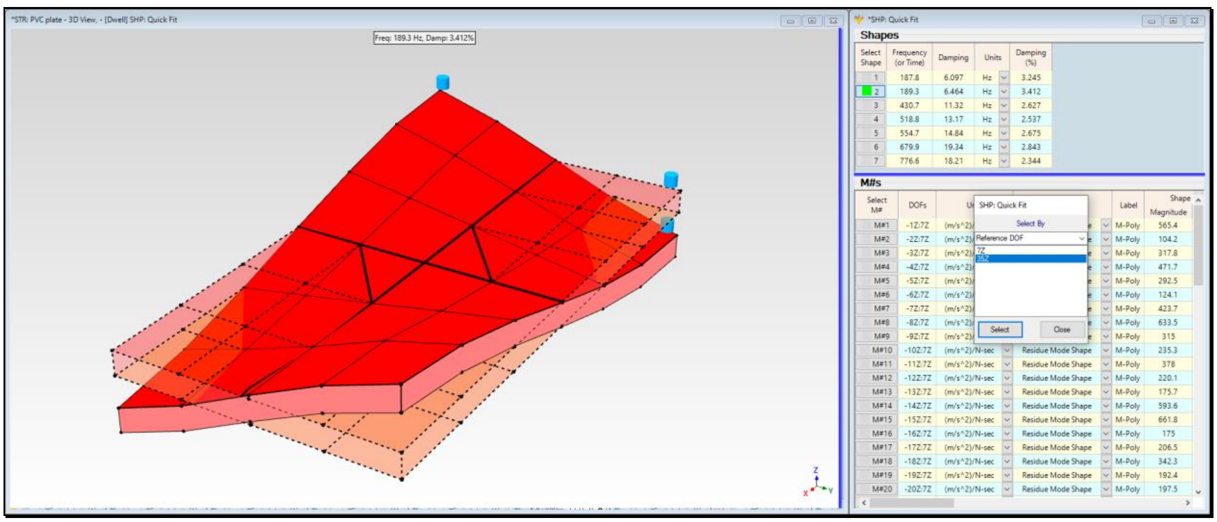
Notice that the ***mode shape does not change*** when displayed from one reference and then from the other.

- ***Press*** the **Select Shape 2** in the **SHP: Quick Fit** window

Of the ***two closely-coupled modes***, the **188 Hz** mode is the expected ***first bending mode***, and the **189 Hz** mode is the expected ***first torsional mode*** of the PVC plate.



First Bending Mode at 188 Hz



First Torsional Mode at 189 Hz

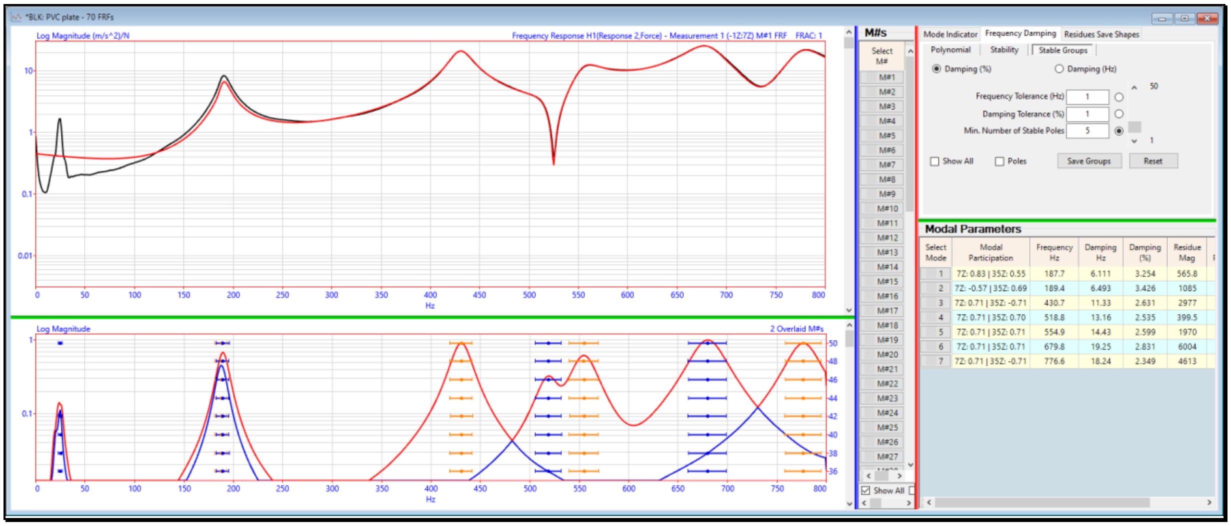
STEP 5 - CURVE FITTING WITH A STABILITY DIAGRAM

As an alternative to counting peaks on **Multi-Reference Mode Indicator** curves, a Stability diagram can be used to determine both the ***number of modes*** and a ***stable estimate of the frequency & damping*** of each mode in a set of FRFs.

- ***Press*** **Hotkey 5 Multi-Ref Stability Diagram**

The Stability diagram consists of Poles overlaid on the **Mode Indicator** graph, as shown below. The Poles are the results of multiple curve fitting solutions where the number of modes used for each curve fitting solution ranges from **1 mode to 50 modes**.

- The numbers ***on the right side*** of the **Stability** diagram are the number of modes used for each curve fitting solution, and the stable Pole estimates from each solution are displayed on the same line



Stability Diagram Showing Stable Pole Groups.

STABLE POLE GROUPS

A **Stable Pole Group** is a group of Poles that fall within the criteria listed on the **Stable Groups** tab.

Frequency Tolerance: Poles in a **Stable Pole Group** must have frequency estimates that lie within the Frequency Tolerance in Hz.

Damping Tolerance: Poles in a **Stable Pole Group** must have damping estimates that lie within the Damping Tolerance in Hz or %.

Min. Number of Stable Poles: A **Stable Pole Group** must have at least the minimum number of Stable Poles in it.

Each **Stable Pole Group** of Poles has the **same color**, with the colors alternating between adjacent Stable Groups.

Each **Stable Pole Group** of Poles is colored using **Contour Colors two & three** in the **File | Data Block Options** box.

STEP 6 - COMPARING MODE SHAPES

To compare the **Quick Fit** mode shapes with the **Stability diagram** mode shapes,

- ***Press* Hotkey 6 Compare Mode Shapes**

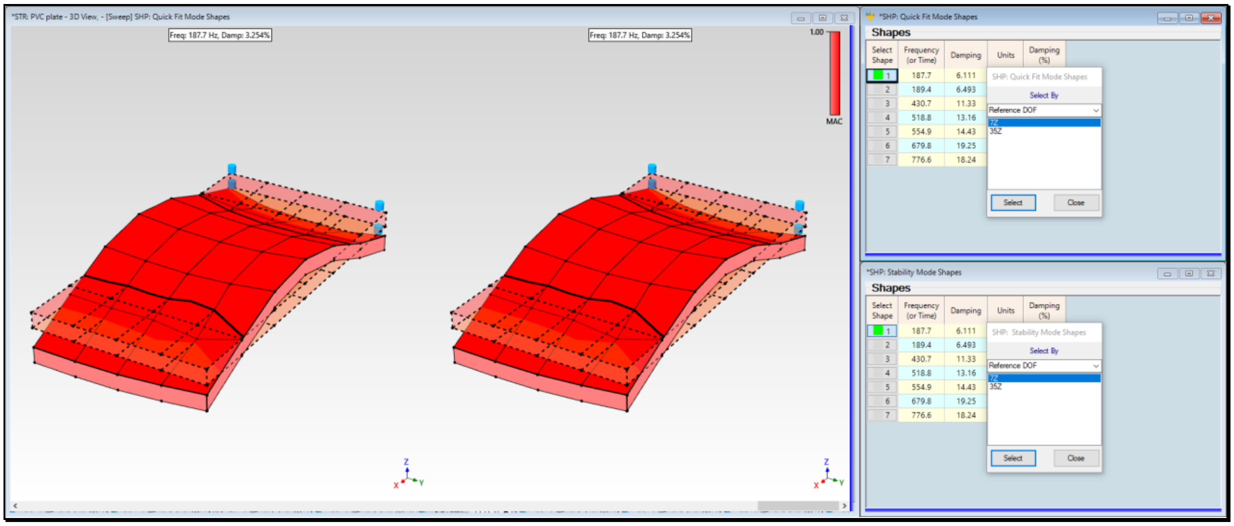
Side-by-side sinusoidal animation of mode shapes from two Shape Tables **SHP: Quick Fit Mode Shapes** & **SHP: Stability Mode Shapes** will begin.

- The **Quick Fit** mode shapes are displayed ***on the left***
- The **Stability** mode shapes are displayed ***on the right***
- **Sweep** animation of the mode shapes in **SHP*:* Quick Fit Mode Shapes** is begun
- The pair of shapes with **Maximum MAC** are displayed together, and their **MAC** value is also displayed

MODAL ASSURANCE CRITERION (MAC)

- **MAC** measures the ***co-linearity*** (or ***linear dependence***) between to shapes
- **MAC** has values ***between* 0 &1**
- **MAC >= 0.9 ➔** two shapes are ***co-linear***, or ***strongly correlated***

During animation, all **Maximum MAC** values for each mode shape pair are ***greater than 0.98***, indicating that the mode shapes obtained from the **Multi-Reference Polynomial method** used by Quick Fit ***are essentially the same*** as those obtained with the the **Multi-Reference AF Polynomial** method using by the Stability Diagram.



Quick Fit versus Stability Mode Shapes.

STEP 7 - REVIEW STEPS

To review the steps of this App Note,

- ***Press*** **Hotkey 7 Review Steps**

SUMMARY

In this App Note, the animated display of ODS's directly from a set of Multi-Reference FRFs revealed the presence of two ***closely-coupled modes***.

Closely-Coupled modes are indicated ***when the* ODS *changes*** when displayed from one Reference DOF versus another.

For lightly damped structures such as this PVC plate, each ODS is typically ***dominated by a single mode*** at or near its resonant frequency. If the ODS at a resonance peak frequency is dominated by a single mode, ***the ODS should look like the same dominant mode shape***, no matter which reference is selected during the ODS display.

In this App Note, when the ODS at the **189 Hz** resonance peak was displayed from reference **7Z** it looked ***distinctly different*** from the ODS that was displayed from reference **35Z**. On the other hand, when the cursor was placed on one of the higher frequency resonance peaks, the ODS ***did not change when a different reference* DOF *was selected***.

A set of **Multi-Reference FRFs** and a **Multi-Reference curve fitting method** are necessary to correctly estimate the modal parameters of ***closely-coupled modes***.

Two different Multi-Reference curve fitting methods were used on the Multi-Reference FRFs and the results of both methods compared very closely with one another.

Multi-Reference curve fitting correctly estimated the modal parameters of two ***closely-coupled modes*** at **188** & **189 Hz**. The **188 Hz** mode was the ***first bending mode***, and the **189 Hz** mode was the ***first torsional mode*** of the PVC plate.

Most structures exhibit a **first bending** and **first torsional** mode shape when tested under ***free-free boundary conditions***.