MEscope Application Note 08

Peak & Power Values in an Auto Spectrum & PSD

The steps in this Application Note can be carried out using any Package that includes the **VES-3600 Advanced Signal Processing** option. They can also be carried out using the **AppNote08** project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 08 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote08.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click* on a Hotkey** to display its Script window

INTRODUCTION

The Auto Spectrum shows how the power of a signal is distributed in the frequency domain. The FFT algorithm is used to calculate the **Digital Fourier Transform** (**DFT**) of a time waveform.

- An Auto Spectrum is calculated by ***multiplying the* DFT *by its own complex conjugate***

A **Power Spectral Density** (**PSD**) is an Auto Spectrum ***normalized*** to a **1 Hz** bandwidth.

- A PSD is calculated by dividing an Auto Spectrum by its frequency resolution (Δf)

POWER

An MEscope Data Block has a command (**Tools | Statistics**) for displaying the power in a **M#**, or the power in a band if the **Band cursor** is displayed. The power in a time domain waveform **f(t)** is calculated as the integral of a time period **T** as,

$$\text{Power} = \frac{1}{T}\int_{t=0}^{T}(f(t))^2\,dt$$
$$= \frac{1}{N}\sum_{i=1}^{N}\left(\text{Real}_i^{\,2}\right)$$

N = the number of samples of M# data

Real_i = the time waveform data for the i^{th} sample

The power in a **Linear** spectrum (also called a **Root Mean Squared** or **RMS** spectrum) is calculated as,

$$\text{Power} = \frac{1}{2}\sum_{i=1}^{N}\left(\text{Real}_i^{\,2} + \text{Imag}_i^{\,2}\right)$$
$$= \frac{1}{2}\sum_{i=1}^{N}\left(\text{Mag}_i^{\,2}\right)$$

Real $_i$ = the Real part of the i^{th} sample

Imag $_i$ = the Imaginary part of the i^{th} sample

Mag $_i$ = the magnitude of the i^{th} sample.

The power in a **Power** spectrum (also called **Mean Squared** or **MS** spectrum), is calculated as,

$$\text{Power} = \frac{1}{2}\sum_{i=1}^{N}\left(\text{Mag}_i\right)$$

In MEscope, DFT values are calculated using either a ***one sided*** **FFT** or a ***two sided*** **FFT**.

- DFT values from a ***two sided*** **FFT** are **1/2 *of the values of a one sided*** **FFT**
- Power spectrum values from a ***two sided*** **FFT *must be multiplied by* 4** to compare them with Power spectrum values from a ***one sided*** **FFT**

Power can be displayed as a Linear (or RMS) power spectrum by taking the square root of a Power spectrum.

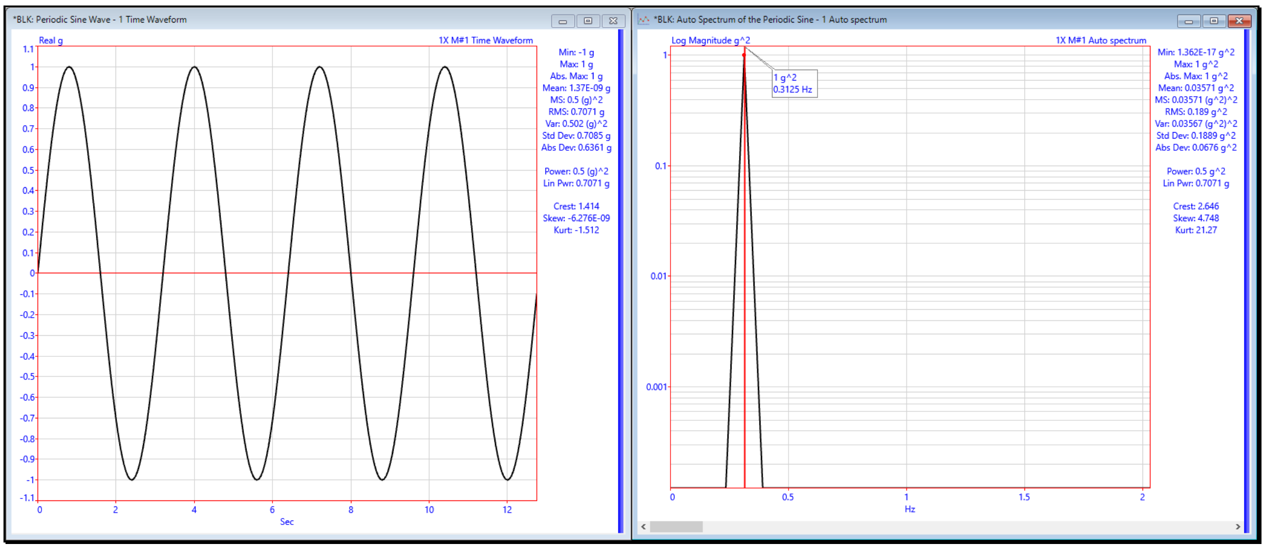
STEP 1 - PERIODIC SINE WAVEFORM

In this Step the power in a sine waveform and in its Auto spectrum are compared. The sine waveform was synthesized using the **File | New | Data Block** command. To eliminate ***leakage effects in its Auto spectrum***, the sine waveform was synthesized so that is ***periodic in its sampling window***. (***See*** **App Note 01** for details on leakage.)

- ***Press*** **Hotkey 1 Auto Spectrum of Periodic Sine**

The window ***on the left side*** of the figure below contains a **0.3125** Hz sine waveform with a peak magnitude of **1.0 g**.

- ***Exactly 4 cycles*** of the sine waveform are displayed in Data Block **BLK: Periodic Sine Wave**



Periodic Sine Waveform and Its Auto Spectrum

POWER IN THE TIME WAVEFORM

The waveform ***statistics*** are displayed ***to the right*** of the **M#**.

- **Power ➔ 0.5 $(g)^2$**
- **Linear Power ➔ 0.707 g**

AUTO SPECTRUM OF A PERIODIC SINE WAVE

The Auto spectrum of the periodic sine waveform is displayed ***on the right side*** of the figure above.

Since an integer number of cycles are contained within its sampling window, this signal is said to be ***periodic in its sampling window***.

- Since the sine waveform is ***periodic in its sampling window***, its Auto Spectrum is a ***leakage-free spectrum***

In this leakage-free **Auto Spectrum**, all the signal is represented at the single peak at **0.3125 Hz**.

The waveform ***statistics*** are displayed **to the right** of the **M#**.

- **Power ➔ 0.5 $(g)^2$**
- **Linear Power ➔ 0.707 g**

PEAK VALUE OF THE AUTO SPECTRUM

Because all of the periodic sine waveform is represented by a single frequency in its leakage-free Auto Spectrum, the magnitude of the peak is,

- Peak magnitude at **0.3125 Hz ➔ 1 g^2**

This is the ***magnitude squared*** of the time waveform peak value.

ONE-SIDED FFT

- **One-sided** in chosen in the **FFT** column of the **M#s** spreadsheet.

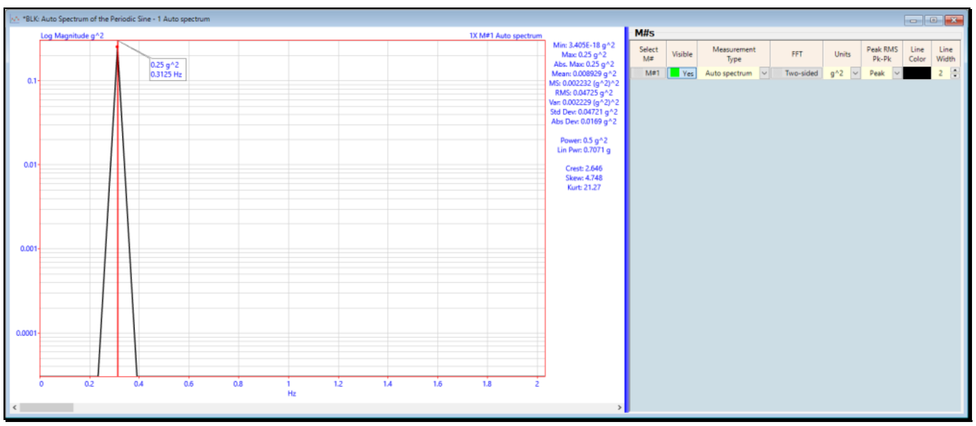
A ***one-sided*** **FFT** distributes ***all the power*** to the positive frequencies in the spectrum.

- Choose **Two-sided** in the **FFT** Column of the the **M#s** spreadsheet in **BLK: Auto Spectrum of the Periodic Sine**
- A ***two-sided*** **FFT** distributes ***half the power*** to the positive frequencies and ***half to the negative frequencies*** in the Auto Spectrum

When **Two-sided** is chosen in the **FFT** Column of the the **M#s** spreadsheet,

- Peak magnitude at **0.3125 Hz ➔ 0.25 (g)^2 ➔ 0.5 g**

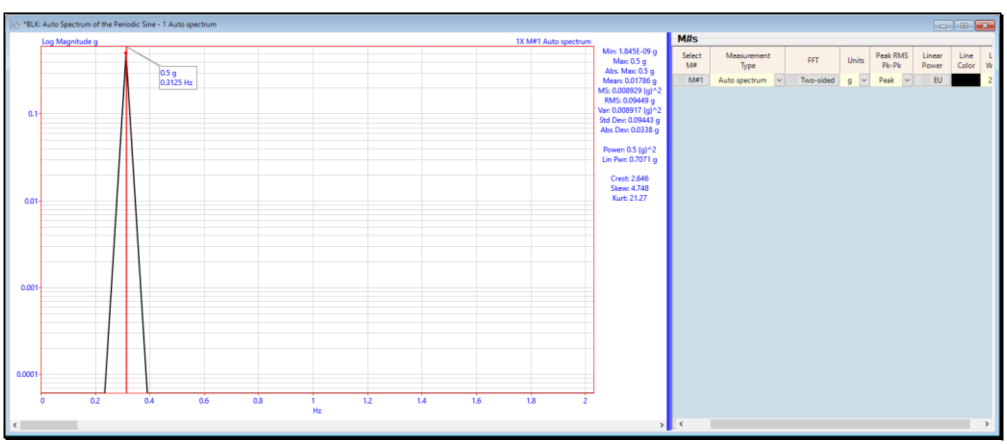
Half of the power is represented by a ***negative frequency peak at*** **-0.3125 Hz** and half of the power is represented by the ***positive frequency peak at*** **0.3125 Hz**.



Peak Magnitude Power of a Two-sided Auto Spectrum.

- Choose **EU** in the **Linear Power** Column of the the **M#s** spreadsheet

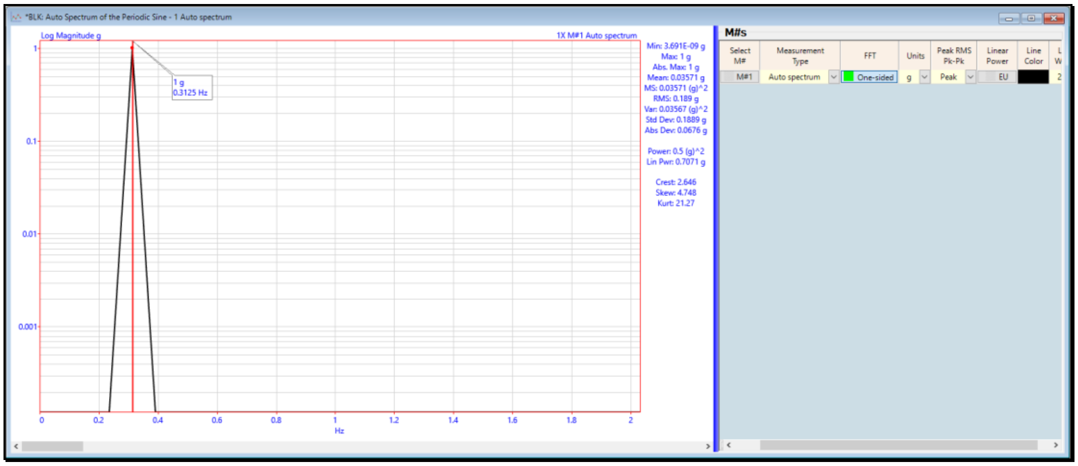
The cursor value at **0.3125 Hz** is **0.5 g**, the ***square root*** of the Power value and half of the time waveform maximum.



Peak Magnitude Linear Power of a Two-sided Auto Spectrum.

- Choose **One-sided** in the **FFT** Column of the the **M#s** spreadsheet

Now the magnitude has returned to **1 g,** the maximum magnitude of the time waveform.



Peak Magnitude Linear Power of a One-sided Auto Spectrum.

POWER IN THE TIME WAVEFORM & AUTO SPECTRUM

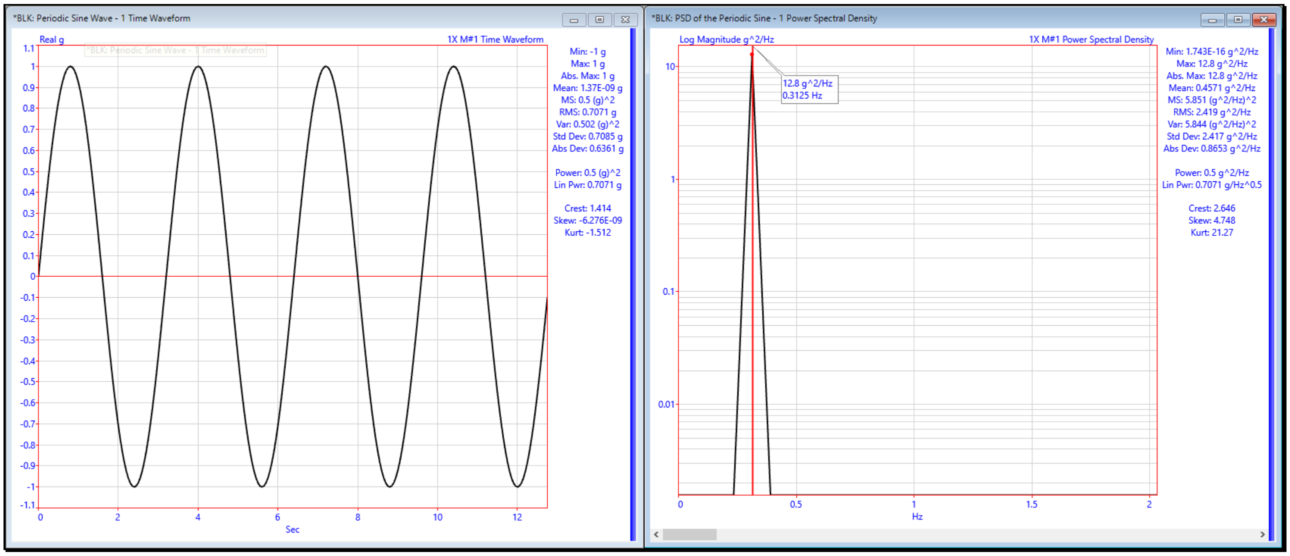
Throughout the changes in the **FFT** & **Linear Power** above, the **Power** & **Lin Power** in the Auto Spectrum remained the same as the **Power** & **Lin Power** of the time waveform. The power in a signal is the same in both the time & frequency domains.

- Power is not changed by using a ***one-sided*** versus a ***two-sided*** FFT
- A **Linear** spectrum (with **EU** units) is simply ***the square root*** of a **Power** spectrum (with **EU^2** units)
- Power units are ***always squared units***
  - **Power → 0.5 (g)^2**
  - **Linear Power → 0.707 g**

STEP 2 - PSD OF A PERIODIC SINE WAVEFORM

To compare the periodic sine waveform with its **Power Spectral Density (PSD)**,

- ***Press* Hotkey 2 PSD of Periodic Sine**



Periodic Sine Waveform and its PSD.

- The magnitude of the **PSD** at **0.3125 Hz** is **12.8 $(g)^2$/Hz**

That value is equal to the peak value of the Auto spectrum divided by the frequency resolution of the spectrum,

Δf = 1/T = 1/(12.8 sec) ➔ 0.078125 Hz

Peak magnitude ➔ 1 $(g)^2$/(0.078125 Hz) ➔ 12.8 $(g)^2$/Hz

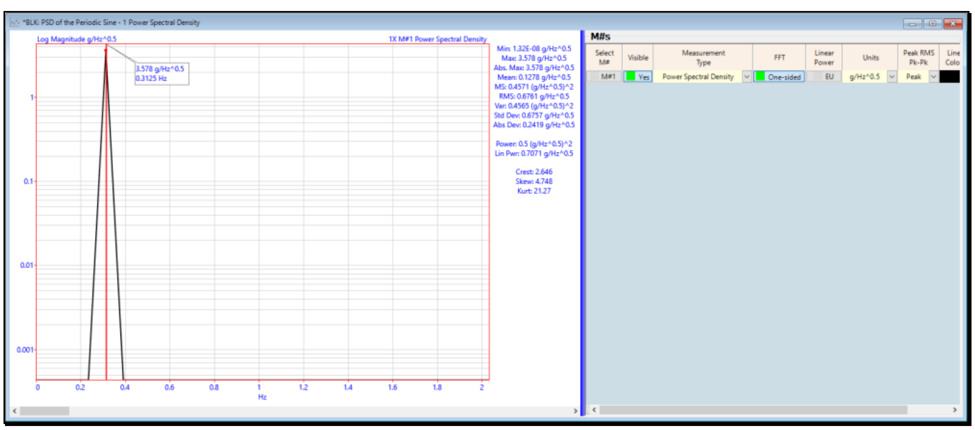
The **Power** quantities in the PSD are ***numerically the same*** as the time waveform power quantities, except they are divided by Hz.

- **Power ➔ 0.5 $(g)^2$/Hz**
- **Linear Power ➔ 0.707 (g)/Hz**.

LINEAR PSD

A **Linear PSD** (with linear units) is the ***square root*** of the **PSD** (with squared or power units). To convert the PSD to linear units,

- In the **Linear Power** column of the **M#s** spreadsheet, select **EU** (***linear engineering units***)



Peak Magnitude Linear Power of a One-sided PSD.

The peak magnitude at **0.3125 Hz** is now **3.578 $(g/(Hz)^{1/2})$,** which is the ***square root*** of the PSD in power units.

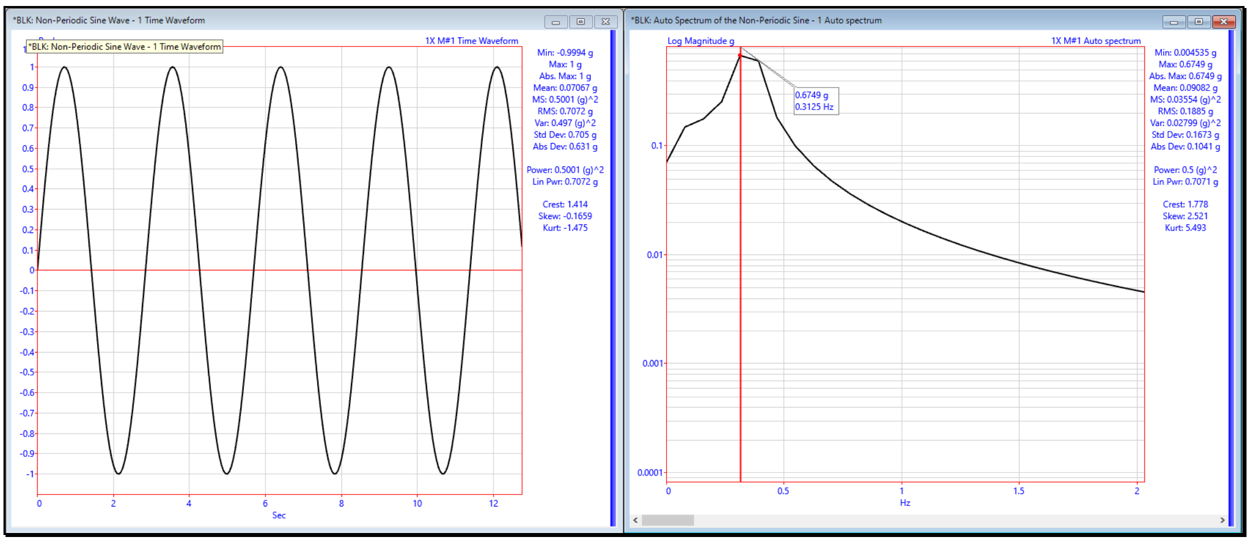
- But the **Power** & **Lin Power** have remained the same as those in the time waveform

STEP 3 - NON-PERIODIC SINE WAVEFORM

- ***Press* Hotkey 3 Non-Periodic Auto Spectrum**

In this Step, the power in a ***non-periodic*** sine waveform and in its Auto spectrum are compared. This sine waveform was synthesized using the **File | New | Data Block** command, but a ***slightly different sinusoidal frequency*** was used.

- The time waveform frequency was changed from **0.3125 Hz** to **0.3515 Hz** to make it ***non-periodic in its sampling window***
- This sine waveform ***on the left*** below completes **4 ½ cycles** in its sampling window and is therefore ***non-periodic*** in the window



Non-Periodic Sine Waveform and its Auto Spectrum.

***Non-periodic* time waveforms** yield different results than ***periodic* time waveforms** when transformed to the frequency domain.

- The FFT assumes that a time waveform is ***periodic in its sampling window***

When the time waveform is not periodic, ***leakage effects will occur*** in its spectrum. Instead of having a single peak at the frequency of the sine waveform, the spectrum of a ***non-periodic*** sine waveform is ***"smeared"*** over several frequencies.

- Leakage causes the power in a spectrum to ***leak out of its resonance peaks into the side bands*** surrounding its resonance peaks
- ***Leakage spreads the power*** from the peak to other frequencies in the spectrum

Leakage ***drastically affects the peak value*** of a sine waveform in its frequency spectrum. Both the magnitude and the frequency at Line cursor in the Data Block ***on the right above are incorrect***.

POWER IN THE TIME WAVEFORM & AUTO SPECTRUM

In **BLK: Auto Spectrum of the Non-Periodic Sine** above, the power ***is still the same as the power in both the periodic and non-periodic the time waveforms***.

- **Power → 0.5 $(g)^2$**
- **Linear Power → 0.707 g**

However, with a **One-sided FFT** the peak amplitude of the Auto spectrum is only **0.675 $(g)^2$** instead of **1.0 $(g)^2$**. The peak in the Auto spectrum is at **0.3125** Hz, and there is ***no sample at the frequency of the sine waveform*** which is **.3515 Hz**.

CONCLUSIONS

The following can be concluded about sine waveforms that are periodic versus non-periodic in their sampling window.

- If a sine waveform is ***periodic in its sampling window*** and a **One-sided FFT** is used, the peak value of its Auto spectrum ***is equal to*** its time waveform peak value
- If a sine waveform is ***periodic in its sampling window*** and a **Two-sided FFT** is used, the peak value of its Auto spectrum ***is one half*** of its time waveform peak value
- The power in the **Linear (RMS)** Auto spectrum is equal to ***the square root*** of the power in its **Power (MS)** Auto spectrum, or its or **PSD**
- If a sine waveform is ***non-periodic in its sampling window***, the peak value of its Auto Spectrum ***is always be less*** than its time waveform peak value ***due to leakage***
- Whether the time waveform is ***periodic*** or ***non-periodic in its sampling window***, the ***Power & Linear Power*** in its Auto spectrum is ***equal to the Power & Linear Power*** in the time waveform

STEP 4 - REVIEW

To review the steps of this App Note,

- ***Press*** **Hotkey 4 Review Steps**