MEscope Application Note 04

Using SDM for Sub-Structuring

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-5000 SDM, VES 4000 Modal Analysis,** and **VES-8000 FEA** options. Without these options, you can still carry out the steps in this App Note using the **AppNote04** project file. These steps might also require MEscope software with ***the most recent release date***.

APP NOTE 04 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote04.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down*** **the Ctrl key** and ***click*** **on a Hotkey** to display its Script window

WHAT IS SUB-STRUCTURING?

In this App Note, two structures are joined together using the VES-5000 Structural Dynamics Modification (**SDM**) option in **MEscope**. A simple two degree-of-freedom (2-DOF) mass-spring substructure ***is mounted*** on a flat plate substructure, and the mode shapes of the combined substructures are calculated using SDM.

To perform sub-structuring with SDM, the following is needed:

- A **modal model** of scaled mode shapes for each substructure
- A scaled 3D model of each Substructure, with **FEA elements** added between the Substructures which connect them together

MODAL MODEL

A set of scaled mode shapes is called a **modal model**. A modal model of each ***un-attached substructure*** can contain either ***EMA or OMA mode shapes*** (obtained experimentally), or ***FEA mode shapes*** (obtained from an FEA model).

- Each **modal model** must be an ***adequate description of the dynamics*** of each substructure, including its boundary conditions

If a substructure is modeled in the ***free-free*** condition, its modal model must include its ***six rigid-body*** mode shapes. If a Substructure is ***attached to ground***, no ***rigid-body*** modes are required.

- If one Substructure is to be ***mounted onto another*** and is not attached to ground, its ***rigid-body mode shapes*** must be included in its modal model
- To perform sub-structuring with SDM, the modal model must consist of mode shapes that are scaled to **Unit Modal Masses** called **UMM mode shapes**

TWO SUBSTRUCTURES

One substructure is a **Plate-on-Springs**. It is an (**8 x 10 x ¼ inch**) steel plate supported vertically by four springs of **100 lbf/in stiffness** and restrained in-plane by four more springs of **5000 lbf/in stiffness** as shown in the figure below.

- The **modal model** for the **Plate-on-Springs** substructure has **12 FEA mode shapes**

The FEA mode shapes are calculated as solutions to the FEA model of the plate with **48 FEA Quads** in it. Each mode shape has **63 DOFs**, describing ***only Z-axis (vertical) deflection*** at 63 Points in the 7 by 9 grid of Points shown below. All DOFs in the **X, Y, rX, rY, rZ** directions are deleted from the mode shapes by the script for **Hotkey 1**.

The second substructure is a **Mass-Spring** substructure, also shown below. This FEA model is two point-masses connected with a single FEA spring (**Spring 1**) between them.

- The **modal model** for the **Mass-Spring** substructure has **two FEA mode shapes**

Each mode shape has **9 DOFs**, describing ***only Z-axis (vertical) motion*** at 3 Points. All DOFs in the **rX, rY, rZ** directions are deleted from the mode shapes by the script for **Hotkey 2**.

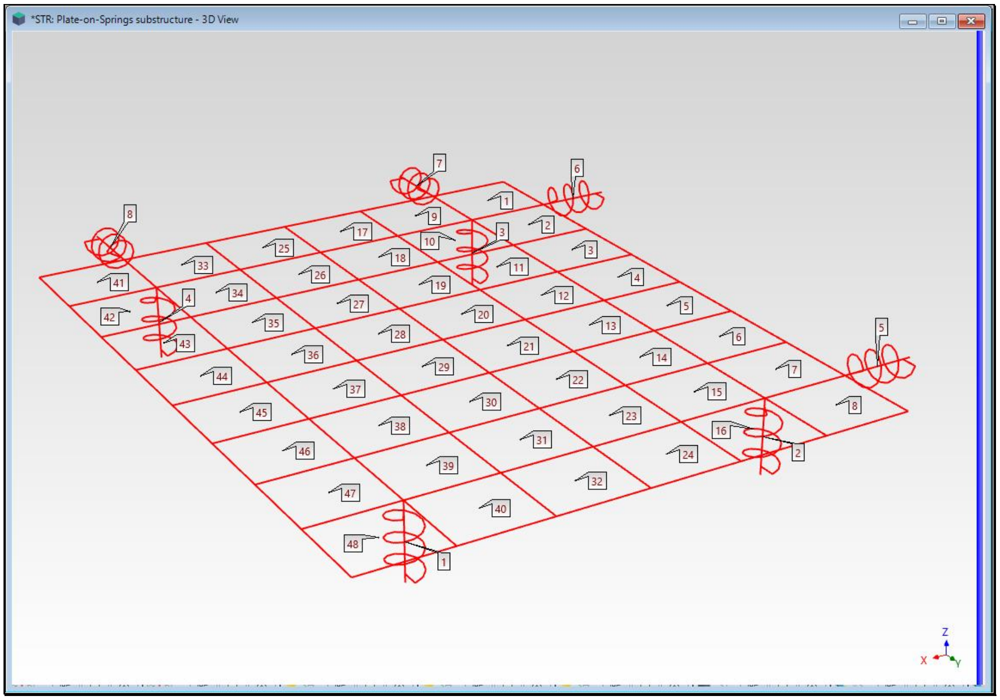
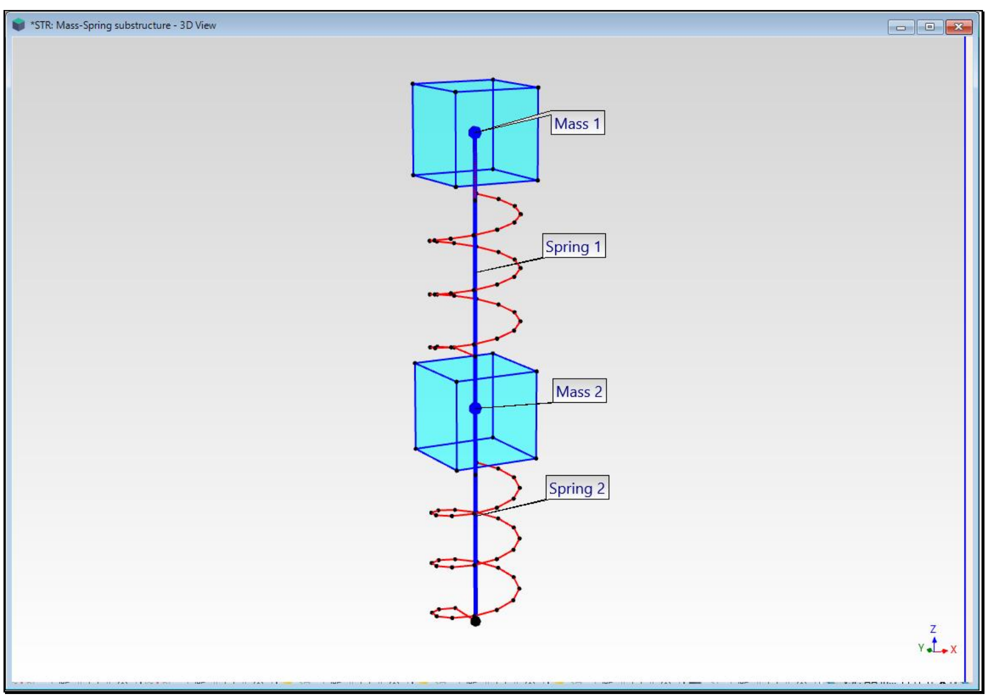


Plate-on-Springs Showing FEA Quads & FEA Springs.



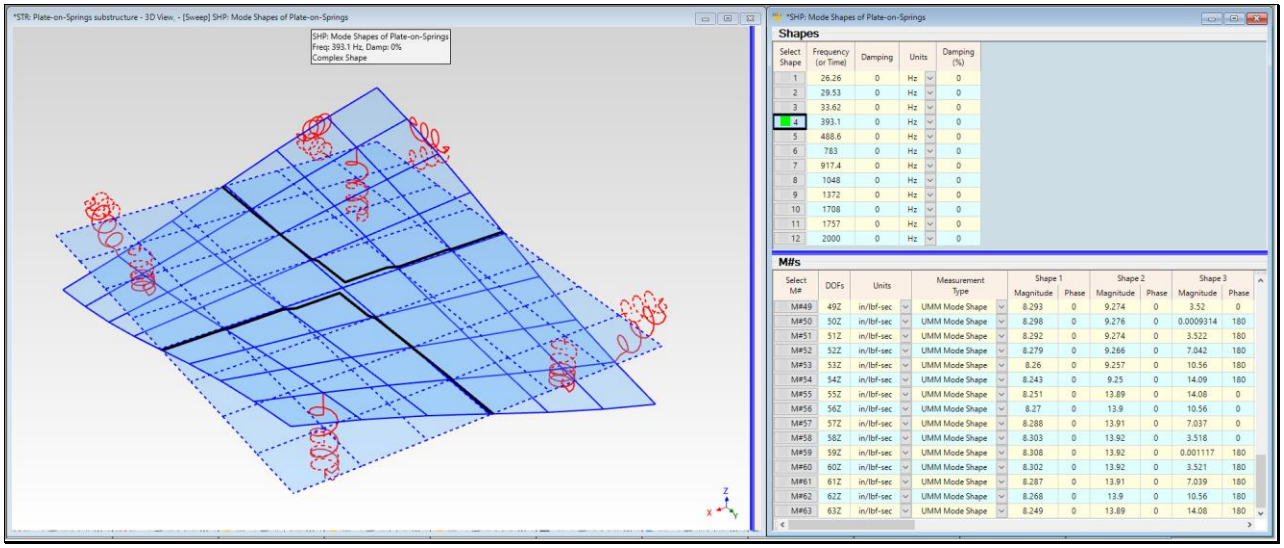
Mass-Spring Substructure Showing FEA Masses & FEA Springs.

A second FEA spring (**Spring 2**) is used to model the connection of the **Mass-Spring** substructure to the **Plate-on-Springs** substructure

- To perform sub-structuring, SDM uses this single FEA spring (**Spring 2**) ***to model the connection*** of the **Mass-Spring** substructure to the center of the **Plate-on-Springs**
- The deflection of all interior Points of **each red coil spring** is interpolated from the deflection of its end Points

STEP 1 - MODE SHAPES OF THE PLATE-ON-SPRINGS

- ***Press* Hotkey 1 Plate-on-Springs Mode Shapes**



First Flexible-Body Mode Shape of the Plate-on-Springs.

When **Hotkey 1 is *pressed***, the FEA mode shapes of the **Plate-on-Springs** are calculated and sweep animation of the mode shapes in the Shape Table **SHP: Mode Shapes of Plate-on-Springs** is begun.

- The first three mode shapes (**26**, **29** & **33** Hz) are ***rigid-body mode shapes***

The ***plate merely bounces*** as a rigid body on its springs.

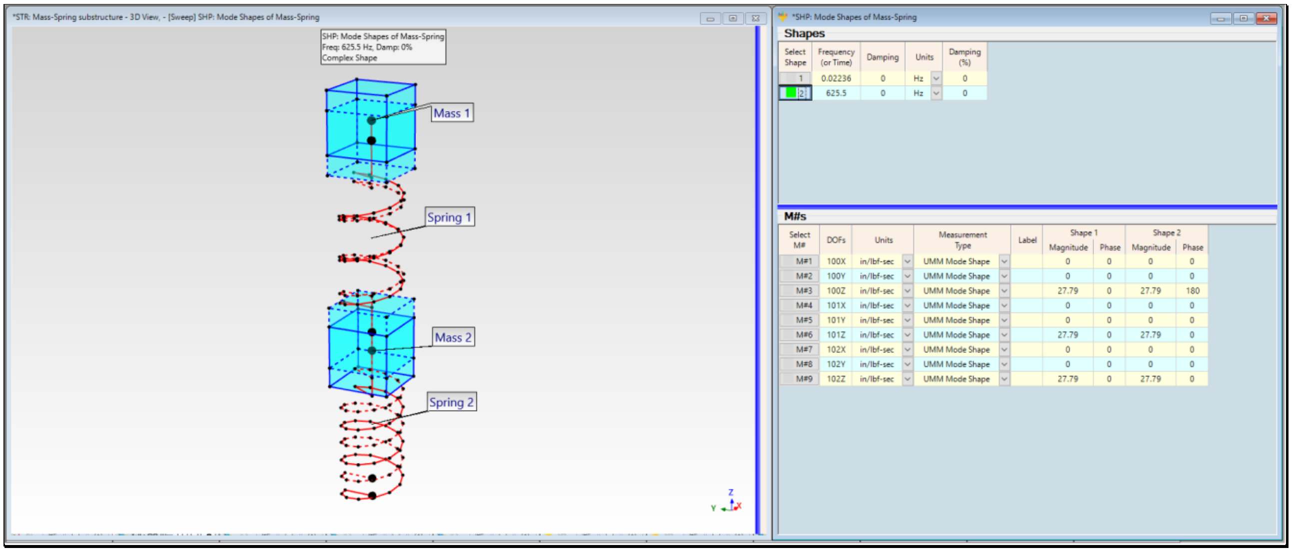
- The fourth mode shape is the ***first flexible-body mode shape***, a ***torsional (twisting)*** mode shape

Many of the mode shapes have ***node lines passing through the center point*** of the plate.

- When SDM is be used to model the attachment of the **Mass-Spring** substructure to the ***center point*** of the plate, all modes of the plate that have a node point (***no deflection***) at the center point ***will not be influenced*** by the Mass-Spring substructure

STEP 2 - MODE SHAPES OF THE MASS-SPRING SUBSTRUCTURE

- ***Press* Hotkey 2 Mass-Spring Mode Shapes**



Mode Shape of the Mass-Spring Substructure.

Because it only has two masses, this structure only has two modes of vibration.

- The first mode has a ***rigid-body mode shape*** with the masses ***moving in-phase*** with one another
- Thee second mode has a ***flexible-body mode shape*** with the masses ***moving out-of-phase*** with one another

Each mode shape is initially calculated with **6 DOFs** (three translational & three rotational **DOFs**) at each mass, but only the ***Z-axis (vertical)* translational** of the two masses is ***non-zero***. The rotational **DOFs** were deleted by the script for **Hotkey 2**, and the X & Y **DOFs** of the mode shapes ***are zero* (0)**.

The spring connected to the **Bottom Mass** simple moves with the motion of the bottom mass. The **Bottom Spring** is merely ***"floating in air"*** because it is not connected to anything yet. The ***bottom of the* Bottom Spring** will be connected to the center of the Plate-on-Springs to model the coupling of the two substructures together.

STEP 3 - DRIVING POINT FRFs FOR THE PLATE-ON-SPRINGS

- ***Press* Hotkey 3 Driving Point FRFs**

SDM will be used to calculate new mode shapes when the Mass-Spring substructure is ***attached to the center*** of the Plate-on-Springs substructure.

- Some of the mode shapes of the Plate-on-Springs will be influenced by the attachment of the Mass-Spring sub-structure to the center of the Plate-on-Springs
- Only those mode shapes that ***are non-zero at the center of the plate***, will be influenced by the Mass-Spring sub-structure

One way to determine which mode shapes are ***participating*** in the ***response at the center*** of the Plate-on-Springs is to synthesize a **Driving Point FRF** at the center using the ***modal model*** of the Plate-on-Springs.

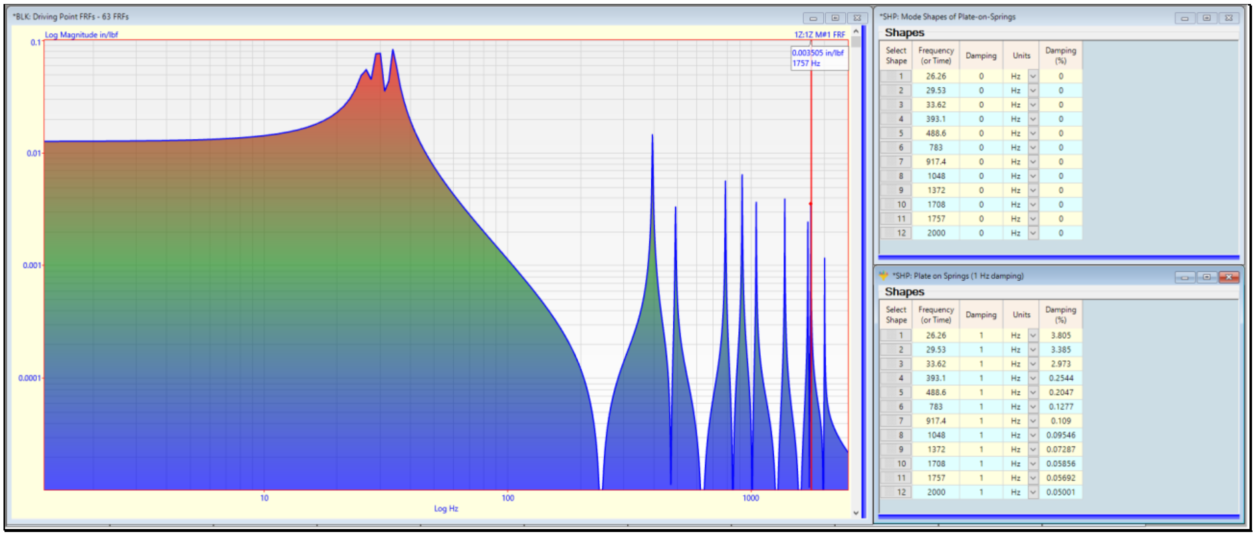
- In a **Driving Point FRF** the Roving DOF ***is the same as*** the Reference DOF

The FEA mode shapes of the Plate-on-Springs were calculated from an FEA model which had ***no damping*** in it. Therefore, the FEA mode shapes have ***no modal damping***.

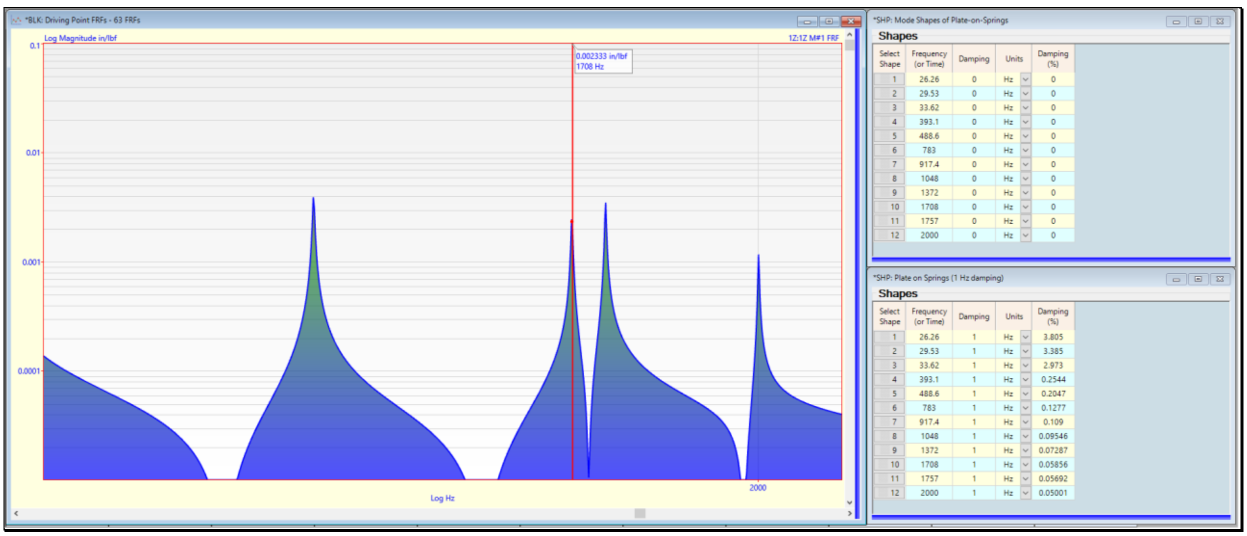
- To synthesize an FRF using mode shapes, each mode shape must have ***non-zero modal damping***

When **Hotkey 3 is *pressed*, 1 Hz modal damping** is added to each mode shape, and a **Driving Point FRF** is synthesized for each DOF of the of the Plate-on-Springs.

The **Driving Point FRF 1Z:1Z** is shown below. It has ***12 resonance peaks*** in it, meaning that ***all 12 modes are participating*** in the response at **DOF 1Z**.



Driving Point FRF 1Z:1Z.



Zoomed FRF 1Z:1Z Showing Two Peaks Near 1700 Hz.

- ***Click near the* 1700 Hz *peak*** and ***spin the mouse wheel*** to zoom the display of the ***closely spaced resonance peaks*** at **1708 Hz** and **1757 Hz**

STIFFNESS LINE

The Mass-Spring substructure will be attached to **Point 32** at the center of the plate.

- ***Scroll*** the **M#** display to display **Driving Point FRF 32Z:32Z**, as shown below
- **FRF 32Z:32Z** has only ***four resonance peaks***

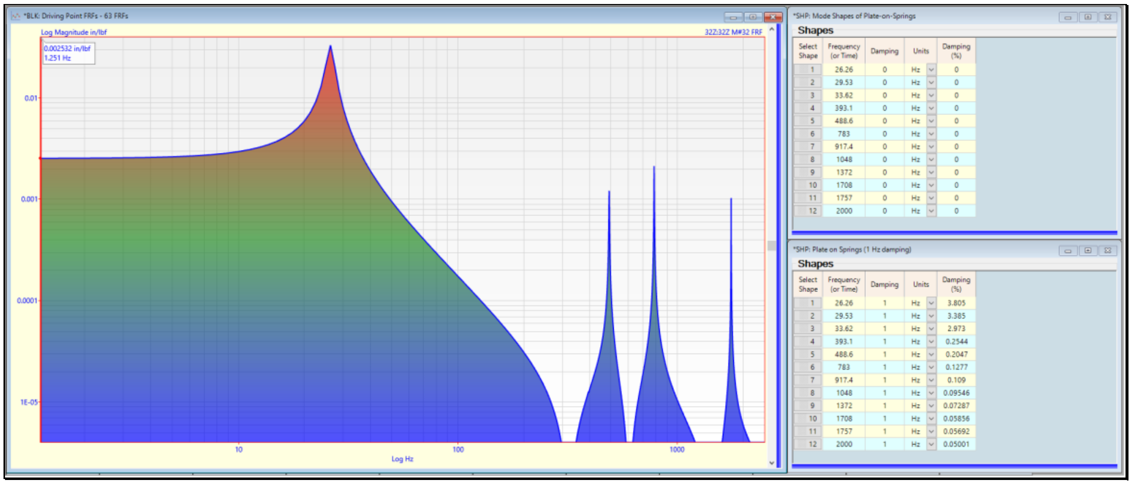
This means that ***only four flexible-body modes*** plus the ***first rigid-body mode participate*** in the plate dynamics at Point 32.

- O***nly five modes*** of the Plate-on-Springs ***will be influenced*** by the attachment of the Mass-Spring substructure at Point 32

The Plate-on-Springs is supported by ***four vertical springs***, each with a **stiffness of 100 lbf/in**.

- The sum of the stiffnesses of the four springs supporting the plate is (**4 x 100 lbf/in) ➔ 400 lbf/in**

The un-zoomed display of **FRF 32Z:32Z** shows the horizontal **stiffness line** of the **26 Hz *rigid-body mode shape***. This is the mode shape of the plate ***bouncing on its four springs***.



Line cursor on the Stiffness Line of FRF 32Z:32Z.

The stiffness of the springs can be ***estimated from the stiffness line*** as it approaches ***DC (0 frequency)*** in **FRF 32Z:32Z**.

- ***Drag*** the **Line** cursor ***near* 0 Hz** to display the Cursor value as shown above
- The FRF value ***near 0 Hz*** is called the ***flexibility***. Flexibility is the ***inverse of stiffness***
- The cursor value **near 0 Hz** is **0.002532 in/lbf**
- The ***inverse*** of **0.002532 in/lbf** is **395 lbf/in**, which is ***close to the sum of the stiffnesses*** of the four supporting springs

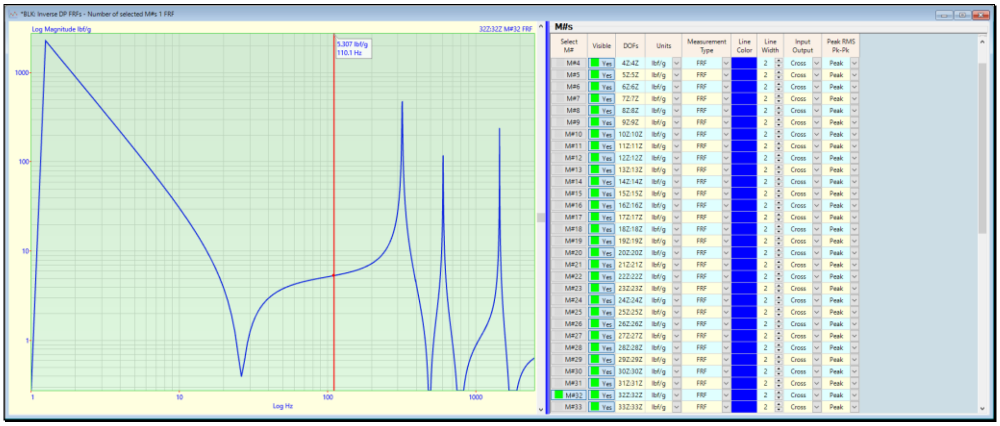
STEP 4 - MASS LINE

- ***Press* Hotkey 4 Mass Line**
- An (**acceleration/force**) FRF is obtained by ***double differentiating*** a (**displacement/force**) FRF
- The weight of the Plate-on-Springs is (**8 x 10 x 0.25) in^3 x 0.283 lbm/in^3 ➔ 5.66 lbm**

When **FRF 32Z:32Z** is ***twice differentiated*** from ***displacement/force units*** to ***acceleration/force units*** and ***inverted***, the weight pf the **Plate-on-Springs** can be estimated from the ***mass line***.

- The ***mass line*** should ***approximate a horizontal line*** in an (acceleration/force) driving point FRF at a ***frequency higher than the rigid-body mode at 26 Hz***

When **Hotkey 4 is *pressed***, ***the inverse*** of **FRF 32Z:32Z** is calculated and the Line cursor is displayed at ***about 110 Hz*** in.



Mass Line in the Inverse of FRF 32Z:32Z.

- In a 1g gravitational field, the weight of the Plate-on-Springs is **5.3 lbf/g ➔ 5.3 lbm**

The mass line is ***in close agreement*** with the weight of the Plate-on-Springs, even though the other higher frequency modes contribute residual effects at 110 Hz.

The following conclusions can be made from the properties of the **FRF 32Z:32Z** which was synthesized from the modal model of the Plate-on-Springs,

- The **modal model** is a ***complete representation of the dynamics*** of the Plate-on-Springs
- The **stiffness & mass lines** have values that ***closely match the stiffness & mass*** of the Plate-on-Springs

The modal model of the Plate-on-Springs can be confidently used with SDM for sub-structuring.

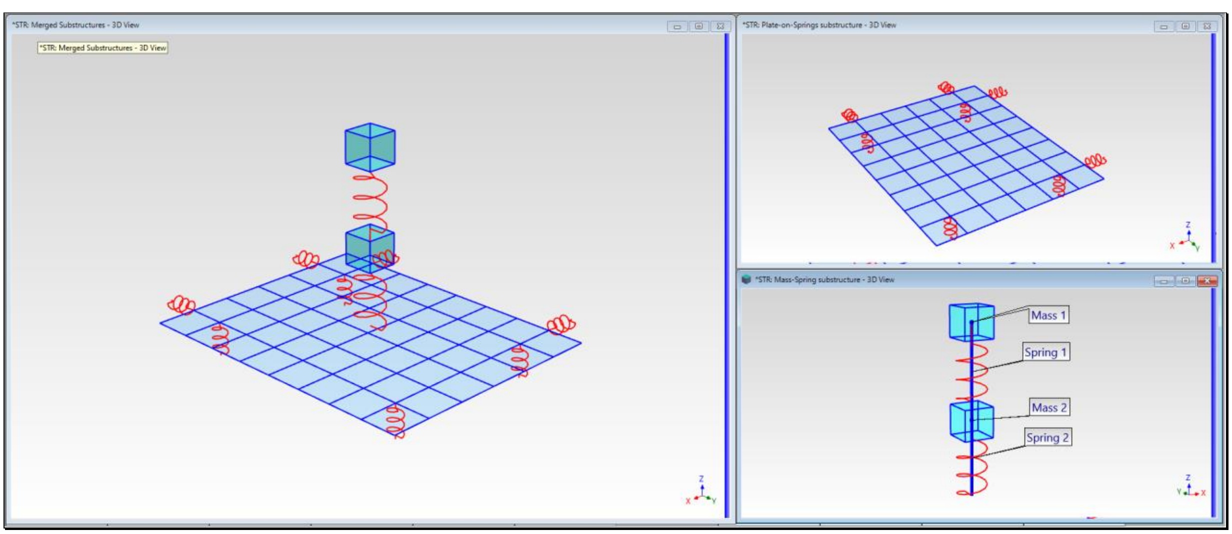
STEP 5 - MERGING THE SUBSTRUCTURE MODELS

- ***Press*** **Hotkey 5 Merge Substructures**

To use SDM for sub-structuring, there are three requirements,

1. The substructure models ***must be merged*** into the same Structure window
2. The substructures ***must be coupled together*** using one or more FRF Objects
3. The mode shapes of the substructures ***must be merged*** into the same Shape Table

When **Hotkey 5 is *pressed***, the new **STR: Merged Structures** window displayed ***on the left*** together with the two sub-structures windows ***on the right***.

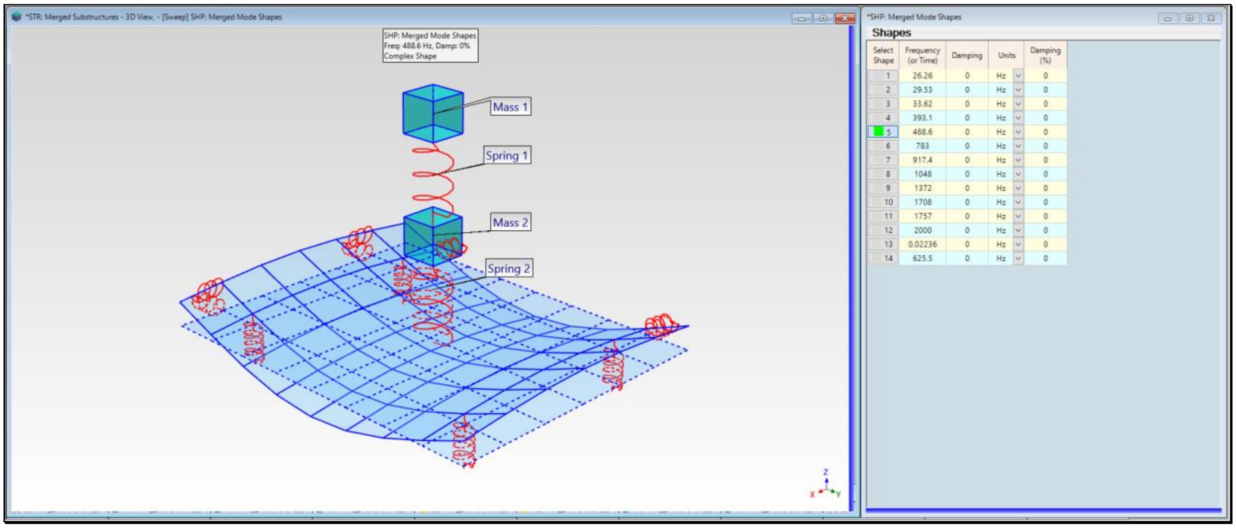


Merged Plate-on-Springs & Mass-Spring Substructure Models.

STEP 6 - MERGING THE MODE SHAPES OF THE SUBSTRUCTURES

- ***Press* Hotkey 6 Merge Substructure Mode Shapes**

When **Hotkey 6 is *pressed***, the two mode shapes of the Mass-Spring substructure are added to the mode shapes of the Plate-on-Springs substructure and ***sweep animation*** of the mode shapes of the two ***unmodified substructures*** is begun.



Animation of the Mode Shapes of the Unmodified Structures.

MODE SHAPES OF THE UNMODIFIED SUBSTRUCTURES

- ***Click*** on each **Select Shape** button in **SHP: Merged Mode Shapes** to display its mode shape
- Mode shapes **1 to 12** have deflection ***only on the*** **Plate-on-Springs substructure**
- Mode shapes **13 & 14** have deflection ***only on the*** **Mass-Spring substructure**

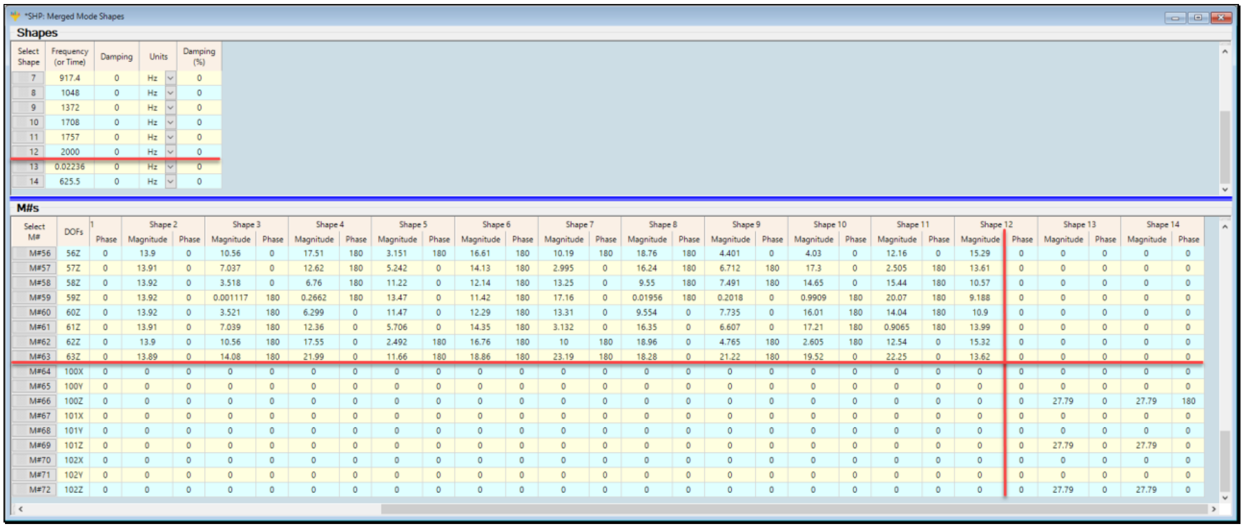
The bottom Point of the **Bottom Spring** was also attached to the center point (**Point 32**) of the plate.

- When SDM calculates new modes for the coupled substructures, the bottom Point of the Bottom Spring will have the same deflection as Point 32 on the plate

SHAPE TABLE BLOCK DIAGONAL FORMAT

Because the Mass-Spring substructure mode shapes ***have different* DOFs** than the DOFs of the Plate-on-Springs mode shapes, when the two sets of mode shapes are merged into the same Shape Table, a "***block diagonal***" format is created, as shown below

- **Block diagonal format:** When shapes are added together in the same Shape Table, they ***must share the same* DOFs**. Any shape component that does not share a common DOF is given a ***zero value***

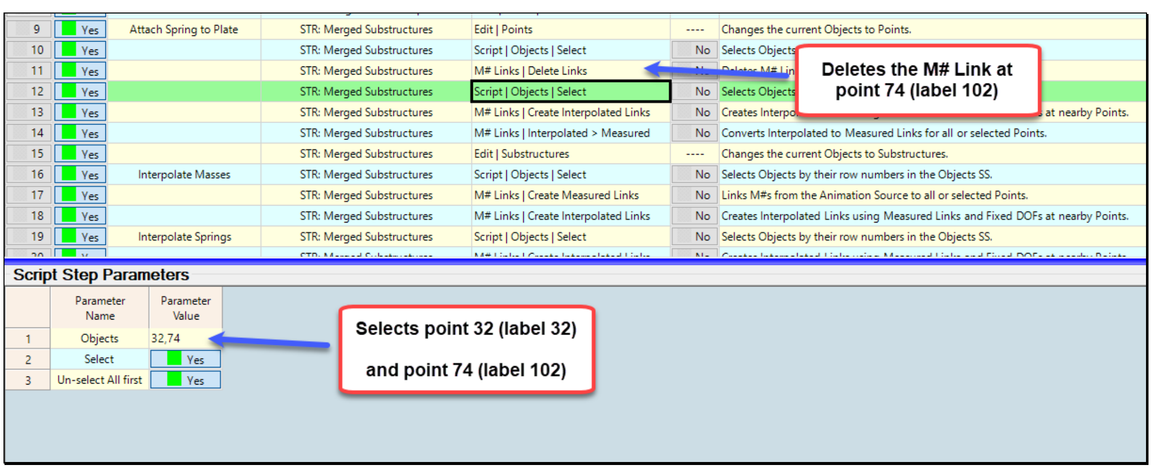


Merged Mode Shapes in Block Diagonal Format.

CONNECTING THE BOTTOM SPRING TO THE PLATE CENTER POINT

The script for **Hotkey 6 Merge Substructure Mode Shapes** the script that ***links the M# of Point 32 to Point 102*** is shown below.

- **Script line 11** deletes the **M# Link** at **Point 74** (with **Label 102**)
- **Script line 12** selects Point **32** (the ***center point*** on the plate) and Point **74** (labeled **102**) on the Mass-Spring
- **Script line 13** creates and **Interpolated Link** for **Point 74** equal to the **Measured Link** at **Point 32**
- **Script line 14** converts the **Interpolated Link** to a **Measured Link** for Point **74**



Script to Create a Measured Link at Point 102 Equal to the Link at Point 32.

STEP 7 - SUB-STRUCTURING WITH SDM

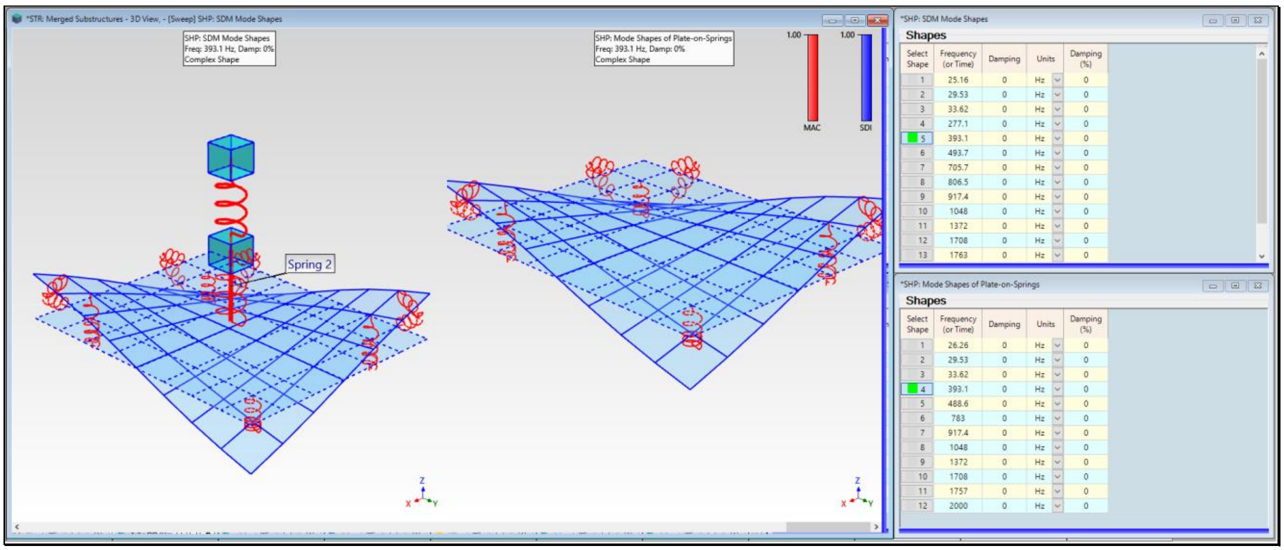
In Step 6, the Mass-Spring substructure was attached to the Plate-on-Springs substructure with an **FEA Spring** connected between the ***Bottom Mass*** of the Mass-Spring substructure and the ***center point*** of the Plate-on-Springs.

The model in **STR: Merged Structures** and the mode shapes in **SHP: Merged Mode Shapes** are now ready to calculate the new modes of the Mass-Spring substructure attached to the Plate-on-Springs.

- ***Press*** **Hotkey 7 Calculate New Mode Shapes with SDM**

An SDM mode shape of the model in **STR: Merged Substructures** is displayed ***on the left*** and the ***closest matching*** mode shape of the ***unmodified*** **Plate-on-Springs** is displayed ***on the right***, as shown below.

- Each ***closest matching mode shape pair*** is a mode shapes from **SHP: SDM Mode Shapes** that has the **Maximum MAC** value among all mode shapes in **SHP: Mode Shapes of Plate-on-Springs**
- The Modal Assurance Criterion (**MAC**) and Shape Different Indicator (**SDI**) are displayed with each shape pair
- Both MAC & **SDI** have ***values that range between*** **0 & 1**
- **MAC** indicates the ***co-linearity*** of two mode shapes. **SDI** indicates the ***difference*** between two mode shapes



SDM Mode Shape on the Left Versus Mode Shape of the Unmodified Plate on the Right.

COMPARING MODE SHAPES

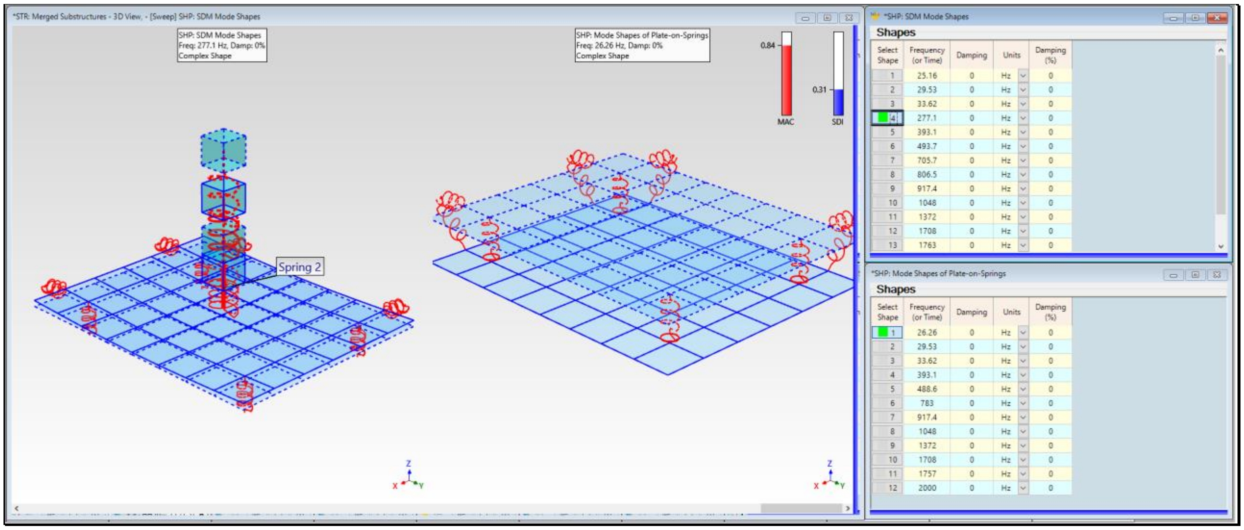
- **Execute Animate | Pause Continue** in **STR: Merged Structures** to pause the animation
- ***Click*** on any **Select Shape** button in **SHP: SDM Mode Shapes** to display a ***closely matching pair*** of mode shapes

As you click through the new mode shapes, ***all mode shape pairs have MAC & SDI values close to*** **"1"**, ***except modes 4 &7***.

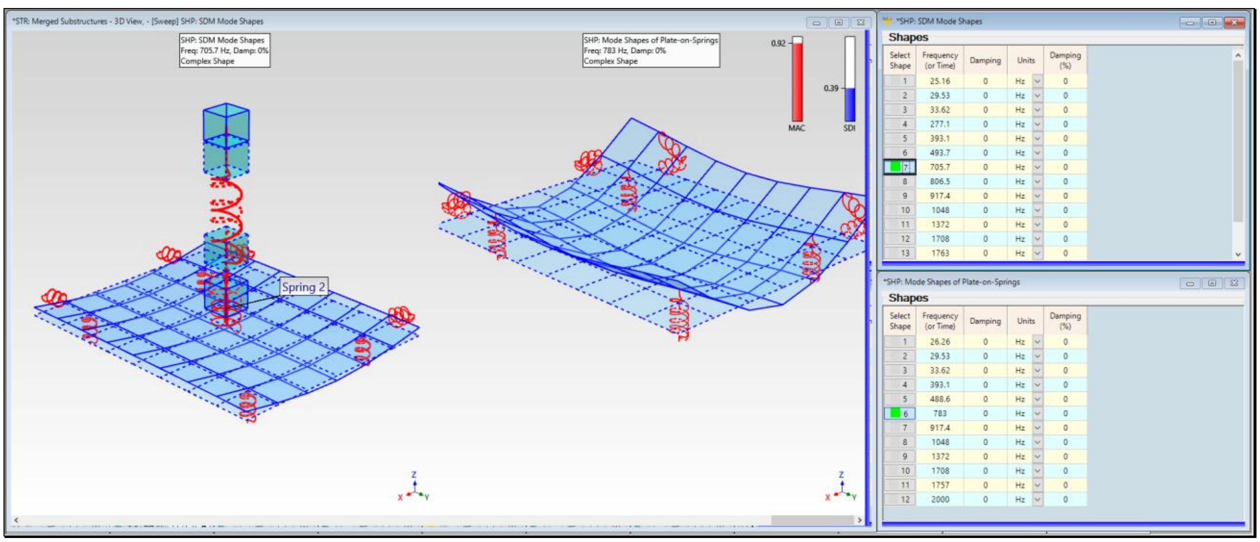
- Attaching the Mass-Spring substructure to the Plate-on-Springs ***did not change 10 of the 12 mode shapes*** of the unmodified plate
- **Mode shape 4 (217 Hz)** and **mode shape 7 (665 Hz)** are both new mode shapes

The addition of the Mass-Spring substructure to the Plate-on-Springs acted like a tuned vibration absorber.

- SDM mode **Shape 4** had the ***highest MAC*** with **Shape 1** of the ***unmodified*** Plate-on-Springs, but its ***low*** **SDI** indicated that it ***suppressed*** **Shape 1**
- SDM mode **Shape 7** had the ***highest MAC*** with **Shape 6** of the ***unmodified*** Plate-on-Springs, but its ***low*** **SDI** indicated that it ***suppressed*** **Shape 6**



SDM Mode Shape 4 (217 Hz) Suppressed Mode Shape (26 Hz) of the Plate-on-Springs.



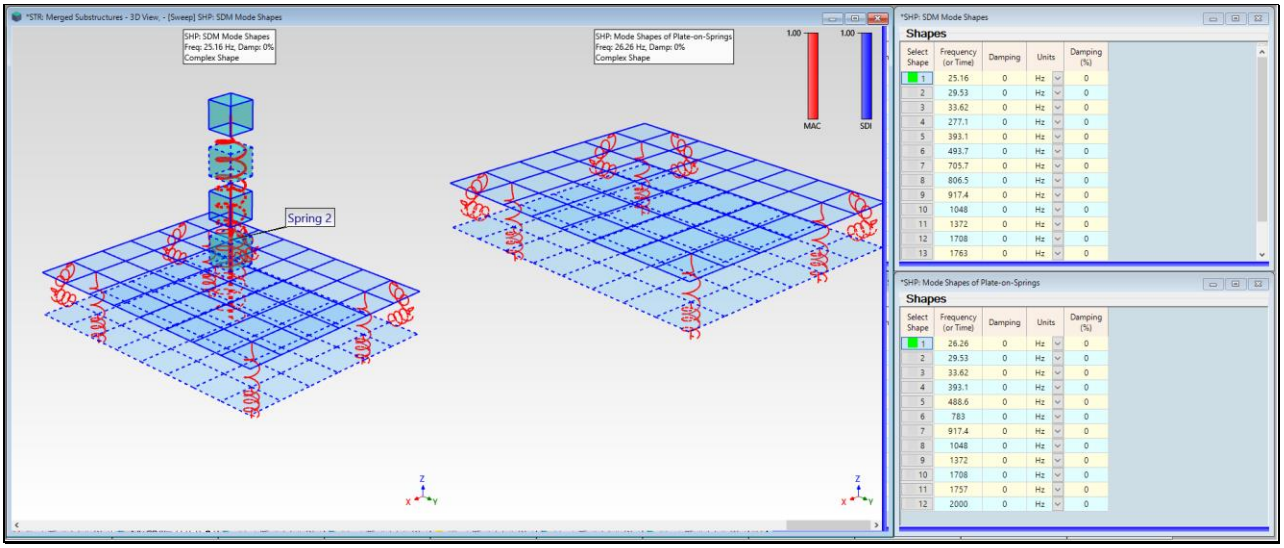
SDM Mode Shape 7 (706 Hz) Suppressed Mode Shape 6 (783 Hz) of the Plate-on-Springs.

ADDED MASS LOWERED THE FREQUENCY OF THE FIRST MODE SHAPE

The first mode shape of the coupled substructures is the same ***rigid-body mode shape*** as the first mode shape of the un-modified Plate-on-Springs.

- Both **MAC** & **SDI** equal **1.0**

The two masses of the Mass-Spring substructure had the effect of adding a ***rigid mass*** (2 x 0.25 lb = **0.5 lb**) to the center of the Plate-on-Springs.



Rigid-body Mode Shape of the Coupled Substructures.

The frequency of the ***first*** SDM ***mode shape*** can be calculated using a simple formula for an SDOF mass on a spring.

$$\mathbf{f_{substructures}} = \mathbf{f_{plate}}\sqrt{\frac{\mathbf{mass_{plate}}}{\mathbf{mass_{substructures}}}} = \mathbf{26.26}\sqrt{\frac{5.66}{5.66+0.5}} = \mathbf{25.17\ Hz}$$

STEP 8 - REVIEW

To review the steps of this App Note,

- ***Press*** **Hotkey 8 Review Steps**

SUMMARY

SDM was used to dynamically couple together two Structures using a single FEA Spring element. In this App Note the following steps were carried out:

- Two FEA Masses and an FEA Spring were used to create a Mass-Spring substructure model
- The FEA mode shapes of Mass-Spring substructure were merged with FEA mode shapes of a Plate-on-Springs structure in ***block diagonal*** format
- The **Bottom Mass** of the Mass-Spring substructure was connected to the ***center Point*** of the Plate-on-Springs model using a ***single*** **FEA Spring**
- **SDM** calculated the **new mode shapes** of the coupled Substructures
- Comparing mode shapes using both **MAC** & **SDI** showed that ***10 out of the 12 mode shapes*** of the Plate-on-Springs ***were not influenced*** by the Mass-Spring substructure
- Comparing mode shapes using both **MAC** & **SDI** showed that SDM mode shapes **4 &7** suppressed the deflection of mode shapes **1 & 6** of the Plate-on-Springs. Therefore, the Mass-Spring substructure ***acted like a tuned absorber*** on the Plate-on-Springs
- **Conclusion:** Attaching substructures together at DOFs where the mode shapes of either substructure have ***nodal points (zero deflection) has no effect*** on the mode shapes of the coupled substructures