MEscope Application Note 14

Multi-Reference Curve Fitting to Find Repeated Roots

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** & **VES-4600 Advanced Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the AppNote14 project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 14 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote14.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click* on a Hotkey** to display its Script window

INTRODUCTION

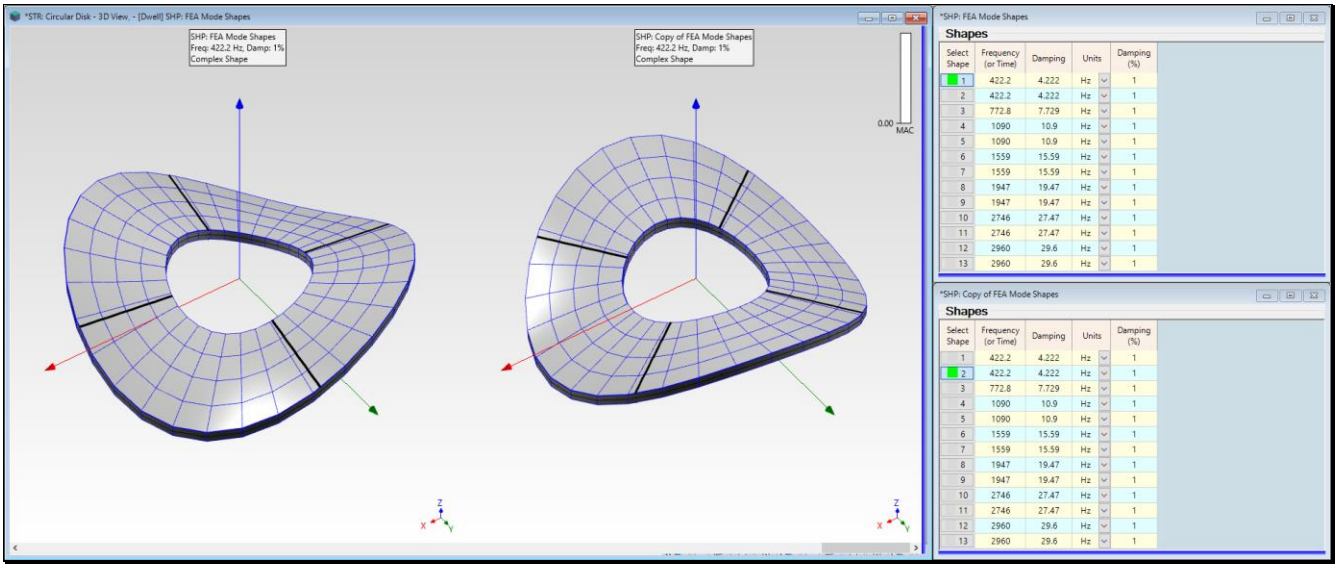
In this note, the **VES-4600 Advanced Modal Analysis** option is used to curve fit a set of multiple-reference FRFs to estimate the modal parameters of the repeated roots of the circular plate structure shown below.

- **Repeated Root:** A structure has a repeated root if ***two or more of its modes*** have the ***same frequency but different mode shapes***

A finite element analysis (**FEA**) model of a circular disk shown below was created using the **VES-8000 FEA** option in MEscope. The natural frequencies & mode shapes were found as an eigen-solution to the differential equations of motion for the disk obtained from the **FEA** model. An eigen-solution consists of the ***natural frequency*** (called an ***eigenvalue***) and the ***mode shape*** (called an ***eigenvector***) of each mode in the solution.

Damping was not modeled in the **FEA** model so the **FEA** mode shapes had no modal damping.

Mode shapes from an **FEA** model without damping are called ***normal modes***.



***FEA** Mode Shapes of a Pair of Repeated Roots*

STEP 1 - MODE SHAPES OF REPEATED ROOTS

- ***Press* Hotkey 1 FEA Mode Shapes**

The figure above is a display of the mode shapes of a pair of repeated roots of the circular disk. Notice that the two modes have the same frequency, but **their node lines** show that their ***mode shapes are different***. The mode shapes are similar-looking ***but are rotated 45 degrees*** from one another.

MODAL ASSURANCE CRITERION (MAC)

In the figure above, **MAC = 0** between these two shapes. This means that they are linearly independent of one another, even though the look similar to one another. Another way of saying it is that the ***mode shapes are orthogonal*** to one another

- **MAC** measures the ***co-linearity* (or *linear dependence*)** between two shapes
- **MAC** values range ***between* 0 &1**
- **MAC >= 0.9 ➔**two shapes ***are nearly co-linear***, or ***strongly correlated***
- **MAC < 0.9 ➔**two shapes ***are different***

MULTI-REFERENCE FRF SYNTHESIS

The **FEA** model has six pairs of repeated roots, indicated by the pairs of identical frequencies in the Shape Table **SHP: FEA Modes** below. There is also one ***non-repeated root*** at **773 Hz**. A set of multiple reference FRFs will be synthesized starting with the mode shapes in the Shape Table **SHP: FEA Modes**.

- Modal damping ***is necessary*** to synthesize FRFs
- **FEA** mode shapes typically ***have no modal damping*** because damping is not modeled in the **FEA** model
- To synthesize FRFs, each **FEA** mode shape was given ***1% critical damping***

The synthesized multi-reference FRFs are curve fit using a multi-reference curve fitting method. Finally, the mode shape estimates obtained from curve fitting the multi-reference FRFs are compared with the original **FEA** mode shapes.

*SHP: FEA Mode Shapes

Shapes

Select Shape	Frequency (or Time)	Damping	Units	Damping (%)
1	422.2	4.222	Hz	1
2	422.2	4.222	Hz	1
3	772.8	7.729	Hz	1
4	1090	10.9	Hz	1
5	1090	10.9	Hz	1
6	1559	15.59	Hz	1
7	1559	15.59	Hz	1
8	1947	19.47	Hz	1
9	1947	19.47	Hz	1
10	2746	27.47	Hz	1
11	2746	27.47	Hz	1
12	2960	29.6	Hz	1
13	2960	29.6	Hz	1

M#s

Select M#	DOFs	Units	Measurement Type	Shape 1		Shape 2		Shape 3	
				Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
M#1	1X	in/lbf-sec	UMM Mode Shape	6.437E-15	180	5.49E-16	180	2.064E-16	0
M#2	1Y	in/lbf-sec	UMM Mode Shape	2.345E-14	0	2.437E-16	180	1.208E-15	180
M#3	1Z	in/lbf-sec	UMM Mode Shape	1.33	180	4.065	180	15.25	0
M#4	2X	in/lbf-sec	UMM Mode Shape	6.437E-15	180	5.49E-16	180	2.064E-16	0
M#5	2Y	in/lbf-sec	UMM Mode Shape	2.426E-14	0	2.458E-16	180	1.326E-15	180
M#6	2Z	in/lbf-sec	UMM Mode Shape	2.056	180	6.342	180	8.264	0
M#7	3X	in/lbf-sec	UMM Mode Shape	6.437E-15	180	5.49E-16	180	2.064E-16	0
M#8	3Y	in/lbf-sec	UMM Mode Shape	2.507E-14	0	2.479E-16	180	1.443E-15	180
M#9	3Z	in/lbf-sec	UMM Mode Shape	2.893	180	8.987	180	1.555	0
M#10	4X	in/lbf-sec	UMM Mode Shape	6.437E-15	180	5.49E-16	180	2.064E-16	0
M#11	4Y	in/lbf-sec	UMM Mode Shape	2.588E-14	0	2.5E-16	180	1.56E-15	180
M#12	4Z	in/lbf-sec	UMM Mode Shape	3.829	180	11.96	180	4.899	180
M#13	5X	in/lbf-sec	UMM Mode Shape	6.437E-15	180	5.49E-16	180	2.064E-16	0

***FEA** Mode Shapes Showing Six Pairs of Repeated Roots*

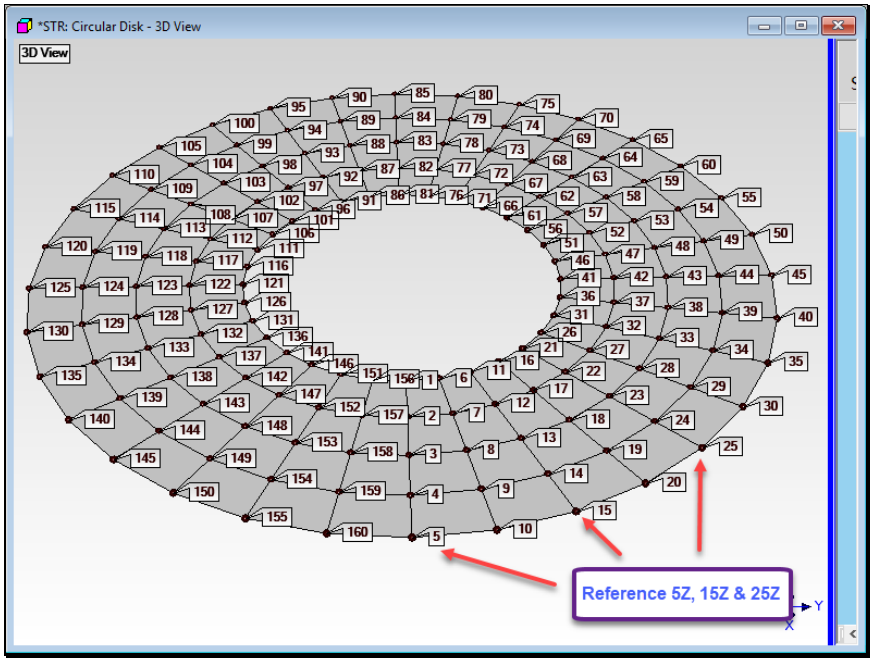
The mode shapes in the Shape Table **SHP: FEA Modes** shown above have several unique properties,

- The **FEA** mode shapes only have **Roving DOFs**
- The **FEA** mode shapes have units of (**in/ bf-sec**). Their response units are **displacement** units
- The **FEA** mode shapes have **Measurement Type = UMM Mode Shape** indicating their scaling. **Unit Modal Mass (UMM)** scaling preserves the structural dynamics
- The ***dominant direction of motion*** of the **FEA** mode shapes is the **Z-direction**
- The phases of the **FEA** mode shapes are ***0 or 180 degrees***

Mode shapes with **0 or 180 degree** phases are called ***normal mode shapes***.

In **Step 3**, a set of multi-reference FRFs will be synthesized from the mode shapes in Shape Table **SHP: FEA Mode Shapes** shown above. To be used for FRF synthesis, the **FEA** mode shapes must be converted from **UMM mode shapes** to **Residue mode shapes** that have Roving & Reference DOFs.

Three suitable DOFs (**5Z, 15Z, 25Z**) ***were chosen as Reference DOFs*** for synthesizing the FRFs. The **UMM** mode shapes are converted to **Residue** mode shapes by executing **Tools | Scaling | UMM to Residue Shapes** in the **SHP: FEA Modes** window.



***FEA** Model Showing Reference DOF*

DIFFERENTIATING THE RESIDUE MODE SHAPES

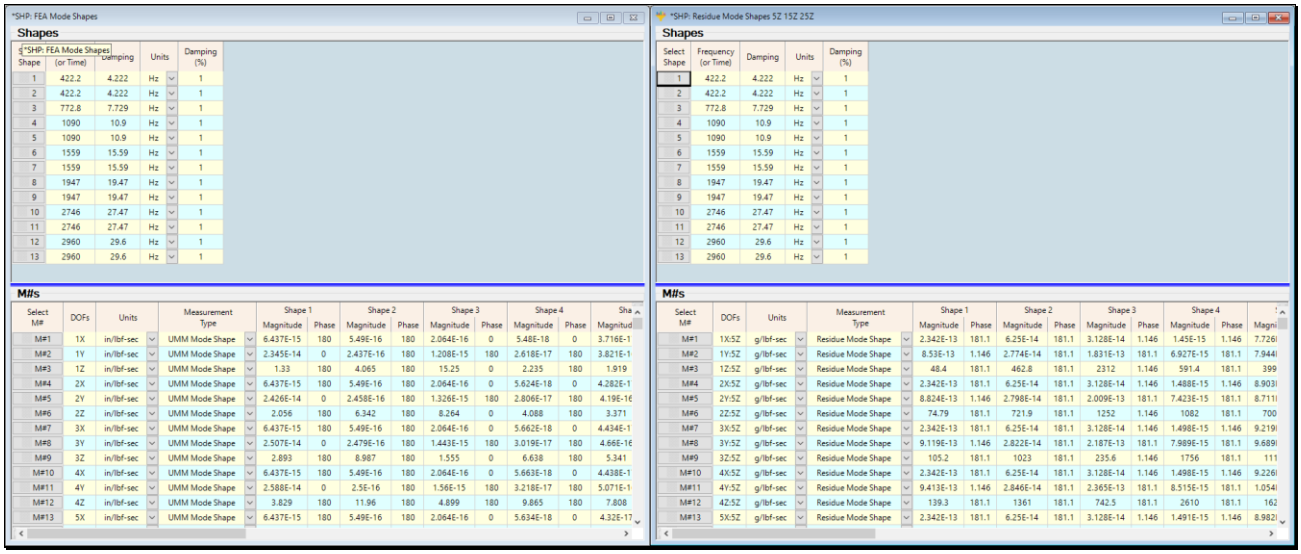
Since the units of the **UMM** mode shapes are (**displacement/force-seconds**), after the Residue mode shapes have been created, they will be ***double differentiated*** by executing **Tools | Differentiate *twice*** in the **SHP: Residues 5Z 15Z 25Z** window. This converts the Residue mode Shapes to (**acceleration/force-seconds**) units, which are then used to synthesize FRFs with (**acceleration/force**) units.

STEP 2 - CONVERTING UMM TO RESIDUE MODE SHAPES

- ***Press* Hotkey 2 Residue Mode Shapes**

After the **FEA** mode shapes are converted to Residue mode shapes the two Shape Tables, **SHP: FEA Modes** and **SHP: Residues 5Z 15Z 25Z** are display, as shown below.

- Scroll through the **M#**s in **SHP: Residues 5Z 15Z 25Z**



UMM Mode Shapes & Residue Mode Shapes

There are several differences between the properties of the **FEA** mode shapes in **SHP: FEA Modes** and the **Residue** mode shapes in **SHP: Residues 5Z 15Z 25Z**,

- **SHP: Residues 5Z 15Z 25Z** has three times as many **M#**s as **SHP: FEA Modes**
- The mode shapes in **SHP: Residues 5Z 15Z 25Z** have ***Roving & Reference* DOFs**
- The mode shapes in **SHP: Residues 5Z 15Z 25Z** have **Measurement Type = Residue Mode Shapes**
- The Residue Mode Shapes have **units of (g/lbf-sec)**
- The Residue Mode Shapes have phases that are ***slightly different from 0 or 180 degrees***
- The ***dominant motion*** in the **Residue** mode shapes is ***still in the Z-direction***

STEP 3 - SYNTHESIZING MULTI-REFERENCE FRFs

- ***Press* Hotkey 3 Multi-Ref FRFs**

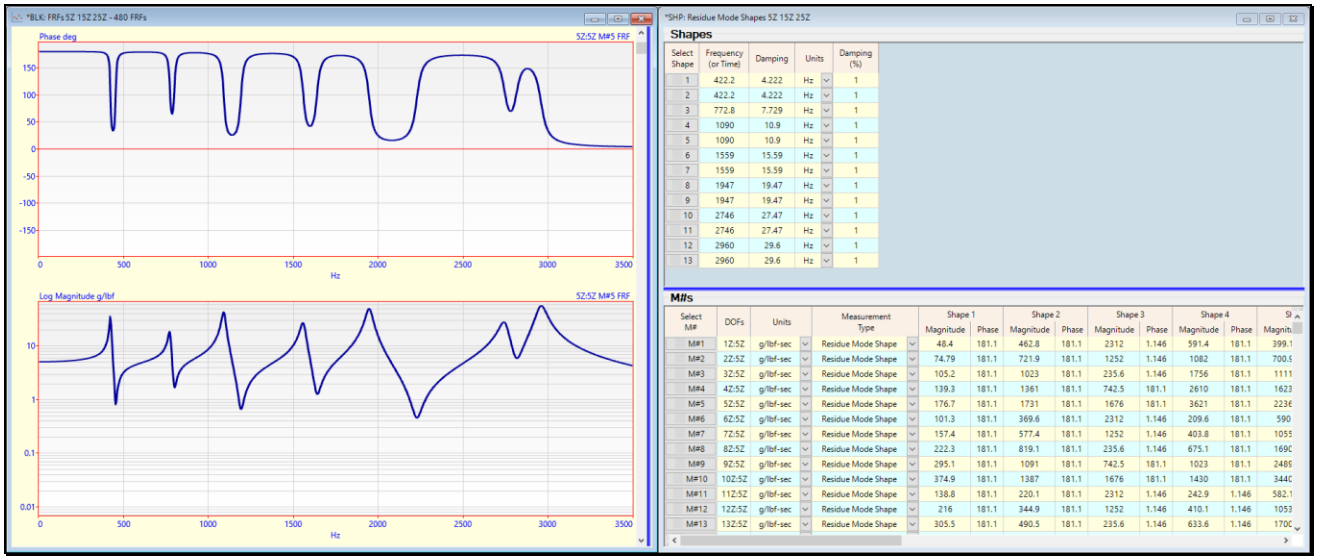
When **Hotkey 3 is *pressed***, **480 FRFs** are synthesized and stored in **BLK: FRFs 5Z 15Z 25Z**. These FRFs have **160 Roving DOFs** each paired with one of the three Reference DOFs, (**5Z, 15Z, 25Z**.)

- Scroll through the FRFs using the vertical scroll bar ***on the right side*** of the graphics area

There are only ***seven resonance peaks*** in the FRFs. Also the phase only transitions ***though 180 degrees*** from either side of a resonance peak.

If these FRFs were curve fit ***using a single-reference curve fitter***, mode shape estimates for seven modes would be found. The mode shapes of the repeated roots would ***look like a summation of two mode shapes***.

Multi-reference curve fitting is required to extract the frequency, damping, & mode shape ***of the repeated roots*** that were used to synthesize the FRFs.



Magnitude & Phase of a Synthesized Driving Point FRF Showing Seven Peaks.

STEP 4 - MULTI-REFERENCE MODE INDICATOR

The ***first step of all curve fitting*** is to determine ***how many modes*** are represented in the FRF data.

To calculate and display a multi-reference Indicator,

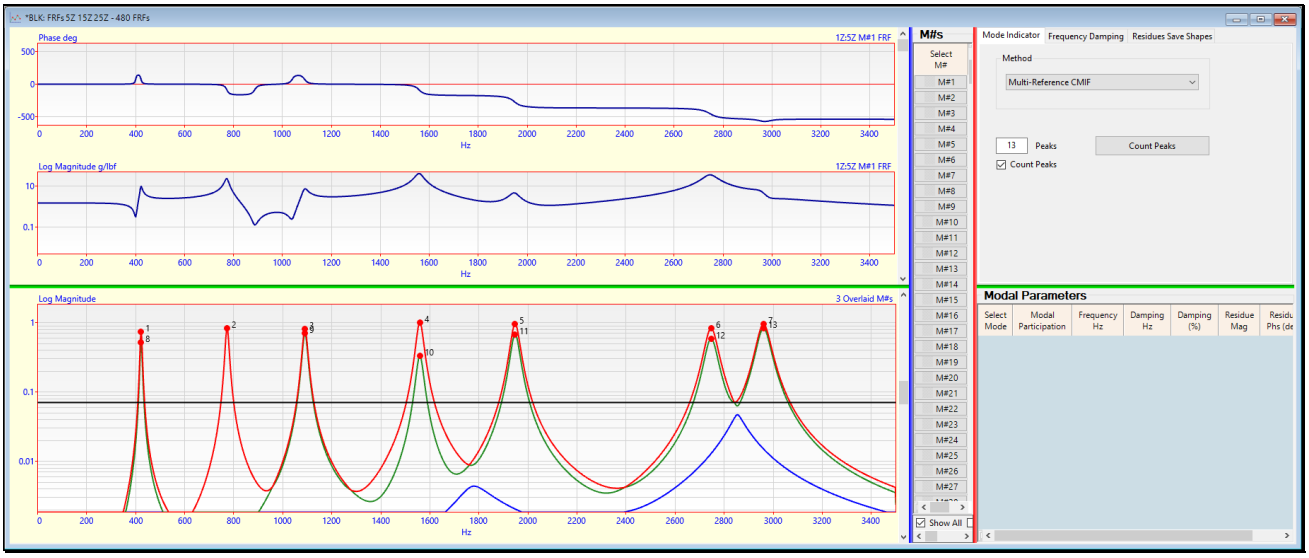
- ***Press* Hotkey 4 Mode Indicator**

The curve fitting tabs open in **BLK: FRFs 5Z 15Z 25Z** and the multi-reference Mode Indicator curves are displayed, as shown below.

- ***Three*** multi-reference CMIF curves are shown ***on the lower left***, one for each reference of FRF data
- ***Thirteen*** peaks are counted on the three Indicator curves and the number of peaks counted is displayed in the **Peaks** box on the Mode Indicator tab
- The resonance peaks are indicated with **red dots** on the Mode Indicator curves

MEscope contains two types of multi-reference Mode Indicators, a **Multi-Reference CMIF** and **Multi-Reference MMIF**.

- A ***separate multi-reference* Indicator** is calculated ***for each reference*** of FRF data
- The Mode Indicator curves are used for ***resonance peak counting***
- A peak ***at or near the same frequency*** in two or more multi-reference Indicators indicates ***closely-coupled modes or repeated roots***
- **Modal Participation** curves are also calculated along with each multi-reference Indicator
- The **Modal Participation** curves are used for ***weighting the* FRF *data*** during curve fitting



Multi-Reference CMIF Showing 13 Resonance Peaks.

STEP 5 - MULTI-REFERENCE QUICK FIT

When a multi-reference Indicator is ***chosen*** on the **Mode Indicator** tab, the multi-reference Polynomial curve fitting method is ***automatically chosen*** on the **Frequency Damping** tab and the Polynomial method is ***chosen*** on the **Residues Save Shapes** tab.

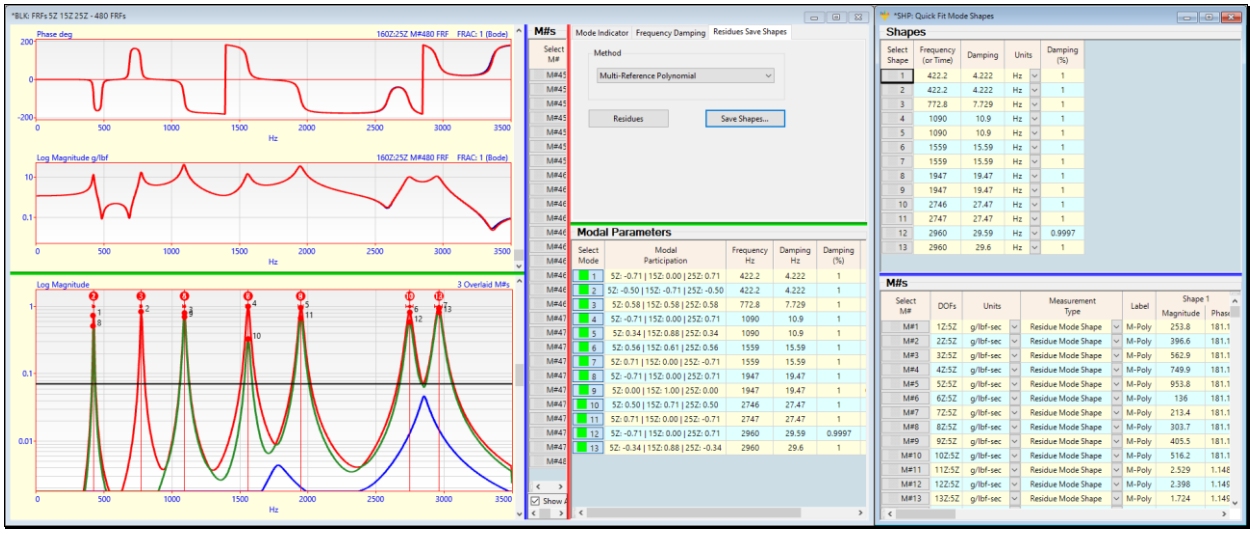
The Quick Fit command completes the curve fitting process ***in one step*** by ***successively executing*** the methods chosen on the **Mode Indicator**, **Frequency Damping**, and **Residues Save Shapes** tabs.

In the **Step 4**, the multi-reference Mode Indicator was already selected, and its resonance peaks counted.

- ***Press* Hotkey 5 Quick Fit**

The modal parameters for the ***thirteen resonance peaks*** counting on the multi-ref CMIF Indicator are estimated and are listed as shown below. A **red Fit Function** has also been calculated and overlaid on its corresponding FRF.

- ***Scroll*** the bar ***on the right*** of the FRFs to display each **red Fit Function** overlaid on its FRF



Multi-Reference Quick Fit Results.

MODAL PARTICIPATION

The **Modal Participation** of each mode in each reference of FRF data is also listed in the **Modal Parameters** spreadsheet shown ***on the lower right*** above. Modal Participation was used to ***weight the FRF data*** during curve fitting.

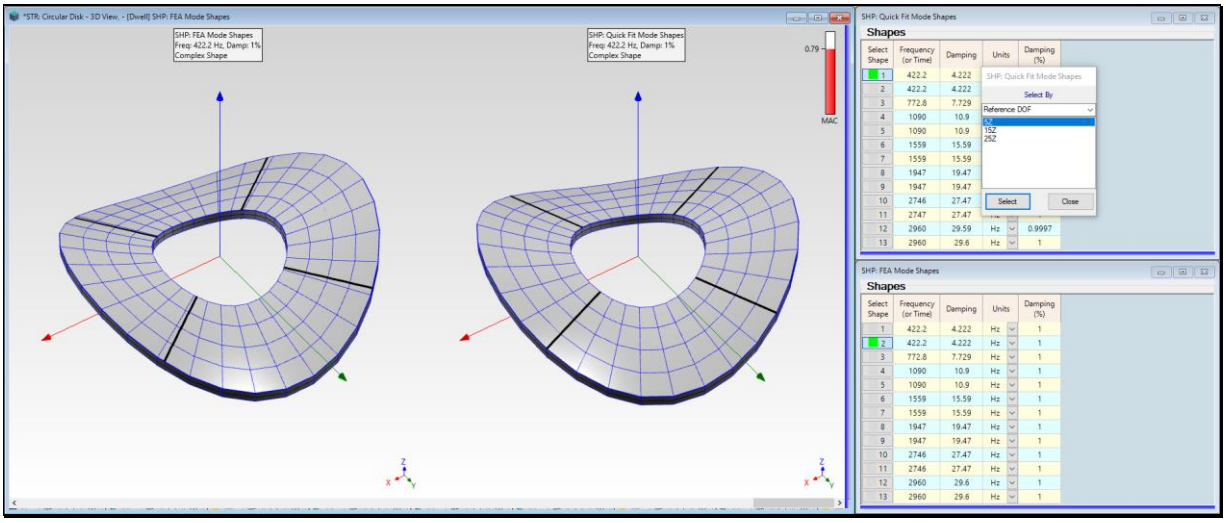
- Modal Participation is a measure of ***how much each mode participates in each reference*** of FRF data
- Modal Participation has ***magnitudes between* 0 & 1**
- Modal Participation ***near* 1.0** ➔ the resonance is represented with a ***large peak*** in that reference of FRFs
- Modal Participation ***near* 0.0** ➔ the resonance is represented with a ***small peak*** in that reference of FRFs

STEP 6 - COMPARING *FEA* & QUICK FIT MODE SHAPES

Six pairs of repeated roots and ***one non-repeated root*** were found from multi-reference curve fitting of the multi-reference FRFs. The frequency & damping of all ***13*** modes were recovered. But were the **FEA** mode shapes recovered?

- ***Press* Hotkey 6 Quick Fit vs. FEA Mode Shape**
- **Select** a different shape in either Shape Table ***on the right***

The Modal Assurance Criterion (**MAC**) is used to ***select and display*** the mode shape from the other Shape Table that has the **Maximum MAC** with the mod shape that is selected.



*Comparison of a Quick Fit with its Closest Matching **FEA** Mode Shape.*

- ***Select*** a different **Reference DOF** in **SHP: Quick Fit Modes**

Notice that the Quick Fit mode shape does not change when a different Reference DOF is selected. This is because ***each reference*** of mode shapes in **SHP: Quick Fit Mode Shapes *contains the same mode shape***.

DIFFERENT MODE SHAPES

- Some of the Quick Fit mode shapes closely match with an FEA mode shape (**MAC > 0.9**), and some Quick Fit mode shapes do not (**MAC < 0.9**)
- All the repeated root mode shape pairs ***"look alike"***, but those pairs with **MAC < 0.9** are ***rotated about 25 degrees*** from the matching mode shape in the other Shape Table

This ***"round trip"*** showed that multi-reference curve fitting can recover the same mode shapes that were used to synthesize the multi-reference FRFs, but in some cases the Quick Fit mode shapes ***"are rotated"*** from the original **FEA** shapes used to synthesize the FRFs.

The mismatch of some repeated root mode shapes is due to the ***arbitrary nature of repeated root mode shapes*** that occur in structures with axi-symmetric geometries.

STEP 7 - REVIEW STEPS

To review all the steps of this App Note,

- ***Press* Hotkey 7 Review Steps**