VIBRANT MEscope Application Note 14

Multi-Reference Curve Fitting to Find Repeated Roots

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing & VES-4600 Advanced Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the AppNote14 project file. These steps might also require MEscope software with a *more recent release date*.

APP NOTE 14 PROJECT FILE

• To retrieve the Project for this App Note, <u>click here</u> to download AppNote14.zip

This Project file contains numbered Hotkeys & Scripts for carrying out the steps of this App Note.

• Hold down the Ctrl key and click on a Hotkey to display its Script window

INTRODUCTION

In this note, the **VES-4600 Advanced Modal Analysis** option is used to curve fit a set of multiple-reference FRFs to estimate the modal parameters of the repeated roots of the circular plate structure shown below.

• **Repeated Root:** A structure has a repeated root if *two or more of its modes* have the *same frequency but different mode shapes*

A finite element analysis (FEA) model of a circular disk shown below was created using the VES-8000 FEA option in MEscope. The natural frequencies & mode shapes were found as an eigen-solution to the differential equations of motion for the disk obtained from the FEA model. An eigen-solution consists of the *natural frequency* (called an *eigenvalue*) and the *mode shape* (called an *eigenvector*) of each mode in the solution.

Damping was not modeled in the **FEA** model so the **FEA** mode shapes had no modal damping.

Mode shapes from an FEA model without damping are called normal modes.



FEA Mode Shapes of a Pair of Repeated Roots

STEP 1 - MODE SHAPES OF REPEATED ROOTS

• Press Hotkey 1 FEA Mode Shapes

The figure above is a display of the mode shapes of a pair of repeated roots of the circular disk. Notice that the two modes have the same frequency, but **their node lines** show that their **mode shapes are different**. The mode shapes are similar-looking **but are rotated 45 degrees** from one another.

MODAL ASSURANCE CRITERION (MAC)

In the figure above, MAC = 0 between these two shapes. This means that they are linearly independent of one another, even though the look similar to one another. Another way of saying it is that the *mode shapes are orthogonal* to one another

- MAC measures the *co-linearity* (or *linear dependence*) between two shapes
- MAC values range between 0 &1
- MAC >= 0.9 → two shapes *are nearly co-linear*, or *strongly correlated*
- MAC < 0.9 → two shapes *are different*

MULTI-REFERENCE FRF SYNTHESIS

The **FEA** model has six pairs of repeated roots, indicated by the pairs of identical frequencies in the Shape Table **SHP: FEA Modes** below. There is also one *non-repeated root* at **773 Hz**. A set of multiple reference FRFs will be synthesized starting with the mode shapes in the Shape Table **SHP: FEA Modes**.

- Modal damping *is necessary* to synthesize FRFs
- FEA mode shapes typically *have no modal damping* because damping is not modeled in the FEA model
- To synthesize FRFs, each FEA mode shape was given 1% critical damping

The synthesized multi-reference FRFs are curve fit using a multi-reference curve fitting method. Finally, the mode shape estimates obtained from curve fitting the multi-reference FRFs are compared with the original **FEA** mode shapes.

SHP: FEA Mode Shapes Shapes														
Select Shape	Frequency (or Time)	Damping	Unit	ts	Damping (%)								^	
1	422.2	4.222	Hz	~	1									
2	422.2	4.222	Hz	~	1									
3	772.8	7.729	Hz	~	1									
4	1090	10.9	Hz	~	1									
5	1090	10.9	Hz	~	1									
6	1559	15.59	Hz	~	1									
7	1559	15.59	Hz	~	1									
8	1947	19.47	Hz	~	1									
9	1947	19.47	Hz	~	1									
10	2746	27.47	Hz	~	1									
11	2746	27.47	Hz	~	1									
12	2960	29.6	Hz	~	1									
12	2960	20.6	Hz	V	1									
15	2500	23.0	114											
M#s	2500	29.0	112											
M#s Select	DOEs	Units	112		Measureme	nt	Shape	1	Shape	2	Shape	3	1	
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FEA Mode Shapes Showing Six Pairs of Repeated Roots

The mode shapes in the Shape Table SHP: FEA Modes shown above have several unique properties,

- The FEA mode shapes only have Roving DOFs
- The FEA mode shapes have units of (in/ bf-sec). Their response units are displacement units
- The FEA mode shapes have Measurement Type = UMM Mode Shape indicating their scaling. Unit Modal Mass (UMM) scaling preserves the structural dynamics
- The *dominant direction of motion* of the **FEA** mode shapes is the **Z-direction**
- The phases of the **FEA** mode shapes are *0 or 180 degrees*

Mode shapes with 0 or 180 degree phases are called normal mode shapes.

In Step 3, a set of multi-reference FRFs will be synthesized from the mode shapes in Shape Table SHP: FEA Mode Shapes shown above. To be used for FRF synthesis, the FEA mode shapes must be converted from UMM mode shapes to Residue mode shapes that have Roving & Reference DOFs.

Three suitable DOFs (5Z, 15Z, 25Z) were chosen as Reference DOFs for synthesizing the FRFs. The UMM mode shapes are converted to Residue mode shapes by executing Tools | Scaling | UMM to Residue Shapes in the SHP: FEA Modes window.



FEA Model Showing Reference DOF

DIFFERENTIATING THE RESIDUE MODE SHAPES

Since the units of the **UMM** mode *s*hapes are (**displacement/force-seconds**), after the Residue mode *s*hapes have been created, they will be *double differentiated* by executing **Tools** | **Differentiate** *twice* in the **SHP: Residues 5Z 15Z 25Z** window. This converts the Residue mode Shapes to (acceleration/force-seconds) units, which are then used to synthesize FRFs with (acceleration/force) units.

STEP 2 - CONVERTING UMM TO RESIDUE MODE SHAPES

• Press Hotkey 2 Residue Mode Shapes

After the **FEA** mode shapes are converted to Residue mode shapes the two Shape Tables, **SHP: FEA Modes** and **SHP: Residues 5Z 15Z 25Z** are display, as shown below.

• Scroll through the **M**#s in **SHP: Residues 5Z 15Z 25Z**

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*SHP: FEA Mode Shapes											🕐 "SHP: Residue Mode Shapes 52 152 252																			
Shapes											Shapes																			
STHP: FE	A Mode Sh	apes .	(Larke	Damping											Select	Frequen	cy o	and an	11-2-	Damping										
Shape	(or Time)	Damping	Units	(%)											Shape	(or Tim	e) U	amping	Units	(%)										
1	422.2	4.222	Hz	× 1														4.222	Hz	× 1										
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3	772.8	7.729	Hz	× 1														7.729	Hz	× 1										
4	1090	10.9	Hz	× 1														10.9	Hz	× 1										
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6	1559	15.59	Hz	✓ 1														15.59	Hz	✓ 1										
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M#3	1Z	in/lbf-sec	~	UMM Mode Shape	\sim	1.33	180	4.065	180	15.25	0	2.235	180	1.919	M	#3 1	Z:5Z	g/lbf-sec	\sim	Residue Mode Sha	spe 🗸	48.4	181.1	462.8	181.1	2312	1.146	591.4	181.1	399
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M#5	2Y	in/lbf-sec	~	UMM Mode Shape	\sim	2.426E-14	0	2.458E-16	180	1.326E-15	180	2.806E-17	180	4.19E-16	M	≠5 2°	/:5Z	g/lbf-sec	\sim	Residue Mode Sha	spe 🗸	8.824E-13	1.146	2.798E-14	181.1	2.009E-13	181.1	7.423E-15	181.1	8.7111
M#6	2Z	in/lbf-sec	~	UMM Mode Shape	~	2.056	180	6.342	180	8.264	0	4.088	180	3.371	M	F6 2	Z:5Z	g/lbf-sec	~	Residue Mode Sha	spe 🗸	74.79	181.1	721.9	181.1	1252	1.146	1082	181.1	700
M#7	3X	in/lbf-sec	~	UMM Mode Shape	\sim	6.437E-15	180	5.49E-16	180	2.064E-16	0	5.662E-18	0	4.434E-1	M	#7 3	<:5Z	g/lbf-sec	~	Residue Mode Sha	ope 🗸	2.342E-13	181.1	6.25E-14	181.1	3.128E-14	1.146	1.498E-15	1.146	9.219
M#8	3Y	in/lbf-sec	~	UMM Mode Shape	~	2.507E-14	0	2.479E-16	180	1.443E-15	180	3.019E-17	180	4.66E-16	M	#8 3 [°]	/:5Z	g/lbf-sec	~	Residue Mode Sha	spe 🗸	9.119E-13	1.146	2.822E-14	181.1	2.187E-13	181.1	7.989E-15	181.1	9.6891
M#9	3Z	in/lbf-sec	~	UMM Mode Shape	\sim	2.893	180	8.987	180	1.555	0	6.638	180	5.341	M	#9 3	Z:5Z	g/lbf-sec	\sim	Residue Mode Sha	ope 🗸	105.2	181.1	1023	181.1	235.6	1.146	1756	181.1	111
M#10	4X	in/lbf-sec	~	UMM Mode Shape	\sim	6.437E-15	180	5.49E-16	180	2.064E-16	0	5.663E-18	0	4.438E-1	M#	10 4	<:5Z	g/lbf-sec	\sim	Residue Mode Sha	spe 🗸	2.342E-13	181.1	6.25E-14	181.1	3.128E-14	1.146	1.498E-15	1.146	9.2261
M#11	4Y	in/lbf-sec	~	UMM Mode Shape	~	2.588E-14	0	2.5E-16	180	1.56E-15	180	3.218E-17	180	5.071E-1	Ma	11 4	/:5Z	g/lbf-sec	~	Residue Mode Sha	ope 🗸	9.413E-13	1.146	2.846E-14	181.1	2.365E-13	181.1	8.515E-15	181.1	1.054
M#12	4Z	in/lbf-sec	~	UMM Mode Shape	~	3.829	180	11.96	180	4.899	180	9.865	180	7.808	M#	12 4	Z:5Z	g/lbf-sec	~	Residue Mode Sha	spe 🗸	139.3	181.1	1361	181.1	742.5	181.1	2610	181.1	162
M#13	5X	in/lbf-sec	~	UMM Mode Shape	~	6.437E-15	180	5.49E-16	180	2.064E-16	0	5.634E-18	0	4.32E-17	M#	13 5	C:5Z	g/lbf-sec	~	Residue Mode Sha	spe 🗸	2.342E-13	181.1	6.25E-14	181.1	3.128E-14	1.146	1.491E-15	1.146	8.9821
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UMM Mode Shapes & Residue Mode Shapes

There are several differences between the properties of the FEA mode shapes in SHP: FEA Modes and the Residue mode shapes in SHP: Residues 5Z 15Z 25Z,

- SHP: Residues 5Z 15Z 25Z has three times as many M#s as SHP: FEA Modes
- The mode shapes in SHP: Residues 5Z 15Z 25Z have Roving & Reference DOFs
- The mode shapes in SHP: Residues 5Z 15Z 25Z have Measurement Type = Residue Mode Shapes
- The Residue Mode Shapes have **units of (g/lbf-sec)**
- The Residue Mode Shapes have phases that are slightly different from 0 or 180 degrees
- The *dominant motion* in the **Residue** mode shapes is *still in the Z-direction*

STEP 3 - SYNTHESIZING MULTI-REFERENCE FRFs

• Press Hotkey 3 Multi-Ref FRFs

When Hotkey 3 is *pressed*, 480 FRFs are synthesized and stored in BLK: FRFs 5Z 15Z 25Z. These FRFs have 160 Roving DOFs each paired with one of the three Reference DOFs, (5Z, 15Z, 25Z.)

• Scroll through the FRFs using the vertical scroll bar *on the right side* of the graphics area

There are only *seven resonance peaks* in the FRFs. Also the phase only transitions *though 180 degrees* from either side of a resonance peak.

If these FRFs were curve fit *using a single-reference curve fitter*, mode shape estimates for seven modes would be found. The mode shapes of the repeated roots would *look like a summation of two mode shapes*.

Multi-reference curve fitting is required to extract the frequency, damping, & mode shape *of the repeated roots* that were used to synthesize the FRFs.



Magnitude & Phase of a Synthesized Driving Point FRF Showing Seven Peaks.

STEP 4 - MULTI-REFERENCE MODE INDICATOR

The *first step of all curve fitting* is to determine *how many modes* are represented in the FRF data.

To calculate and display a multi-reference Indicator,

Press Hotkey 4 Mode Indicator

The curve fitting tabs open in **BLK: FRFs 5Z 15Z 25Z** and the multi-reference Mode Indicator curves are displayed, as shown below.

- Three multi-reference CMIF curves are shown on the lower left, one for each reference of FRF data
- *Thirteen* peaks are counted on the three Indicator curves and the number of peaks counted is displayed in the **Peaks** box on the Mode Indicator tab
- The resonance peaks are indicated with **red dots** on the Mode Indicator curves

MEscope contains two types of multi-reference Mode Indicators, a Multi-Reference CMIF and Multi-Reference MMIF.

- A separate multi-reference Indicator is calculated for each reference of FRF data
- The Mode Indicator curves are used for *resonance peak counting*
- A peak *at or near the same frequency* in two or more multi-reference Indicators indicates *closely-coupled modes or repeated roots*
- Modal Participation curves are also calculated along with each multi-reference Indicator
- The Modal Participation curves are used for weighting the FRF data during curve fitting



Multi-Reference CMIF Showing 13 Resonance Peaks.

STEP 5 - MULTI-REFERENCE QUICK FIT

When a multi-reference Indicator is *chosen* on the **Mode Indicator** tab, the multi-reference Polynomial curve fitting method is *automatically chosen* on the **Frequency Damping** tab and the Polynomial method is *chosen* on the **Residues Save Shapes** tab.

The Quick Fit command completes the curve fitting process *in one step* by *successively executing* the methods chosen on the **Mode Indicator**, **Frequency Damping**, and **Residues Save Shapes** tabs.

In the Step 4, the multi-reference Mode Indicator was already selected, and its resonance peaks counted.

• Press Hotkey 5 Quick Fit

The modal parameters for the *thirteen resonance peaks* counting on the multi-ref CMIF Indicator are estimated and are listed as shown below. A red Fit Function has also been calculated and overlaid on its corresponding FRF.

• Scroll the bar on the right of the FRFs to display each red Fit Function overlaid on its FRF



Multi-Reference Quick Fit Results.

MODAL PARTICIPATION

The **Modal Participation** of each mode in each reference of FRF data is also listed in the **Modal Parameters** spreadsheet shown *on the lower right* above. Modal Participation was used to *weight the FRF data* during curve fitting.

- Modal Participation is a measure of how much each mode participates in each reference of FRF data
- Modal Participation has *magnitudes between* 0 & 1
- Modal Participation *near* 1.0 \rightarrow the resonance is represented with a *large peak* in that reference of FRFs
- Modal Participation *near* $0.0 \Rightarrow$ the resonance is represented with a *small peak* in that reference of FRFs

STEP 6 - COMPARING FEA & QUICK FIT MODE SHAPES

Six pairs of repeated roots and *one non-repeated root* were found from multi-reference curve fitting of the multi-reference FRFs. The frequency & damping of all *13* modes were recovered. But were the **FEA** mode shapes recovered?

- Press Hotkey 6 Quick Fit vs. FEA Mode Shape
- Select a different shape in either Shape Table on the right

The Modal Assurance Criterion (MAC) is used to *select and display* the mode shape from the other Shape Table that has the Maximum MAC with the mod shape that is selected.



Comparison of a Quick Fit with its Closest Matching FEA Mode Shape.

Select a different Reference DOF in SHP: Quick Fit Modes

Notice that the Quick Fit mode shape does not change when a different Reference DOF is selected. This is because *each reference* of mode shapes in **SHP: Quick Fit Mode Shapes** *contains the same mode shape*.

DIFFERENT MODE SHAPES

- Some of the Quick Fit mode shapes closely match with an FEA mode shape (MAC > 0.9), and some Quick Fit mode shapes do not (MAC < 0.9)
- All the repeated root mode shape pairs *"look alike"*, but those pairs with MAC < 0.9 are *rotated about 25 degrees* from the matching mode shape in the other Shape Table

This "*round trip*" showed that multi-reference curve fitting can recover the same mode shapes that were used to synthesize the multi-reference FRFs, but in some cases the Quick Fit mode shapes "*are rotated*" from the original **FEA** shapes used to synthesize the FRFs.

The mismatch of some repeated root mode shapes is due to the *arbitrary nature of repeated root mode shapes* that occur in structures with axi-symmetric geometries.

STEP 7 - REVIEW STEPS

To review all the steps of this App Note,

• Press Hotkey 7 Review Steps