MEscope Application Note 06

Single-Reference and Multiple-Reference Curve Fitting

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** & **VES-4600 Advanced Modal Analysis** options. Without these options, you can still carry out the steps in this App Note using the **AppNote06** project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 06 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote06.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down*** **the Ctrl key** and ***click*** **on a Hotkey** to display its Script window

INTRODUCTION

In MEscope, experimental modal parameters are typically estimated from a set of Frequency Response Functions (FRFs) that are calculated from data acquired from a test article. Each FRF is an estimate of the dynamic characteristics between two degrees of freedom (**DOF**s) of the structure. Each **DOF** contains both a ***point*** and a ***direction*** of measurement on the structure.

- In this note, curve fitting is used on sets of FRFs to demonstrate the drawbacks and benefits of both ***single reference*** and ***multiple reference curve fitting***

MIMO FRF MODEL

All the FRF measurements that can possibly be made between pairs of **DOF**s on a structure can be assembled into an **FRF matrix**. This FRF matrix is part of a **Multiple Input Multiple Output (MIMO) FRF** model. A MIMO FRF model can then be used to calculate structural responses (or **Outputs**) due to any combination of excitation forces (or **Inputs**). The MIMO model is represented mathematically as,

$$\{X(j\omega)\} = [H(j\omega)]\{F(j\omega)\}$$

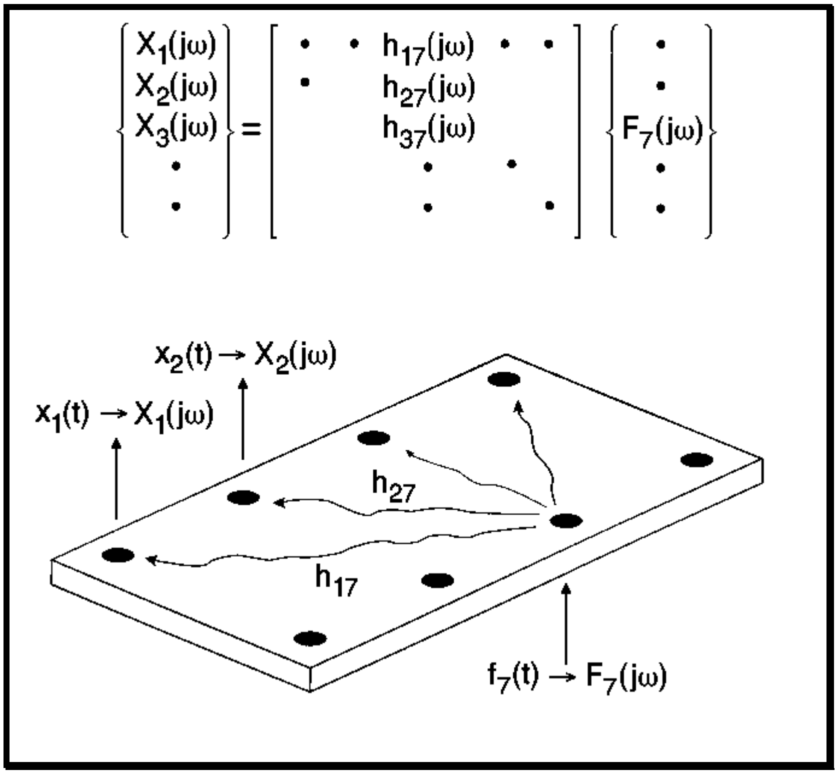
$[H(j\omega)]$ ➔ **FRF matrix** (**N** by **N**)

$\{X(j\omega)\}$ ➔ Fourier transforms of responses, or **Outputs** (N-vector)

$\{F(j\omega)\}$ ➔ Fourier transforms of excitation forces, or **Inputs** (N-vector)

N ➔ number of **DOF**s

Each column of the **FRF matrix** shown below corresponds to an **Input DOF**, and ***each row*** corresponds to an **Output DOF.**



Single Reference Roving Response Test.

ROVING RESPONSE TEST

The figure above depicts a set of FRFs calculated from measurement data acquired from a structure. The **Input *is fixed at point 7*** in the vertical direction, and the **Outputs *are at points 1, 2, 3, etc*.**, each in the vertical direction. Each FRF is calculated between the ***fixed* Input DOF** at point 7 and a ***different* Output DOF.**

- Because the location of the excitation force is fixed, this is a ***single reference* Roving Response Test**

For this test, all the FRF measurements are assembled into ***one column of the* FRF matrix**, corresponding to the **Input** at point 7 in the vertical direction. Each FRF is also placed in the ***row*** corresponding to its **Output** point & direction.

- The FRFs resulting from a **Roving Response** test are assembled into ***one column of the* FRF matrix**

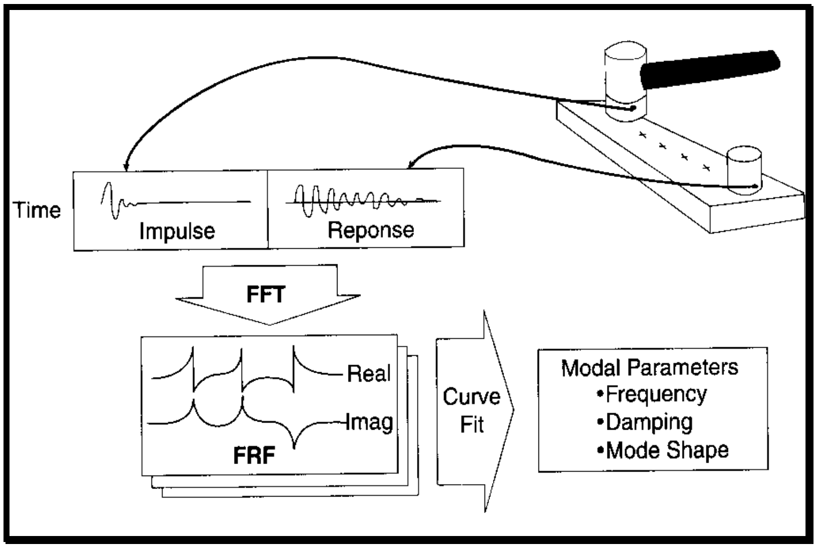
ROVING IMPACT TEST

In a Roving Impact Test is shown in the figure below. In this test, an accelerometer is used to measure a ***single (fixed)* Output**, and an instrumented hammer is used to provide excitation force at ***multiple* Input DOFs**.

- The FRFs resulting from a **Roving Impact** test are assembled into ***one row of the* FRF matrix**

In this case, the ***row*** corresponds to the ***single (fixed) response*** or **Output**, and each FRF corresponds to ***force* Input** at a ***different point*** on the structure. A set of FRFs, each one with the ***same Output* DOF** and a ***different Input* DOF** occupy a ***single row*** of the FRF matrix.

- In most cases, ***a single row*** or ***single column*** of FRFs ***is sufficient*** to estimate all the modal parameters of the structure, in the frequency band of the FRFs



Single Reference Impact Test

MULTIPLE REFERENCE TESTS

During a Multiple Reference modal test, ***two or more (fixed)*** **Input DOF**s or ***two or more (fixed)*** **Output DOF**s are used.

- The FRFs from a **Multiple Reference Test** occupy ***two or more*** columns or ***two or more*** rows of the FRF matrix

If curve fitting ***one row*** or ***one column*** of FRFs in the FRF matrix ***is usually sufficient*** for identifying the modes of a structure, then why measure and curve fit multiple rows or columns? There are several good reasons.

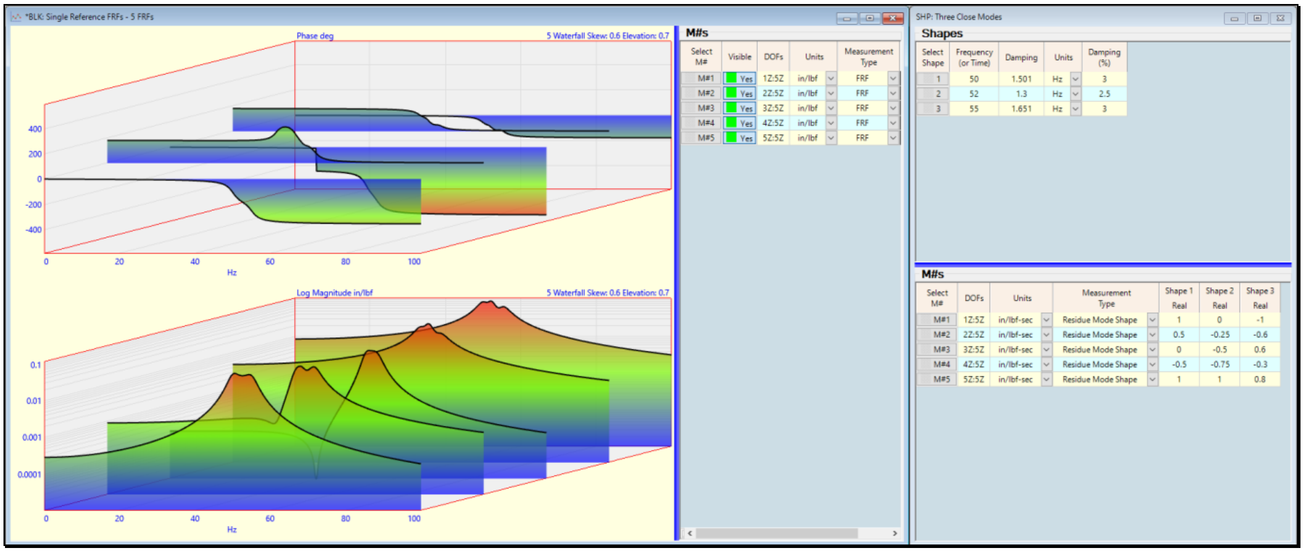
- If the reference **DOF** corresponds to a ***nodal point*** (a **DOF** with zero value) of a mode shape, the resonance peak of that mode ***will not appear in any*** **FRF**. Hence, the resonance might be missed
- Certain references (rows or columns) of FRFs with provide ***larger resonance peaks*** for some modes. Larger resonance peaks provide better data for curve fitting, resulting in better modal parameter estimates
- If the structure undergoes ***non-linear dynamic behavior while being excited***, multiple shakers may be required to create sufficiently large signal levels for effectively calculating ***linear*** **FRFs**
- If a structure contains ***closely-coupled modes*** or ***repeated roots***, Multi-Reference curve fitting of a set of Multi-Reference FRFs is required to identify the mode shapes. ***Closely-coupled modes*** and ***repeated roots*** are addressed in **App Note 15 & App Note 14**)

Multiple reference FRF calculation requires that ***all the fixed reference signals*** (multiple **Inputs** or multiple **Outputs**) be ***simultaneously acquired*** together with ***at least one roving signal***. This requires multiple sensors and a multi-channel acquisition system that can ***simultaneously acquire*** all the reference channels and at least one roving channel of data. Furthermore, if multiple fixed **Inputs** are used, the **Input** ***signals must be uncorrelated*** with one another. In other words, the same excitation signal cannot be used for multiple **Inputs**.

STEP 1 – SINGLE-REFERENCE FRFs

- ***Press*** **Hotkey 1 Single-Reference FRFs**

In this step, a set of mode shapes is used to synthesize FRFs. In later steps, the FRFs are curve fit using single-reference curve fitting, and the mode shape estimates are compared with the original mode shapes. The original mode shapes are listed ***on the right*** in the figure below.



FRFs Synthesized from Residue Mode Shapes.

- Modal damping is expressed both in ***Hz units*** and as a ***percent*** (%) ***of critical damping***
- Each mode shape component (or **M#**) is a **Measurement Type** of **Residue Mode Shape** with **Units** of **in/lbf-sec**
- Each Residue mode shape component has the following **DOFs**,

Residue mode shape **DOFs** ➔ **Roving DOF : Reference DOF**

- Each mode shape component ***has the same*** **Reference DOF** ➔ **5Z**
- The **50 Hz** & **52 Hz** mode shapes each have a ***nodal point*** (point were their mode shape is **"0")**
- **Shape 1** has a nodal point at Roving **DOF 3Z**
- **Shape 2** has a nodal point at Roving **DOF 1Z**

FRF SYNTHESIS

The log magnitude & phase of the synthesized FRFs are displayed in waterfall plot ***on the left side***, shown in the figure above. The five FRFs were synthesized from the Residue mode shapes using the **Tools | Synthesize FRFs** command in the Shape Table **SHP: Three Close Modes**. When **Hotkey 1 was *pressed***, this command was executed by its script.

SINGLE REFERENCE CURVE FITTING STEPS

In MEscope, curve fitting is done in three steps.

1. Count the number of modes by counting resonance peaks on a Mode Indicator curve
2. Estimate **modal frequency & damping** for the number of modes counted
3. Estimate Residue mode shapes for the modes with **modal frequency & damping** estimates

STEP 2 – COUNT PEAKS ON THE MODE INDICATOR

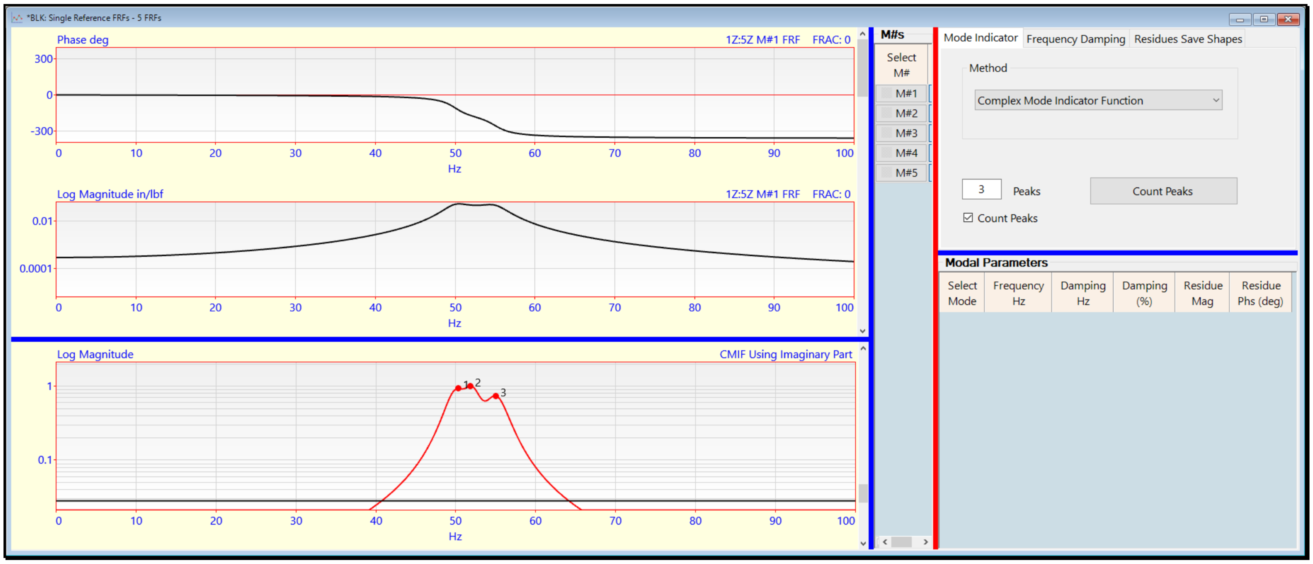
- ***Press*** **Hotkey 2 Count Resonance Peaks**

The first step in curve fitting is to determine how many modes are present in a frequency band of the FRFs by counting resonance peaks. The **Mode Indicator** assists you in counting resonance peaks.

The Indicator curve ***on the lower left*** below shows three resonance peaks with **red dots** on them, indicating the three peaks that were counted.

- Peak counting is performed when the **Count Peaks button is *pressed***, or when the command is ***executed in a script***
- The number of peaks counted is listed in the **Peaks** box on the **Mode Indicator** tab

This is the number of modes (or the model size), that will be used in the next curve fitting step.



Mode Indicator Showing Three Peaks.

- In a real-world situation, the number of modes in a frequency band ***may not always be apparent***
- When ***counting resonance peaks cannot be done***, use of a Stability diagram (part of the **VES-4600 Advanced Modal Analysis** option) is another way to determine the number of modes

RULES FOR CALCULATING MODE INDICATORS

The title on the Mode Indicator graph indicates which Mode Indicator is displayed and which part of the FRFs was used to calculate it. The title on the Mode Indicator above says **CMIF Using Imaginary Part**.

- If the FRFs have ***acceleration*** or ***displacement*** responses (numerators), the **Imaginary Part** of the FRFs should be used to calculate the Mode Indicator
- If the FRFs have ***velocity*** responses (numerators), the **Real Part** of the FRFs should be used
- In all other cases, the **Magnitude** of the FRFs should be used

STEP 3 – ESTIMATE MODAL FREQUENCY & DAMPING

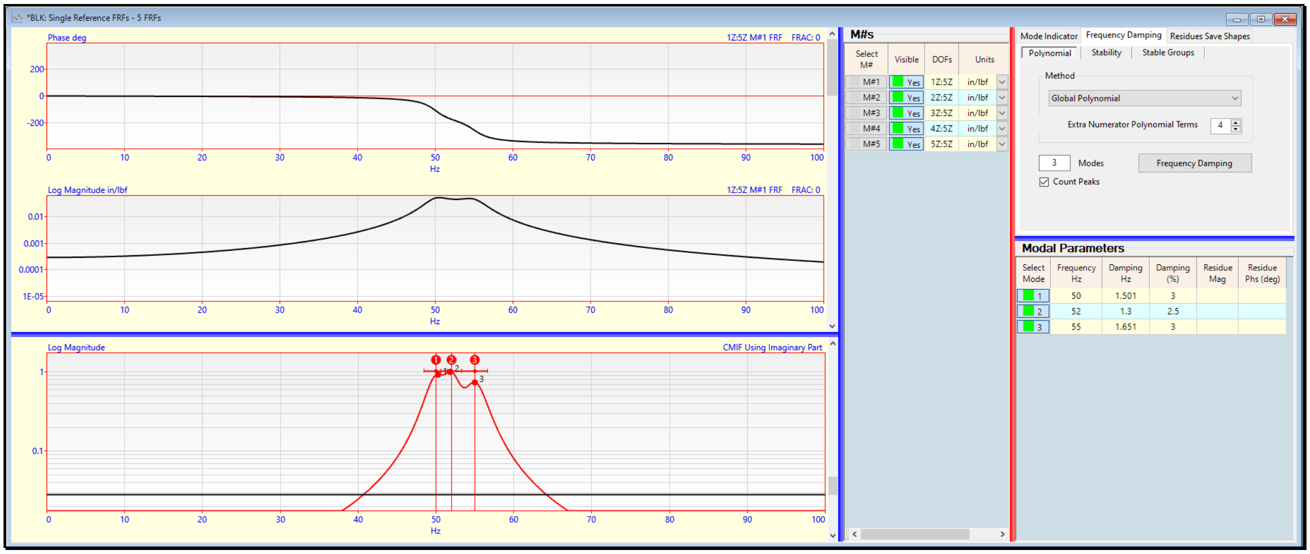
- ***Press* Hotkey 3 Modal Frequency & Damping**

In this curve fitting step, the modal frequency & damping are estimated for three modes.

- Before executing this step, the **Modes** box on the **Frequency Damping** tab should contain a number that is ***at least equal to the number of resonance peaks*** on the Mode Indicator graph

In this case, we know that there are three modes in the FRFs since they were synthesized using the modal parameters for three modes.

- To enter a number of modes, ***uncheck the*** **Count Peaks** box and enter the number of modes



Frequency & Damping Estimates.

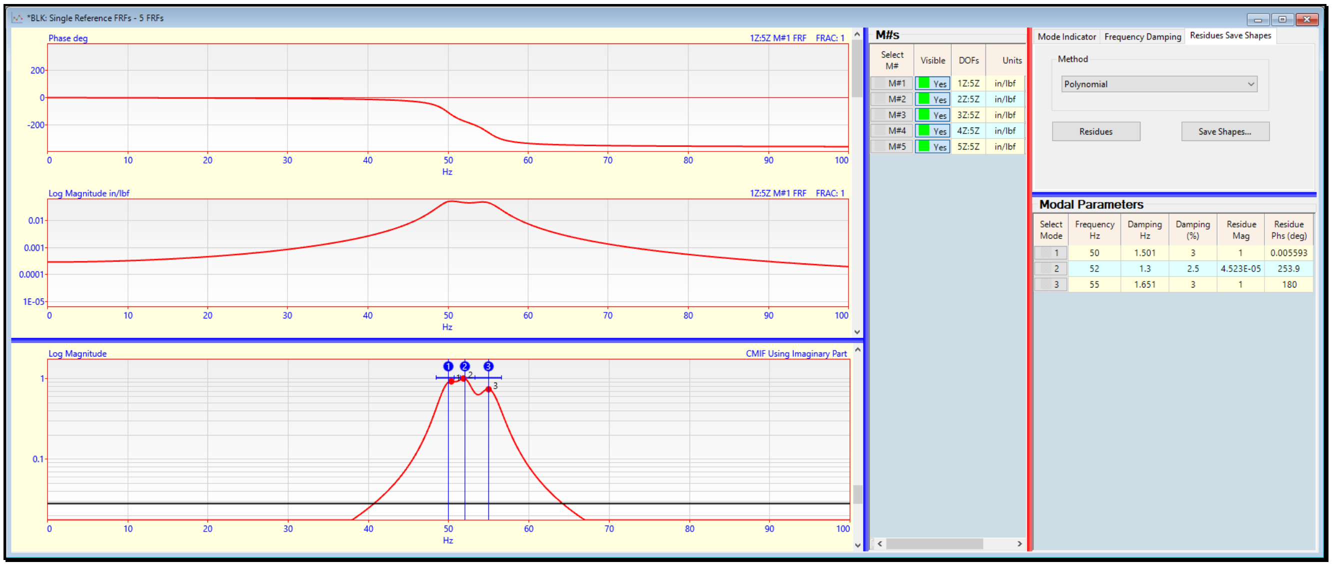
The frequency & damping estimates obtained by curve fitting the FRFs are listed in the **Modal Parameters** spreadsheet ***on the lower right side*** of the window. By inspection of the frequency & damping estimates, it is apparent that they are ***essentially the same as*** the ***original modal frequency & damping*** that were used to synthesize the FRFs.

STEP 4 – ESTIMATE RESIDUE MODE SHAPES

- ***Press* Hotkey 4 Residue Mode Shapes**

After the frequency & damping have been estimated and listed in the **Modal Parameters** spreadsheet, Residue mode shapes are estimated by a ***third curve fitting step***.

When **Hotkey 4 is *pressed***, Residue mode shape estimates are listed in the **Modal Parameters** spreadsheet for ***each mode*** and ***each* FRF**.



Curve Fitting after Residue Mode Shapes are Estimated.

- After modal **frequency, damping, & Residue mode shape** have been estimated for each mode, a **red fit function** is synthesized and overlaid on each FRF
- ***Scroll*** though the FRFs using the scroll bar next to the FRFs

As you scroll through the display of each FRF and its ***overlaid* red fit function**, the Residue mode shapes for each mode in the ***current*** FRF are displayed in the **Modal Parameters** spreadsheet.

- Notice that all the Residue mode shape estimates ***closely match the original*** Residue mode shapes that were used to synthesize the FRFs

SAVED MODE SHAPES

After Residue curve fitting is completed and a Residue has been estimated for ***each mode* and *each* FRF**, the residues are saved into a Shape Table **SHP: Single-Reference Mode Shapes**. In the next step, these mode shape estimates are compared with the original mode shapes in **SHP: Three Close Modes**.

STEP 5 – COMPARE SINGLE-REFERENCE MODE SHAPES

To compare the mode shapes from the previous curve fitting steps with the original shapes in **SHP: Three Close Modes**,

- ***Press* Hotkey 5 Compare Mode Shapes**

A pair of mode shapes can be compared in two ways

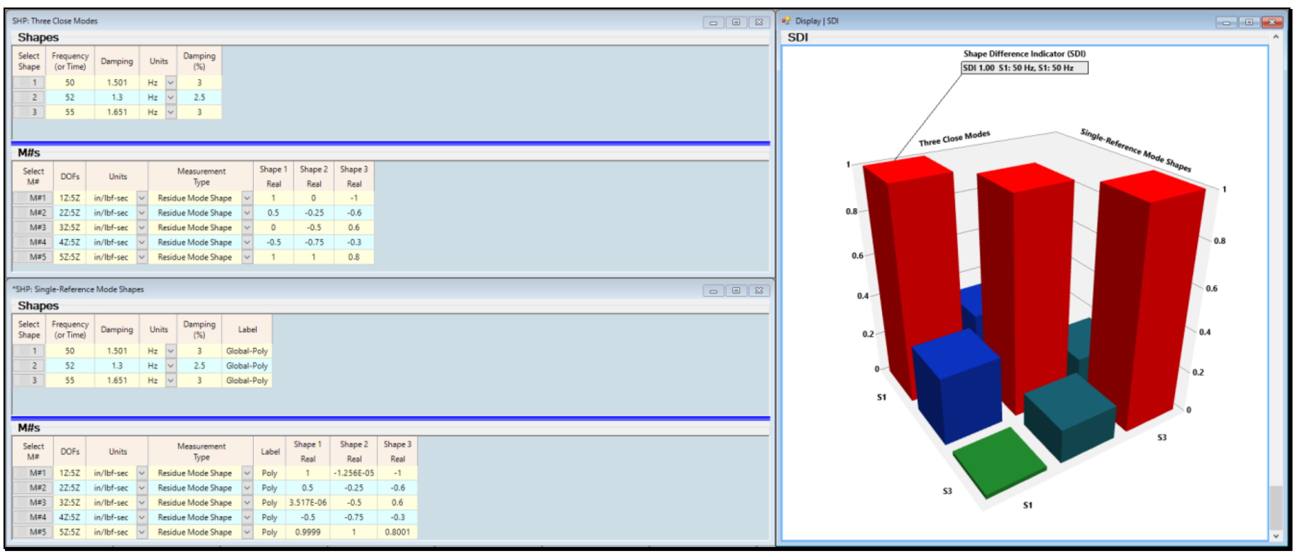
- ***Graphically*** by displaying them side-by-side in animation on two 3D models
- ***Numerically*** by using the Modal Assurance Criterion (**MAC)** or the Shape Difference Indicator **(SDI)** to compare each pair of mode shapes

SHAPE DIFFERENCE INDICATOR (SDI)

SDI is a measure of the ***equality*** of two shape vectors. If two shapes ***have equal components***, their ***SDI value equals 1.0***. If two shapes ***do not have equal components***, their ***SDI value is less than 1.0***. In MEscope, the following ***rules of thumb*** are used with SDI

- **SDI** values ➔ ***between 0 &1***
- **SDI = 1.**0 ➔ two shapes have ***equal components***
- **SDI >= 0.9** ➔ two shapes are ***similar***
- **SDI < 0.9** ➔ two shapes are different (some components are not equal)

When **Hotkey 5 is *pressed***, a Bar Chart of SDI values is displayed next to the two Shape Tables, as shown below.



SDI Bar Chart Between SHP: Single-Reference Mode Shapes & SHP: Three Close Modes

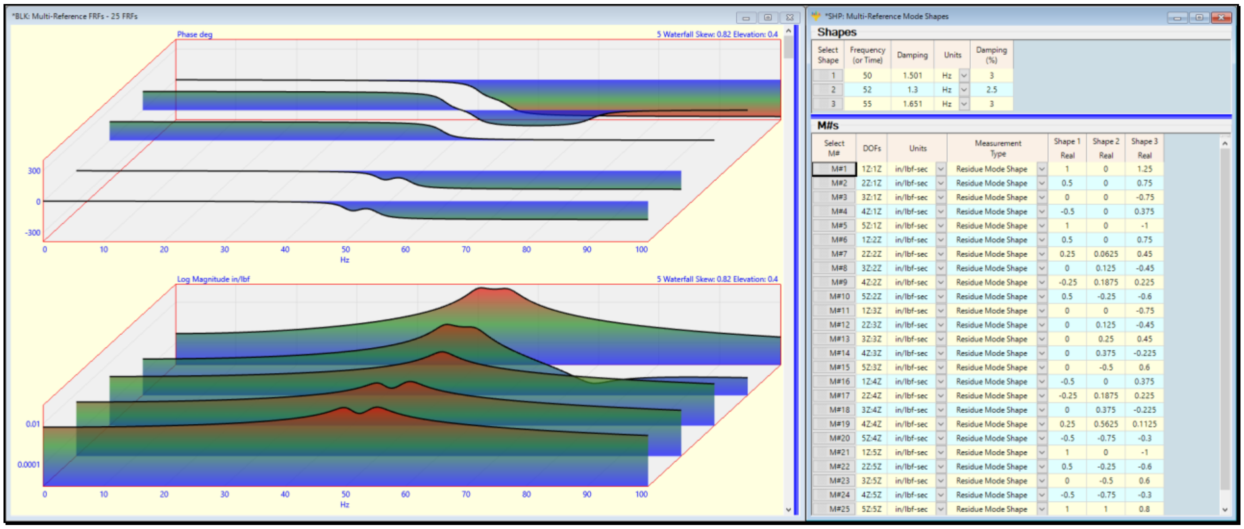
- ***Hover*** the mouse pointer over each SDI bar to display its value
- The **large red bars** each have **SDI = 1.0**, indicating that a mode shape in SHP: Single-Reference Mode Shapes is exactly equal to a mode shape in SHP: Three Close Modes

STEP 6 – MULTI-REFERENCE FRFs

To synthesize Multi-Reference FRFs using the mode shapes in **SHP: Residue Close** Modes**,**

- **Press Hotkey 6 Multi-Reference FRFs**

When **Hotkey 6 is *pressed*** a set of Multi-Reference FRFs is synthesized starting with the Residue mode shapes in **SHP: Three Close Modes**. The mode shapes in **SHP: Three Close Modes** are only defined for Reference **DOF 5Z**. Therefore, the Residue mode shapes must be converted into **UMM** mode shapes from which a new set of Residue mode shapes can be created that include all ***five references*** **DOFs 1Z, 2Z, 3Z, 4Z, 5Z**.



Multi-Reference FRFs & SHP: UMM Mode shapes

The Multi-Reference FRFs and the **Multi-Reference Residue** mode shapes used to synthesize the FRFs are shown above. There are 25 FRFs, each group of five FRFs having a **Reference DOF 1Z, 2Z, 3Z, 4Z, 5Z**.

- ***Scroll*** though the FRFs

MODAL MODELS

Residue mode shapes can be converted back and forth between **UMM** mode shapes when both are Modal Models.

- A **Modal Model** of Residue mode shapes is obtained by curve fitting a set of ***calibrated*** **FRFs**
- A **Modal Model** contains ***all the dynamic properties*** of the real-world structure from which the ***calibrated*** **FRFs** were calculated
- A **Modal Model** of **Residue** mode shapes can be converted to a **Modal Model** of **UMM** mode shapes
- A **Modal Model** of **UMM** mode shapes can be converted to a **Modal Model** of **Residue** mode shapes
- ***Only*** a **Modal Model** of **Residue** mode shapes can be used to synthesize FRFs

When **Hotkey 6 is *pressed***, the following steps are carried out to synthesize the Multi-Reference FRFs starting with the Residue mode shapes in **SHP: Three Close** Modes.

1. The **Single-Reference Residue** mode shapes in **SHP: Three Close Modes** are converted to UMM mode shapes using the **Tools | Scaling | Residues to UMM Shapes** command
2. The **UMM** mode shapes are converted to ***Multi-Reference*** **Residue** mode shapes using the **Tools | Scaling | UMM to Residue Shapes** command
3. The **Multi-Reference Residue** mode shapes are used to synthesize Multi-Reference FRFs

Each **UMM** mode shape component ***only has a*** **Roving DOF** and no Reference **DOF**.

- Each **UMM** mode shape component is multiplied by one of the five **UMM** mode shape components to create a **Residue** mode shape component ***with both Roving & Reference DOFs***

MODE SHAPE UNITS

Each **M#** of the mode shapes of a Modal Model has **engineering units** associated with it.

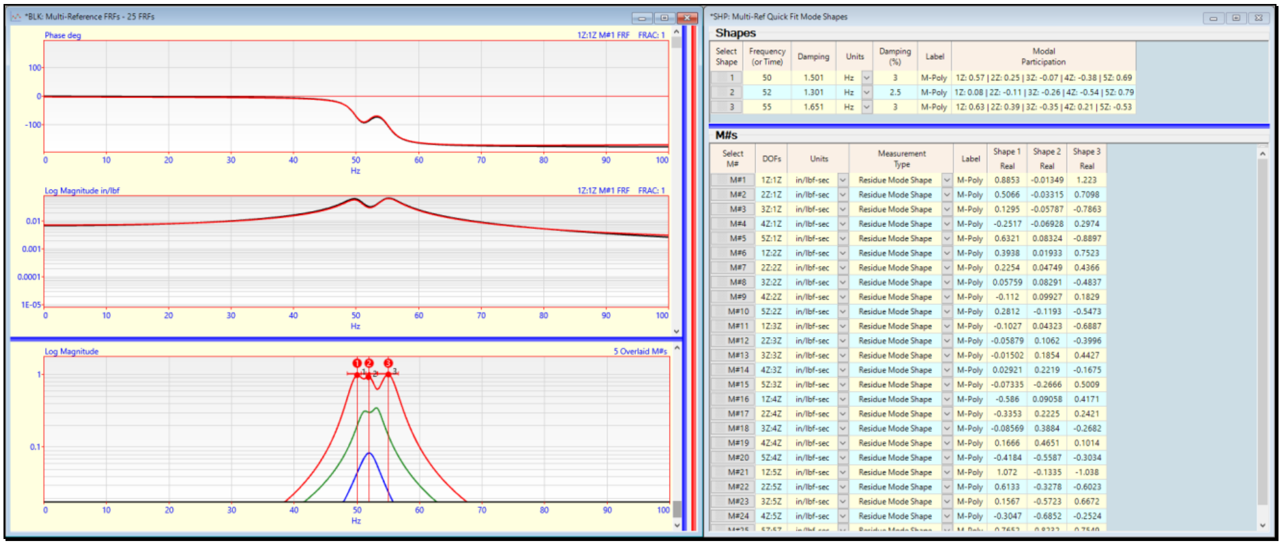
- **Residue** mode shapes have units of **(displacement, velocity, or acceleration) / (force - seconds)**
- **UMM** mode shape have units of **(displacement) / (force - seconds**)

STEP 7- MULTI-REFERENCE CURVE FITTING

To curve fit the Multi-Reference FRFs using the Multi-Reference Quick fit method,

- ***Press* Hotkey 7 Multi-Reference Quick Fit**

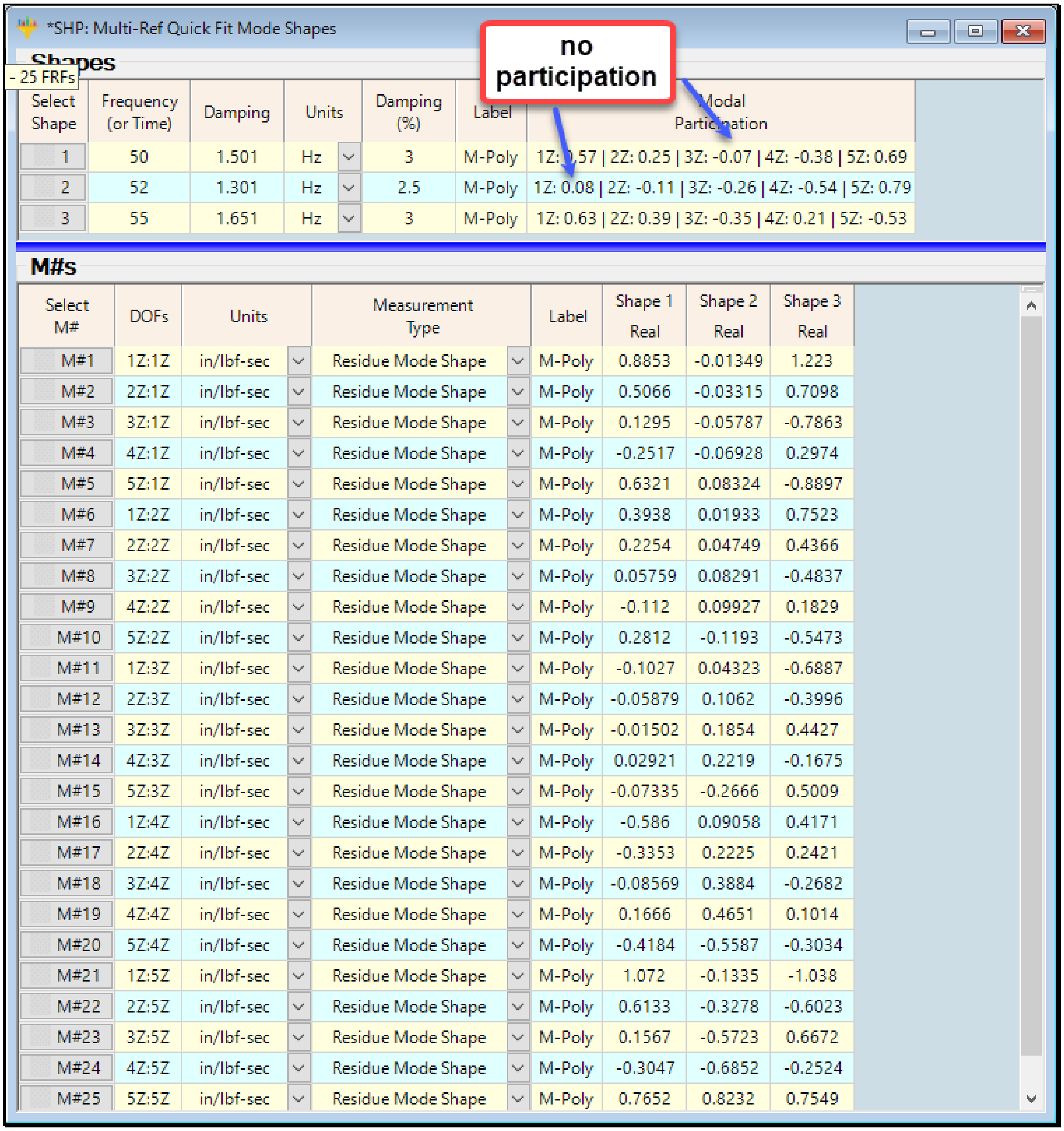
The Quick Fit results, shown below, are obtained by curve fitting all 25 Multi-Reference FRFs. A Residue mode shape was estimated for ***each mode and each* FRF**, the same as with a single reference set of FRFs.



Multi-Reference Quick Fit of Multi-Reference FRFs

MODAL PARTICIPATION

During Multi-Reference curve fitting, the participation of each mode in each reference of FRF data is calculated and used for weighting each reference of FRF data for curve fitting. The modal participation is also saved with each mode shape, as shown below.



Select Shape	Frequency (or Time)	Damping	Units	Damping (%)	Label	Modal Participation
1	50	1.501	Hz	3	M-Poly	1Z: 0.57 | 2Z: 0.25 | 3Z: -0.07 | 4Z: -0.38 | 5Z: 0.69
2	52	1.301	Hz	2.5	M-Poly	1Z: 0.08 | 2Z: -0.11 | 3Z: -0.26 | 4Z: -0.54 | 5Z: 0.79
3	55	1.651	Hz	3	M-Poly	1Z: 0.63 | 2Z: 0.39 | 3Z: -0.35 | 4Z: 0.21 | 5Z: -0.53

Select M#	DOFs	Units	Measurement Type	Label	Shape 1 Real	Shape 2 Real	Shape 3 Real
M#1	1Z:1Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.8853	-0.01349	1.223
M#2	2Z:1Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.5066	-0.03315	0.7098
M#3	3Z:1Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.1295	-0.05787	-0.7863
M#4	4Z:1Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.2517	-0.06928	0.2974
M#5	5Z:1Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.6321	0.08324	-0.8897
M#6	1Z:2Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.3938	0.01933	0.7523
M#7	2Z:2Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.2254	0.04749	0.4366
M#8	3Z:2Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.05759	0.08291	-0.4837
M#9	4Z:2Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.112	0.09927	0.1829
M#10	5Z:2Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.2812	-0.1193	-0.5473
M#11	1Z:3Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.1027	0.04323	-0.6887
M#12	2Z:3Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.05879	0.1062	-0.3996
M#13	3Z:3Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.01502	0.1854	0.4427
M#14	4Z:3Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.02921	0.2219	-0.1675
M#15	5Z:3Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.07335	-0.2666	0.5009
M#16	1Z:4Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.586	0.09058	0.4171
M#17	2Z:4Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.3353	0.2225	0.2421
M#18	3Z:4Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.08569	0.3884	-0.2682
M#19	4Z:4Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.1666	0.4651	0.1014
M#20	5Z:4Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.4184	-0.5587	-0.3034
M#21	1Z:5Z	in/lbf-sec	Residue Mode Shape	M-Poly	1.072	-0.1335	-1.038
M#22	2Z:5Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.6133	-0.3278	-0.6023
M#23	3Z:5Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.1567	-0.5723	0.6672
M#24	4Z:5Z	in/lbf-sec	Residue Mode Shape	M-Poly	-0.3047	-0.6852	-0.2524
M#25	5Z:5Z	in/lbf-sec	Residue Mode Shape	M-Poly	0.7652	0.8232	0.7549

SHP: Multi-Ref Quick Fit Mode Shapes

- **Modal Participation ➔** absolute values **between 0.0 & 1.0**
- **Participation small ➔** small resonance peaks for the mode and Reference **DOF** in the FRFs
- **Participation large ➔** large resonance peaks for the mode and Reference **DOF** in the FRFs

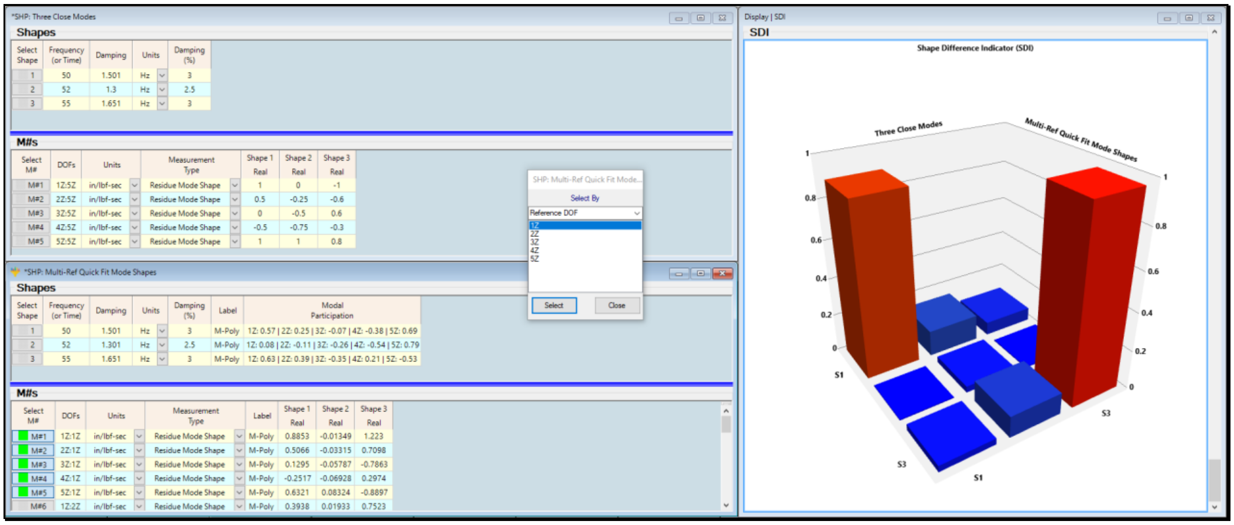
The Modal Participations listed **in SHP: Multi-Ref Quick Fit Mode Shapes** above verify that,

- **Shape 1** has a nodal point at **DOF 3Z**
- **Shape 2** has a nodal point at **DOF 1Z**

STEP 8 – COMPARE MULTI-REFERENCE MODE SHAPES

- ***Press* Hotkey 8 Compare Multi-Ref Mode Shapes**

SHP: Three Close Modes and **SHP: Multi Ref Quick Fit Mode Shapes** are displayed ***on the left***, and the SDI bar chart is displayed ***on the right***.



SHP: Three Close Modes & SHP: Multi-Ref Quick Fit Mode Shapes - Reference 1Z Selected

SHP: Multi-Ref Quick Fit Mode Shapes contains Multi-Reference shapes, so the **Reference DOF Selection** dialog box will open with a **Reference DOF 1Z *selected***.

- The shapes in **SHP: Multi Ref Quick Fit Mode Shapes** can only be ***compared one reference at a time*** with the shapes in **SHP: Three Close Modes**

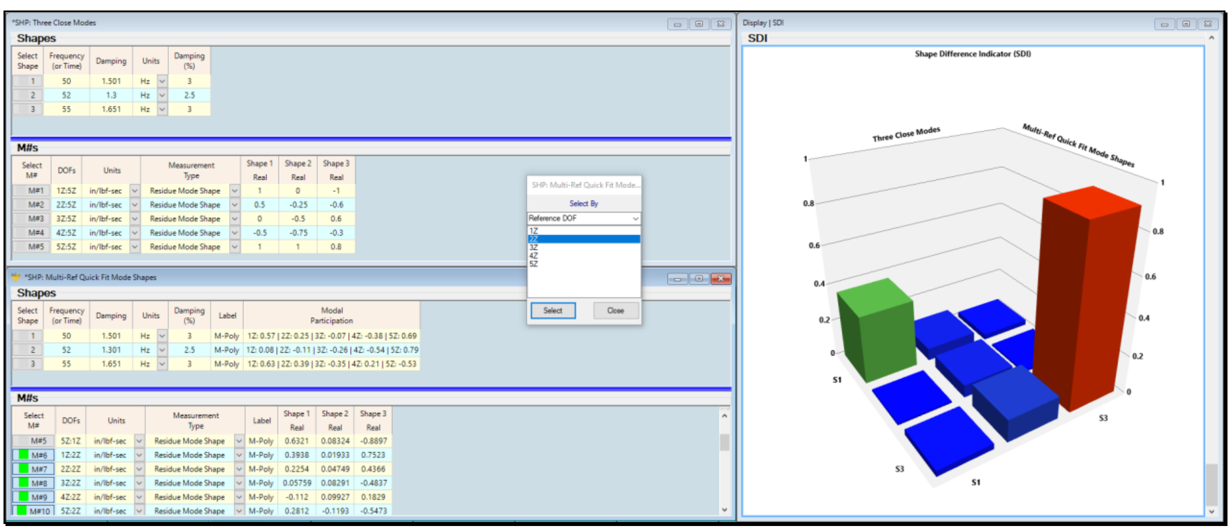
REFERENCE 1Z SELECTED

When **Reference DOF 1Z** is ***selected*** the figure above shows a ***low SDI value for shape pair 2***.

- **Shape 2** has a **nodal point** at **DOF 1Z**

REFERENCE 2Z SELECTED

- Select **Reference DOF 2Z** in the Reference DOF Selection box



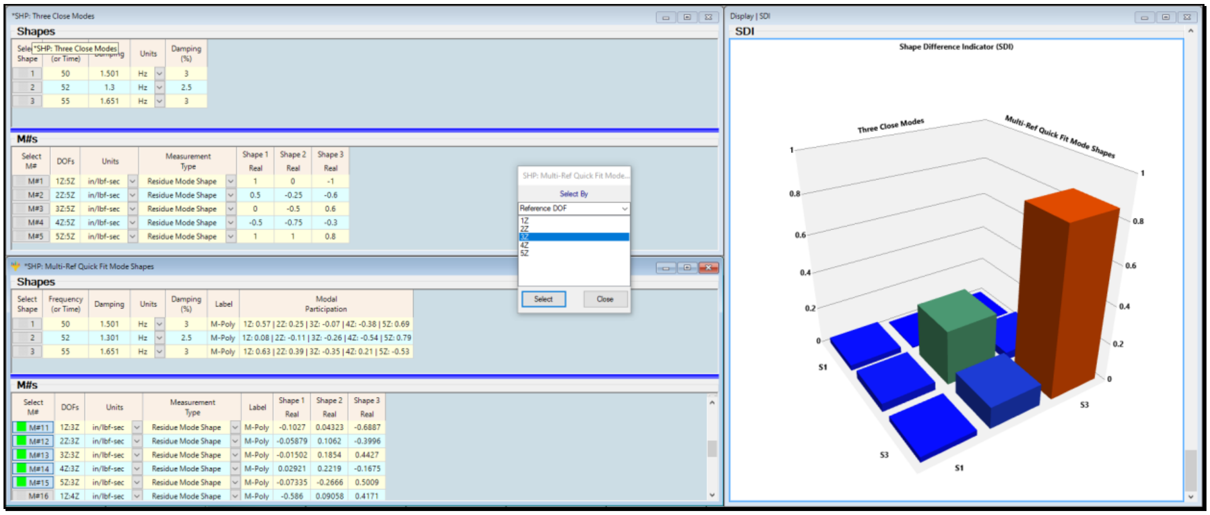
SHP: Three Close Modes & SHP: Multi-Ref Quick Fit Mode Shapes - Reference 2Z Selected

REFERENCE 3Z SELECTED

- Select **Reference DOF 3Z** in the Reference DOF selection box

The following figure shows a ***low SDI value for shape pair 1*** when **Reference DOF 3Z** is selected.

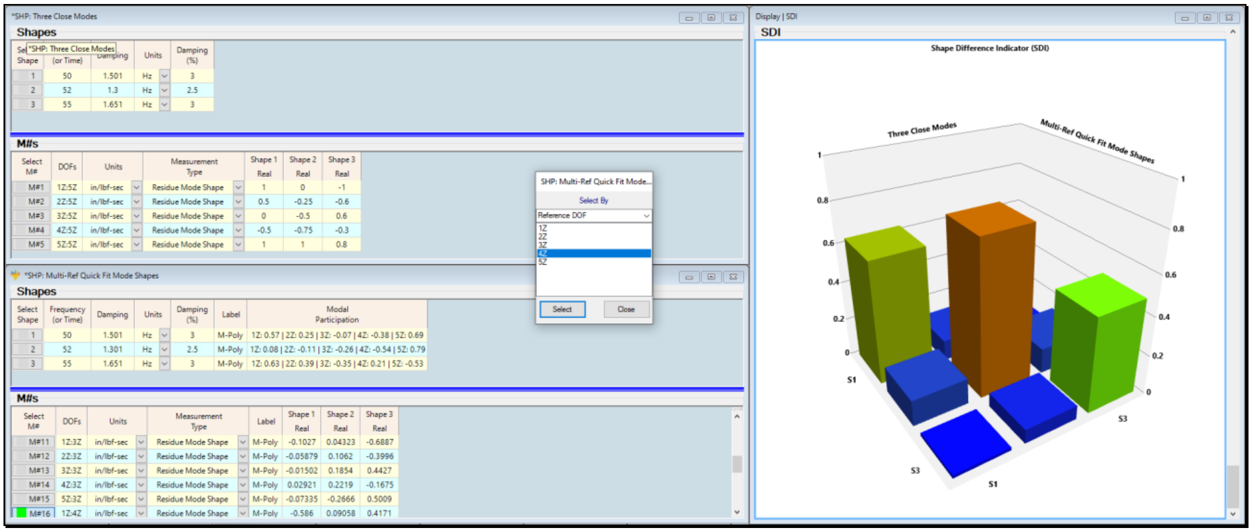
- **Shape 1** has a nodal point at **DOF 3Z**



SHP: Three Close Modes & SHP: Multi-Ref Quick Fit Mode Shapes - Reference 3Z Selected

REFERENCE 4Z SELECTED

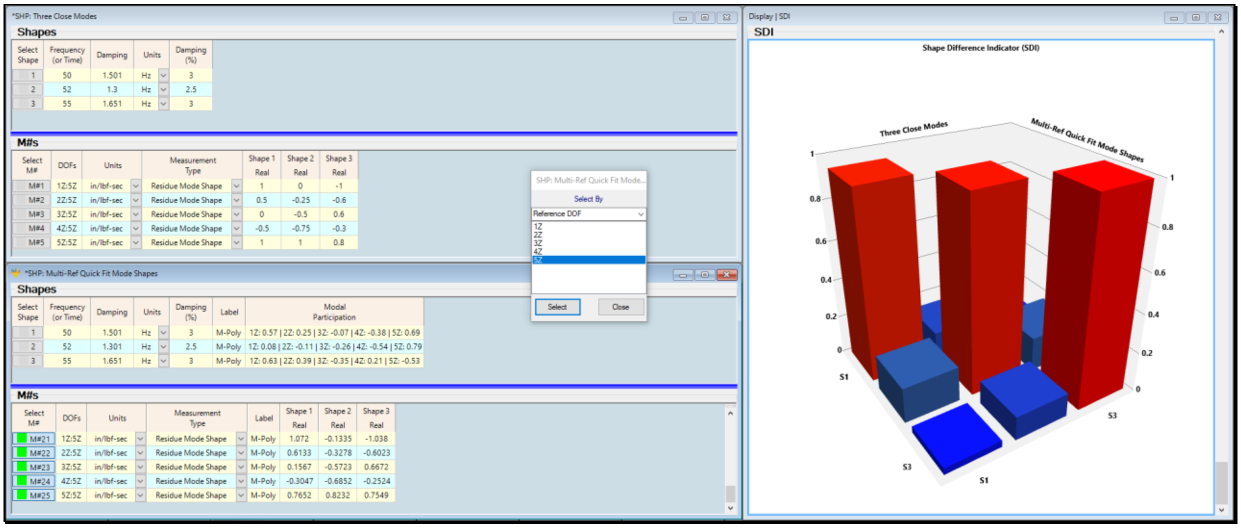
- Select **Reference DOF 4Z** in the Reference DOF selection box



SHP: Three Close Modes & SHP: Multi-Ref Quick Fit Mode Shapes - Reference 4Z Selected

REFERENCE 5Z SELECTED

- Select **Reference DOF 5Z** in the Reference DOF selection box



SHP: Three Close Modes & SHP: Multi-Ref Quick Fit Mode Shapes - Reference 5Z Selected

When Reference 5Z is ***selected***, all three mode shape pairs have ***SDI values of 1.0***, meaning that the ***Multi-Reference*** Residue mode shapes obtained by Multi-Reference curve fitting ***are identical to the original single-reference*** Residue mode shapes.

STEP 9 - REVIEW

To review the steps of this App Note,

- ***Press*** **Hotkey 9 Review Steps**