MEscope Application Note 02

Waveform Integration & Differentiation

The steps in this Application Note can be carried out using any MEscope package that includes the **VES-3600 Advanced Signal Processing** option. Without this option, you can still carry out the steps in this App Note using the **AppNote02** project file. These steps might also require MEscope software with a ***more recent release date***.

APP NOTE 02 PROJECT FILE

- To retrieve the Project for this App Note, **click here** to download **AppNote02.zip**

This Project file contains ***numbered Hotkeys & Scripts*** for carrying out the steps of this App Note.

- ***Hold down* the Ctrl key** and ***click* on a Hotkey** to display its Script window

INTRODUCTION

The most common type of vibration sensor is an accelerometer, which measures acceleration. However, to answer the question ***"How much is the machine or structure really moving?"*** a common requirement of signal processing is to integrate acceleration (or velocity) signals to displacements.

In this note, both the integration & differentiation methods in MEscope are used. Integration & differentiation can be done on either time domain or frequency domain waveforms. It is shown how ***DC offsets and leakage*** can cause errors when integrating or differentiating waveforms, and how these errors can be effectively dealt with.

MEscope has a built-in Fast Fourier Transform (**FFT**). Using the **FFT**, any Data Block of waveforms can be transformed between the time & frequency domains without loss of information.

INTEGRATION & DIFFERENTIATION

- The **Tools | Integrate** command in MEscope can be used to integrate a Data Block of ***either time or frequency waveforms*** from ***acceleration units to velocity units***, and from ***velocity units to displacement units***
- The **Tools | Differentiate** command in MEscope can be used to differentiate a Data Block of ***either time or frequency waveforms*** from ***displacement units to velocity units***, and from ***velocity units to acceleration units***

Time domain waveforms are integrated & differentiated in MEscope by transforming them to the frequency domain using the **FFT**, performing frequency domain integration or differentiation on their Digital Fourier Transform (or **DFT**), and inverse transforming them back to the time domain using the **Inverse FFT**.

The main difficulty with any time or frequency domain integration method is that the lowest frequencies in the signal are amplified. Therefore,

- Any **DC offset** in a signal ***must be removed before integration*** is performed on it. Otherwise, the ***integrated* DC offset** will dominate the result

Also, if a time waveform is ***not periodic (or completely contained) within its sampling window***, ***leakage will occur*** in its **DFT**.

- **Leakage *causes more errors*** in an integrated or differentiated waveform

FREQUENCY DOMAIN INTEGRATION

Frequency domain integration is done by ***dividing each sample of the frequency spectrum*** $\left(\mathbf{X_i(2\pi f_i)}\right)$ by the sample frequency $\mathbf{(j2\pi f_i)}$. Time domain waveforms are integrated by using the following ***equivalent frequency domain operation***.

$$\int \mathbf{x(t)dt} \Leftrightarrow \frac{\mathbf{X_i(2\pi f_i)}}{\mathbf{(j2\pi f_i)}}, \mathbf{i = 1,\ldots,N/2}$$

$\mathbf{X_i(2\pi f_i)}$ ➔ Fourier spectrum (**DFT**) of the signal for the i[th] sample

$\mathbf{(j2\pi f_i)}$ ➔ frequency of the i[th] sample (in radians/sec), **j** - denotes the imaginary operator

$\mathbf{f_i}$ ➔ frequency of the i[th] sample (in Hz)

FREQUENCY DOMAIN DIFFERENTIATION

Frequency domain differentiation is done by ***multiplying each sample*** of the frequency spectrum $\left(\mathbf{X_i(2\pi f_i)}\right)$ by the sample frequency $\mathbf{(j2\pi f_i)}$. Time domain waveforms are differentiation by using the following ***equivalent frequency domain operation***.

$$\frac{\mathbf{d(x(t))}}{\mathbf{dt}} \Leftrightarrow \mathbf{(j2\pi f_i)}\left(\mathbf{X_i(2\pi f_i)}\right), \mathbf{i=1,\ldots,N/2}$$

STEP 1 – INTEGRATION OF A PERIODIC SINE WAVE

- If a time domain waveform is ***periodic in its sampling window***, it can be ***accurately integrated or differentiated***

To demonstrate this, a sine wave that is periodic in its sampling window was synthesized using the **File | New | Data Block** command. The following parameters were entered into the dialog box for this command

Time Domain Block Size ➔ 1000 Samples

Fmax ➔ 100 Hertz

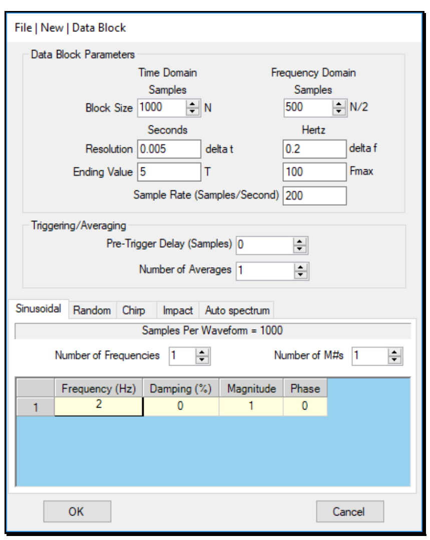
On the **Sinusoidal** tab,

Number of Frequencies ➔1

Number of M#s ➔ 1

Frequency (Hz) ➔ 2.0, Damping (%) ➔0

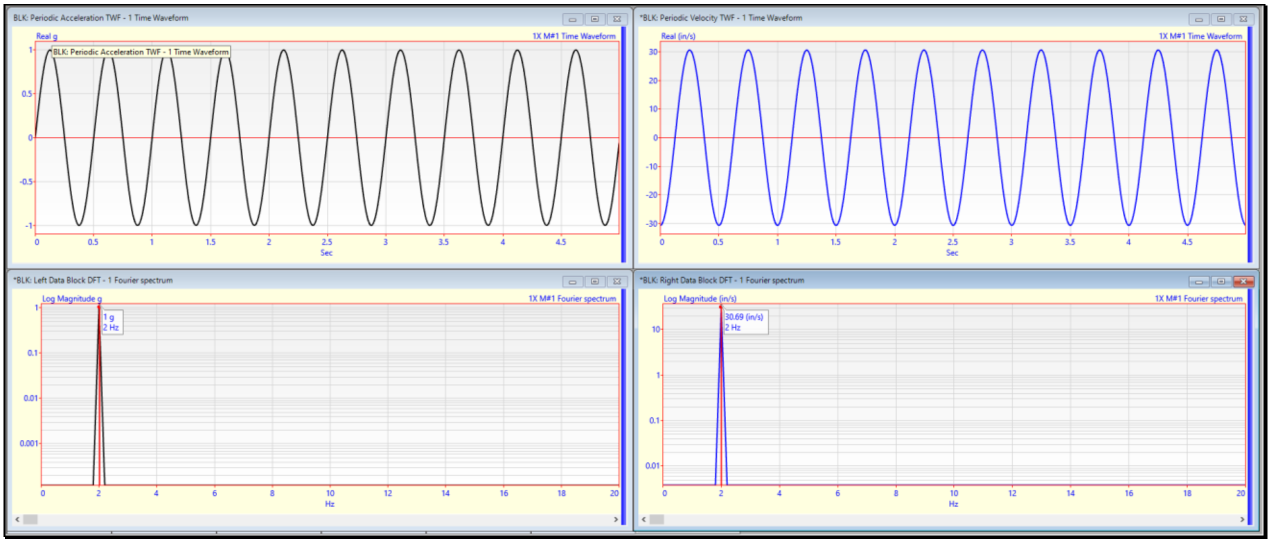
Magnitude ➔1, Phase ➔ 0



File | New | Data Block Dialog Box for Periodic Sine.

- ***Press* Hotkey 1 Integrate Periodic Sine Wave**

Two Data Blocks will be displayed, **BLK: Periodic Acceleration TWF** on the ***upper-left***, its **DFT** on the ***lower-left***, and the integrated acceleration sine wave **BLK: Periodic Velocity TWF** on the ***upper-right***, with its **DFT** on the ***lower-right***.



Periodic Acceleration Sine Wave on the left, Velocity Sine Wave on the Right.

There are ***exactly 10 cycles*** of a sine wave in both the acceleration and velocity **TWFs**.

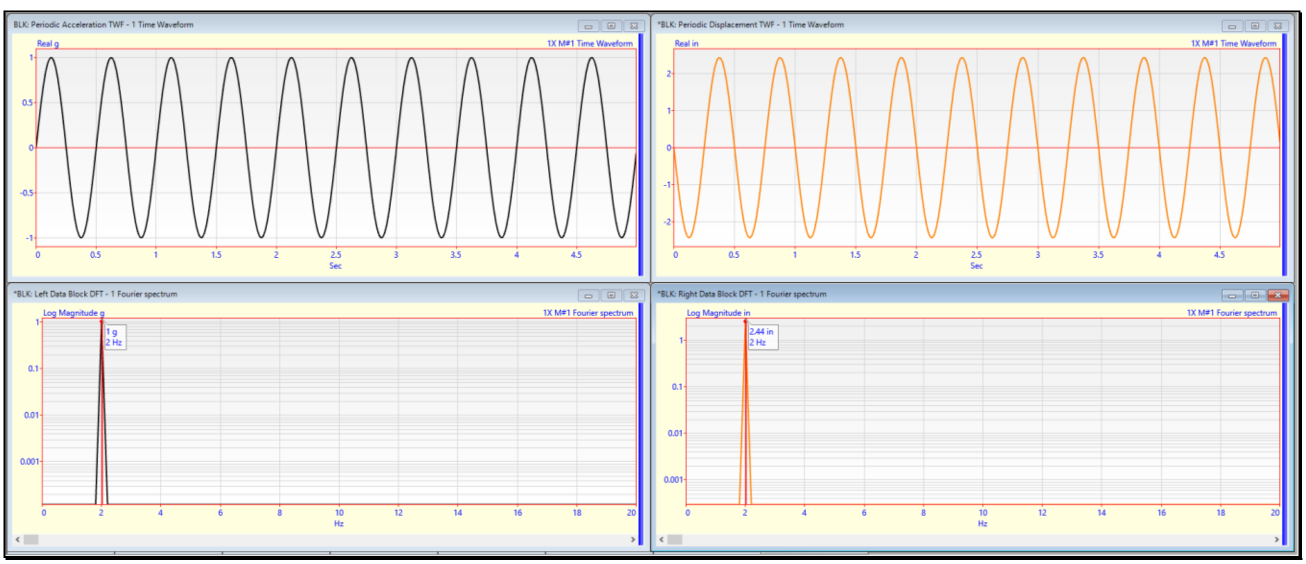
- Since an ***integer number of cycles*** of both TWFs are contained in their sampling windows, both signals are ***periodic in their sampling window***

Since both the acceleration and velocity **TWFs** are periodic in their sampling windows, their respective **DFTs** below them also show a single peak at the 2 Hz frequency of both sine waves.

STEP 2 – DOUBLE INTEGRATION OF A PERIODIC SINE WAVE

- ***Press* Hotkey 2 Double Integrate Periodic Sine Wave**

Two Data Blocks will be displayed, **BLK: Periodic Acceleration TWF** on the ***upper-left***, its **DFT** on the ***lower-left***, and the doubly-integrated acceleration sine wave **BLK: Periodic Displacement TWF** on the ***upper-right***, with its **DFT** on the ***low-er-right***.



Periodic Acceleration Sine Wave on the left, Displacement Sine Wave on the Right.

There are ***exactly 10 cycles*** of a sine wave in both the acceleration and displacement TWFs.

- Since an ***integer number of cycles*** of both **TWFs** are contained in their sampling windows, both signals are ***periodic in their sampling window***

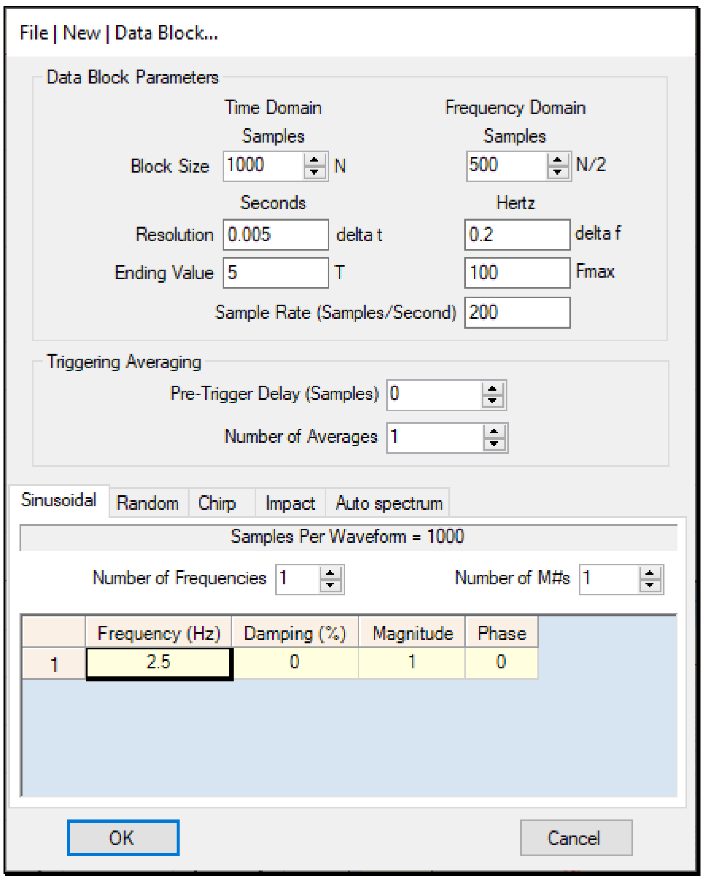
Since both the acceleration and velocity TWFs are periodic in their sampling windows, their respective **DFT**s below them also show a single peak at the 2 Hz frequency of both sine waves.

NON-PERIODIC SINE WAVE

To demonstrate the integration of a non-periodic signal, a sine wave that is non-periodic in its sampling window was synthesized using the **File | New | Data Block** command again.

- To synthesize a sine wave that is non-periodic in its sampled window, its frequency was changed **from 2 Hz to 2.5 Hz**

All the other parameters were the same as those of for the periodic sine wave.

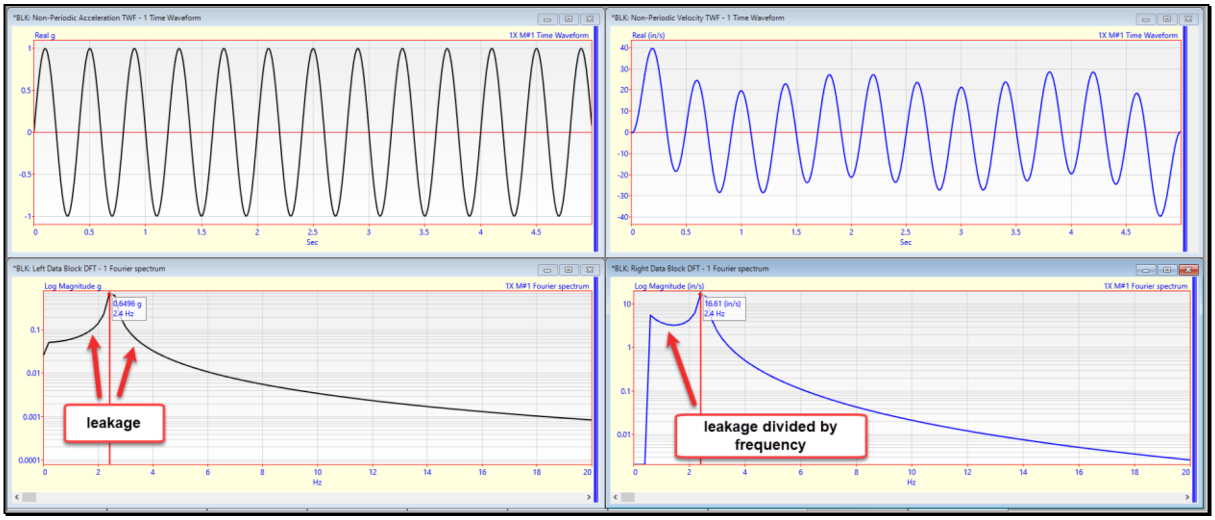


File | New | Data Block Dialog Box for Non-Periodic Sine.

STEP 3 – INTEGRATION OF A NON-PERIODIC SINE WAVE

- ***Press* Hotkey 3 Integrate Non-Periodic Sine Wave**

Two Data Blocks will be displayed, **BLK: Non-Periodic Acceleration TWF** on the ***upper-left***, its **DFT** on the ***lower-left***, and the integrated acceleration sine wave **BLK: Non-Periodic Velocity TWF** on the ***upper-right***, with its **DFT** on the ***lower-right***.



Non-Periodic Acceleration Sine Wave on the left, Velocity Sine Wave on the Right.

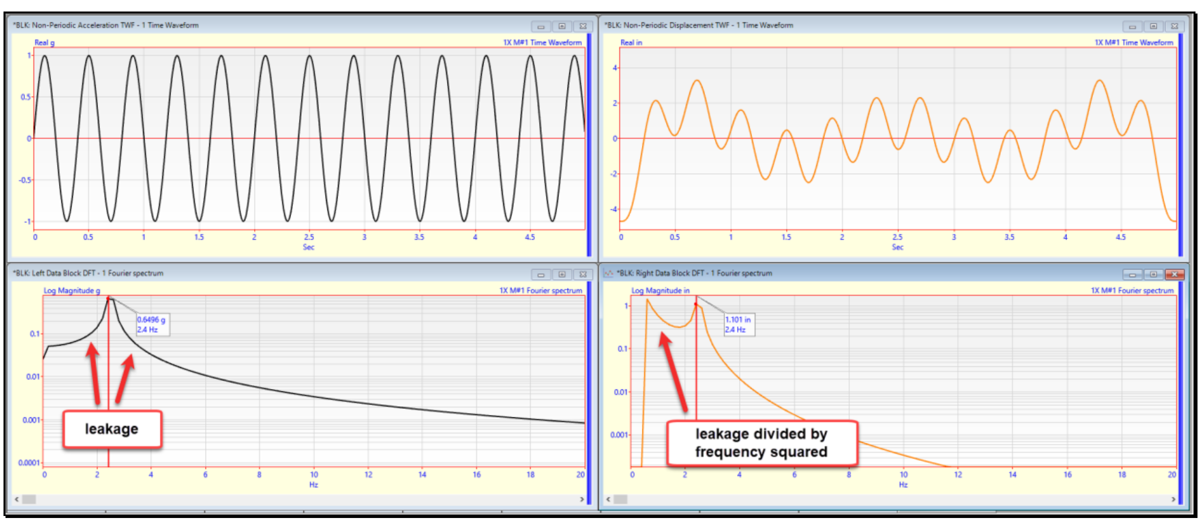
- There are ***not an integer number of cycles*** of the acceleration TWF in its sampling window
- Since the acceleration **TWF** in ***not periodic*** in its sampling window, ***leakage occurred*** around the 2.5 Hz peak in its **DFT**
- The acceleration **DFT** was ***divided by frequency*** to obtain the velocity **DFT**

Then when the velocity **DFT** was Inverse FFT'd, the resulting ***velocity TWF had significant error*** in it.

STEP 4– DOUBLE INTEGRATION OF A NON-PERIODIC SINE WAVE

- ***Press* Hotkey 4 Double Integrate Non-Periodic Sine Wave**

Two Data Blocks will be displayed, **BLK: Non-Periodic Acceleration TWF** on the ***upper-left***, its **DFT** on the ***lower-left***, and the doubly-integrated acceleration sine wave **BLK: Non-Periodic Displacement TWF** on the ***upper-right***, with its **DFT** on the ***lower-right***.



Non-Periodic Acceleration Sine Wave on the left, Displacement Sine Wave on the Right.

The same problem occurs only it is worse with double integration than with single integration

- Since the acceleration **TWF** in ***not periodic*** in its sampling window, ***leakage occurred*** around the 2.5 Hz peak in its **DFT**
- The acceleration **DFT** was ***divided by frequency squared*** to obtain the displacement **DFT**

The leakage in the acceleration **DFT** ***caused significant error*** in the displacement **DFT**. Then when the displacement **DFT** was Inverse FFT'd, the resulting ***displacement TWF had significant error*** in it.

DC REMOVAL

In general, ***integration amplifies the low frequencies*** in a waveform, including DC (zero frequency). Dividing each frequency sample of a waveform's spectrum by frequency is the same as multiplying it by the function (1/frequency).

- **Integration** multiples a spectrum by (**1/frequency**)
- **Double integration** multiples a spectrum by (**1/frequency^2**)

Most real-world signals ***have some amount of DC offset*** in them, even when DC coupling is used to remove DC from a signal during acquisition. When a signal is integrated, even a small amount of DC ***will dominate*** the result, especially when double integration is performed to convert acceleration to displacement units.

- When integration is performed in MEscope, the DC term is deleted from the **DFT** of the waveform before it is divided by frequency
- If the DC term in a **DFT** is small compared with the rest of samples of the **DFT**, removing the DC term will always improve the resulting TWF

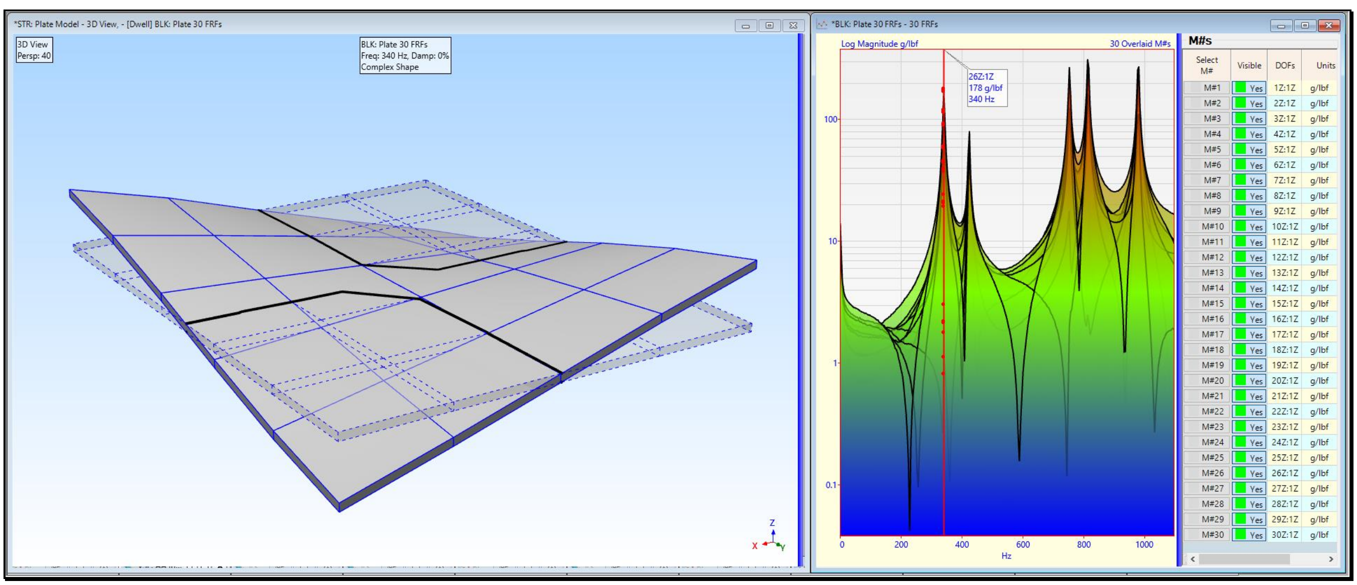
But the previous steps showed that integrating a TWF that is ***non-periodic in its sampling window*** can give ***results with significant errors*** in them.

REMOVAL OF LOW FREQUENCIES BEFORE INTEGRATION

When a time domain signal is ***non-periodic*** in its sampling window or a frequency domain signal has non-zero low frequency components at or near DC those components ***must be removed from its spectrum*** before it can be integrated using frequency domain integration.

STEP 5 – ODS's OF AN ALUMINUM PLATE

- ***Press*** **Hotkey 5 Aluminum Plate ODS's**



Animation of the ODS at a Resonance of an Aluminum Plate.

The ODS at the **Line** cursor position in the FRFs is displayed with sinusoidal animation ***on the left***, and the log magnitudes of 30 overlaid FRFs from which the ODS is displayed are displayed ***on the right***.

- ***Drag*** the **Line** cursor to another resonance peak to display its ODS

All the FRFs have ***non-zero DC*** values plus many other low frequency samples with non-zero values. The low frequency response is the ***rigid body motion*** of the plate on its soft mountings. The plate was impact-tested while resting on a form rubber pad.

- ***Drag*** the **Line** cursor to **DC** to display its rigid-body ODS

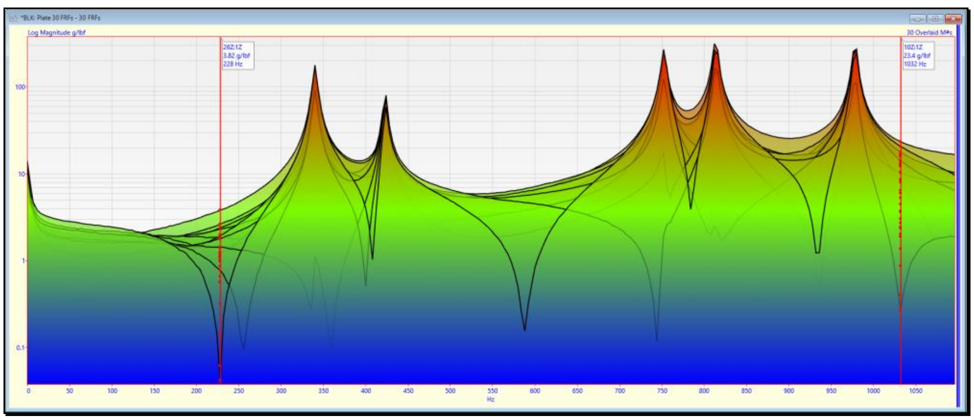
The FRFs have units of **g/lbf** indicating that they were calculated from an acquired accelerometer response divided by the measured impact force that cause the response.

STEP 6 - REMOVING FREQUENCIES WITH A BAND PASS WINDOW

The MEscope **VES-3600 Advanced Signal Processing** option contains several windowing functions that can be applied to either time or frequency domain waveforms. One of the windows is the **Band Pass** window.

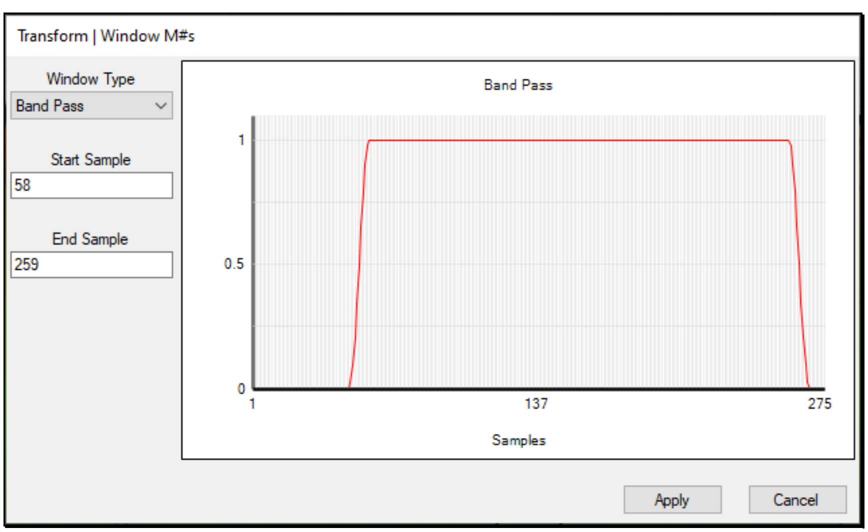
- When the **Band Pass** window is applied to FRFs, all the data ***outside of the cursor band*** is deleted

The figure below shows how the DC offset and other frequencies outside the Band cursor will be deleted when the **Band Pass** window is applied to the FRFs.

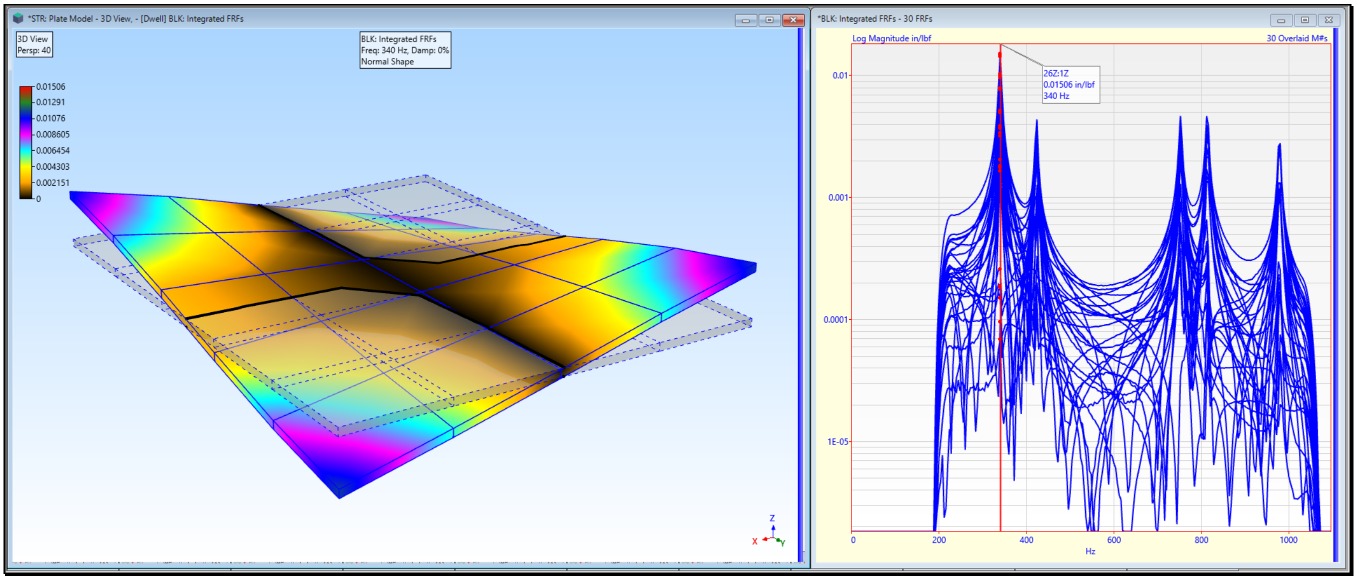


Band Cursor Setup for Band Pass Windowing the FRFs

When the **Transform | Window M#s** command is executed and the **Band Pass** window is selected, the Band Pass window is displayed as shown below



Transform | Window M#s Showing the Band Pass Window



Animation of the ODS at a Resonance in Displacement/Force Units.

- ***Press* Hotkey 6 Double Integrate FRFs**

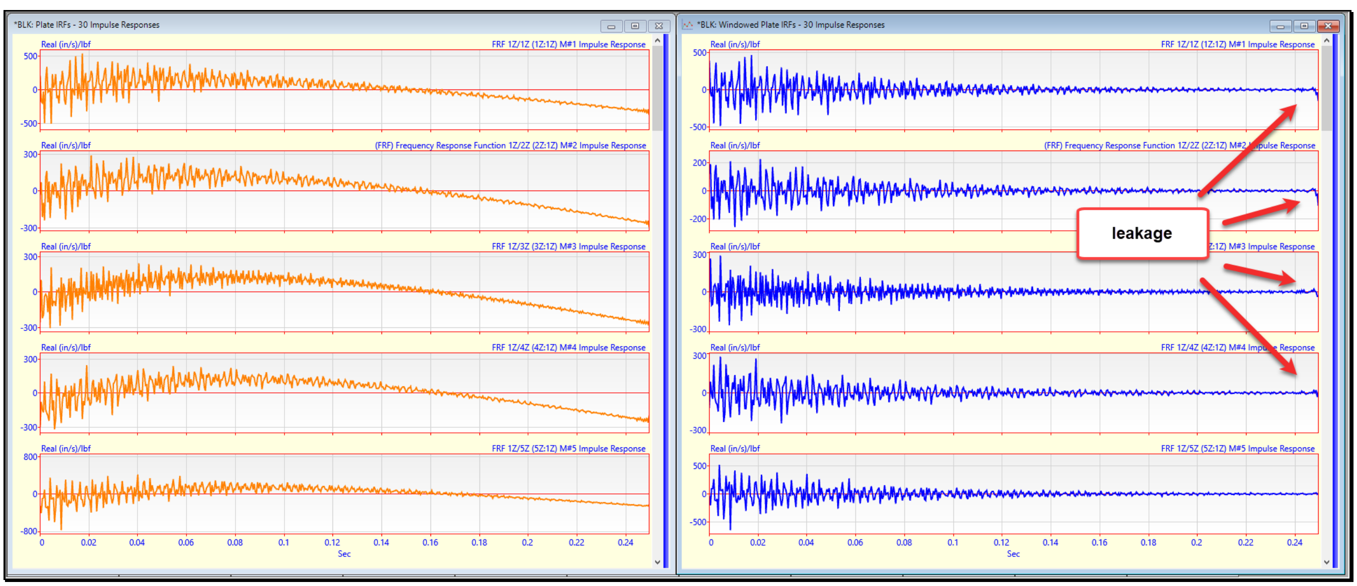
The ODS at the **Line** cursor position in the FRFs is displayed with sinusoidal animation ***on the left***, and the log magnitudes of 30 overlaid FRFs from which the ODS is displayed are displayed ***on the right***.

- ***Drag*** the **Line** cursor to another resonance peak to display its ODS

STEP 7 – COMPARING INTEGRATED IRFs FROM UN-WINDOWED VERSUS WINDOWED FRFs

- ***Press* Hotkey 7 Compare Integrated IRFs**

The IRFs ***on the left*** are the **Inverse FFTs** of the original 30 Plate FRFs that were integrated from **acceleration/force** to **velocity/force**. The IRFs ***on the right*** are the **Inverse FFTs** of the FRFs that were windowed with the same **Band Pass** window used in the previous step, and then integrated from **acceleration/force** to **velocity/force**.



Unwindowed Velocity / Force IRFs on the Left, Windowed Velocity / Force IRFs on the Right.

- The integrated IRFs on the right ***are realistic***

The integrated IRFs ***on the left exhibit the strong influence of the low frequency rigid body motion*** of the aluminum plate which was supported on bubble wrap when it was impact tested to acquire the original FRFs.

WRAP AROUND ERROR

Even though DC and the low frequency ***rigid body motion*** was effectively removed from the FRFs by BAND PASS windowing prior to single or double integration, the resulting IRFs still exhibit a problem.

- The ***error at the end of the IRFs*** is called ***time domain leakage*** or ***wrap around error***

The IRFs from the windowed and integrated FRFs ***on the right above*** exhibit the characteristic ***damped sinusoidal response***, but many of them ***begin to grow in amplitude*** near the end of their sampling window.

- ***Wrap around error*** is not realistic, since real vibration does not damp out and then grow again

Wrap around error is a signal processing error caused by the **FFT**. Just as leakage occurs in its **DFT** when a time domain signal was ***non-periodic (or not completely contained)*** in its sampling window, ***a leakage error also occurs*** when a frequency spectrum is ***not completely contained in its sampling window***.

When an FRF is calculated, it too is multiplied by a rectangular sampling window. If resonances outside of the frequency span of the FRF were excited, the FRF is ***truncated in frequency*** just as if the **true FRF** (defined over a larger frequency span) were windowed with a rectangular window. When the **Inverse FFT** is applied to an FRF that is ***not completely contained in its sampling window***, the result is a ***smeared time domain IRF***, one that has leakage in its beginning and ending samples.

- Some of the IRF ***leaks out of its beginning samples*** and ***wraps around into the ending samples*** of signal

CONCLUSIONS

First, it was shown that a sine wave that was ***periodic in its sampling window*** can be ***integrated once or twice*** using frequency domain integration, and a ***leakage free result*** is obtained.

Next, a sine wave with ***a slightly different frequency*** was integrated and doubly integrated. Because this sine was ***non-periodic in its sampling window,*** noticeable error occurred in the integrated results.

Finally, FRFs that had DC and low frequency ***rigid body dynamics*** in them were doubly integrated. But applying a **Band Pass** window to the FRFs before they were integrated removed the low frequency components as well as high frequency residual components, and the integrated FRFs as well as their IRFs yielded realistic results.

The following conclusions can be drawn from these examples,

- Integration & differentiation can be used on any time domain or frequency domain signal ***if it is periodic*** (or ***completely contained***) within its sampling window
- If a signal is ***non-periodic (or not completely contained)*** within its sampling window, ***leakage effects will occur*** when the signal is integrated, and Fourier transformed into its equivalent waveform in the other domain
- Prior to integrating a ***non-periodic*** signal, to obtain useable results, ***DC and other low frequency components*** as well as the ***higher frequency components***, should be removed by frequency domain **Band Pass** windowing